

OPTICAL SOLITONS WITH HIGHER ORDER DISPERSION BY SEMI-INVERSE VARIATIONAL PRINCIPLE

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Abstract—This paper studies optical solitons, in presence of higher order dispersion terms by the aid of He's semi-inverse variational principle. Both Kerr law and power law are taken into consideration. The numerical simulations are also given to complete the analysis.

1. INTRODUCTION

Optical Solitons is one of the major areas of research in the field of Nonlinear Optics. This area of research has made remarkable progress in the past few decades. One of the most important aspects that is studied in this area is the issue of integrability of the governing equation. There are various methods that are available that are used to carry out the integration of these governing equations. Some of these techniques are G'/G method, tanh-coth method, F -expansion method, exponential function method, Lie symmetry approach and many others [1–40]. These techniques lead to the integration of the governing equation although the Painleve test of integrability will indicate that the equations are not integrable.

The governing equation is the Nonlinear Schrödinger's equation (NLSE), that governs the propagation of solitons through optical fibers, through trans-continental and trans-oceanic distances [6, 8]. The NLSE falls under the category of nonlinear evolution equations in Partial Differential Equations. In this paper, one such method will be used to carry out the integration of the perturbed NLSE. This is called He's variational principle (HVP).

It needs to be noted that this principle was already applied to study optical solitons with Raman scattering, self-steepening, nonlinear dispersion as well as intermodal dispersion terms [17, 35]. Subsequently, this principle was applied to study the same perturbation terms but with full nonlinearity, where the nonlinear terms are generalized to an arbitrary exponent [35]. In this paper, HVP will be used to study optical soliton perturbation with higher order dispersion terms.

2. MATHEMATICAL ANALYSIS

The dimensionless form of the NLSE in a non-Kerr law media is given by [16]

$$iq_t + aq_{xx} + bF(|q|^2)q = 0, \quad (1)$$

where x and t represents the spatial and temporal variables respectively. The first term is the evolution term, The second term is the group velocity dispersion and the third term is the nonlinear term where the function F dictates the type of nonlinearity in question. The dependent variable q represents the wave profile and is a complex valued function. Solitons are the outcome of a delicate balance between dispersion and nonlinearity.

In (1), F is a real-valued algebraic nonlinear function and it is necessary to have the smoothness of the complex function $F(|q|^2)q$:

$C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R^2 , the function $F(|q|^2)q$ is k times continuously differentiable, so that [15–17]

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k((-n, n) \times (-m, m); R^2). \quad (2)$$

2.1. Perturbation Terms

The perturbed NLSE that is going to be studied in this paper is given by

$$iq_t + aq_{xx} + bF(|q|^2)q = -i\gamma q_{xxx} + \sigma q_{xxxx} + \psi q_{xxxxx} \quad (3)$$

Here, in (3), γ , σ and ψ represent the third, fourth and sixth order dispersion terms respectively. It is known that the NLSE, as given by (1), does not give correct prediction for pulse widths smaller than 1 picosecond. For example, in solid state solitary lasers, where pulses as short as 10 femtoseconds are generated, the approximation breaks down. Thus, quasi-monochromaticity is no longer valid and so higher order dispersion terms creep in. If the group velocity dispersion is close to zero, one needs to consider the third and higher order dispersion for performance enhancement along trans-oceanic and trans-continental distances. Also, for short pulse widths where group velocity dispersion changes, within the spectral bandwidth of the signal cannot be neglected, one needs to take into account the presence of higher order dispersion terms. This reasoning leads to the inclusion of the fourth and sixth order dispersion terms in addition to the third order dispersion terms [16, 17]. The effect of third and fourth order dispersion are simultaneously present in many practical cases. Roy et al. [25] pointed out that when the ratio of the third and fourth order dispersion is fixed to a certain value (e.g., $\gamma/\sigma = 10$), both of these perturbation terms contribute. In these situations the higher order dispersion terms have a pronounced effect especially for applications involving ultra-broadband optical source [25].

In this paper, (3) is going to be studied via HVP, for Kerr and power law nonlinearity. It needs to be noted that the NLSE has been studied with third and fourth order dispersions before. In fact, for the case of third order dispersion only, in a Kerr law medium, Equation (3) has been integrated by the aid of Lie symmetries [5, 6]. It must be noted that for the case of fourth order dispersion there has only been numerical studies for this problem [2]. In the case of the sixth order dispersion, it is only the application of soliton perturbation theory that had been achieved in 2008 [16]. This equation with sixth order

dispersion has not been studied from the integration point of view, until now. In fact, Equation (3), with a combination of all higher order dispersion terms both for Kerr and power law nonlinearities, is being integrated, by the aid of HVP, for the first time in this paper. Although Lie symmetry approach can be applied to integrate (3), at least for the Kerr law case, HVP method is immensely simpler as compared to that of Lie symmetry.

3. HE'S VARIATIONAL PRINCIPLE

In this section, HVP will be introduced. Subsequently, it will be applied to carry out the integration of (3) for Kerr and power laws of nonlinearity for F .

The starting point is the solitary wave ansatz that is given by

$$q(x, t) = g(s)e^{i\phi}, \quad (4)$$

where $g(s)$ represents the shape of the pulse and

$$s = x - vt, \quad (5)$$

$$\phi = -\kappa x + \omega t + \theta. \quad (6)$$

Here, v is the velocity of the soliton, κ is the frequency while ω is the soliton wave number and θ is the phase constant. Substituting this ansatz into (3) and decomposing into real and imaginary parts yields the following pair of relations, respectively

$$P_1g - bgF(g^2) - P_2g'' + P_3g^{(iv)} + \psi g^{(vi)} = 0 \quad (7)$$

$$Q_1g - Q_2g'' + Q_3g^{(iv)} = 0 \quad (8)$$

where the notations $g' = dg/ds$, $g'' = d^2g/ds^2$, $g^{(iv)} = d^4g/ds^4$ and $g^{(vi)} = d^6g/ds^6$ are used. Here, in (7) and (8),

$$P_1 = \omega + a\kappa^2 + \gamma\kappa^3 + \sigma\kappa^4 - \psi\kappa^6 \quad (9)$$

$$P_2 = a + 3\gamma\kappa + 6\sigma\kappa^2 - 15\psi\kappa^4 \quad (10)$$

$$P_3 = \sigma - 15\psi\kappa^2 \quad (11)$$

and

$$Q_1 = v + 2a\kappa + 3\gamma\kappa^2 + 4\sigma\kappa^3 - 6\psi\kappa^5 \quad (12)$$

$$Q_2 = \gamma + 4\sigma\kappa - 20\psi\kappa^3 \quad (13)$$

$$Q_3 = -6\psi\kappa \quad (14)$$

Integration of (8) yields

$$g(s) = \exp \left\{ -s \left[\frac{Q_2 - \sqrt{Q_2^2 - 4Q_1Q_3}}{2Q_3} \right]^{\frac{1}{2}} \right\} \quad (15)$$

that leads to the velocity (v) of the soliton.

Now, multiplying both sides of the real part Equation (7), by g' and integrating yields

$$P_1 g^2 - 2bgF(g^2) - P_2 (g')^2 - P_3 (g'')^2 + \psi(g''')^2 = K \quad (16)$$

where K is a constant and $g''' = d^3g/ds^3$. The stationary integral J is then defined as

$$J = \int_{-\infty}^{\infty} K ds = \int_{-\infty}^{\infty} \left[P_1 g^2 - 2bgF(g^2) - P_2 (g')^2 - P_3 (g'')^2 + \psi(g''')^2 \right] ds \quad (17)$$

Finally, the 1-soliton solution ansatz, given by

$$g(s) = Af \left[\frac{1}{\cosh(Bs)} \right], \quad (18)$$

is substituted into (17). Here, in (18), the parameters A and B represent the amplitude and inverse width of the soliton respectively, and the functional f depends on whether the nonlinear function F is Kerr or power. HVP states that the parameters A and B are determined from the solution of the equations [17, 32, 35]

$$\frac{\partial J}{\partial A} = 0 \quad (19)$$

and

$$\frac{\partial J}{\partial B} = 0. \quad (20)$$

The parameters A and B will now be determined for the following cases of nonlinearity in the following subsections. Finally, the velocity v of the soliton can be obtained after substituting (18) in the left hand side of (15) and then solving (15) for v which is located in Q_1 .

Now, to comment that this method is called semi-inverse variational principle it is J. H. He who first coined the terminology, for this method, in his Ph.D. thesis in Mechanical Engineering from Shanghai University in 1997 and this method was later published in 2004 [13]. Since that point onwards, this method is commonly referred to as He's semi-inverse variational principle.

3.1. Kerr Law

The Kerr law of nonlinearity originates from the fact that a light wave in an optical fiber faces nonlinear responses from non-harmonic motion of electrons bound in molecules, caused by an external electric field. Even though the nonlinear responses are extremely weak, their

effects appear in various ways over long distance of propagation that is measured in terms of light wavelength. The origin of nonlinear response is related to the non-harmonic motion of bound electrons under the influence of an applied field. As a result the induced polarization is not linear in the electric field, but involves higher order terms in electric field amplitude [16, 17, 35, 36].

In the case of Kerr law nonlinearity where $F(u) = u$, the perturbed NLSE is given by

$$iq_t + aq_{xx} + b|q|^2q = -i\gamma q_{xxx} + \sigma q_{xxxx} + \psi q_{xxxxx} \tag{21}$$

and therefore (7) reduces to

$$P_1g - bg^3 - P_2g'' + P_3g^{(iv)} + \psi g^{(vi)} = 0 \tag{22}$$

Thus, the stationary integral, from (17), is given by

$$J = \int_{-\infty}^{\infty} \left[P_1g^2 - \frac{b}{2}g^4 - P_2 \left(\frac{dg}{ds} \right)^2 - P_3 \left(\frac{d^2g}{ds^2} \right)^2 + \psi \left(\frac{d^3g}{ds^3} \right)^2 \right] ds \tag{23}$$

For Kerr law nonlinearity, the appropriate form of the soliton is given by

$$g(s) = \frac{A}{\cosh(Bs)} \tag{24}$$

and so J , from (23), simplifies to

$$J = 2P_1 \frac{A^2}{B} - \frac{2b}{3} \frac{A^4}{B} - \frac{2P_2}{3} A^2 B - \frac{14P_3}{15} A^2 B^3 + \frac{62\psi}{21} A^2 B^5 \tag{25}$$

The relations (19) and (20) gives the the relation between the soliton amplitude (A) and the inverse with (B) as

$$A = \left[\frac{155\psi B^6 - 49P_3 B^4 - 35P_2 B^2 + 105P_1}{70b} \right]^{\frac{1}{2}} \tag{26}$$

where the inverse width B is obtained from the algebraic equation

$$1705\psi B^6 - 441P_3 B^4 - 105P_2 B^2 - 105P_1 = 0 \tag{27}$$

whose solution is given by Cardano's method as

$$B = \left[\left(-\frac{3176523P_3^2}{495647765\psi^3} + \frac{3087P_2P_3}{1162810\psi^2} + \frac{21P_1}{682\psi} + \sqrt{D_1} \right)^{\frac{1}{3}} + \left(-\frac{3176523P_3^2}{495647765\psi^3} + \frac{3087P_2P_3}{1162810\psi^2} + \frac{21P_1}{682\psi} - \sqrt{D_1} \right)^{\frac{1}{3}} + \frac{147P_3}{1705\psi} \right]^{\frac{1}{2}} \tag{28}$$

with the discriminant D_1 given by

$$D_1 = - \left(\frac{21609P_3^2}{2907025\psi^2} + \frac{7P_2}{341} \right)^3 + \left(\frac{3176523P_3^2}{4956477625\psi^3} - \frac{3087P_2P_3}{1162810\psi^2} - \frac{21P_1}{682\psi} \right)^2 \quad (29)$$

Finally, the soliton amplitude A can be computed from (26).

The following figures show the numerical simulation of the optical soliton with Kerr law nonlinearity where the perturbation parameters are chosen as $\gamma = 0.014$, $\sigma = 0.012$ and $\psi = 0.001$. Figure 1(a) shows the comparison between the solutions between the variationally obtained solution and asymptotically obtained solution.

3.2. Power Law

This law of nonlinearity arises in nonlinear plasmas that solves the problem of small K -condensation in weak turbulence theory. It also arises in the context of nonlinear optics. Physically, various materials, including semiconductors, exhibit power law nonlinearities. Moreover highly nonlinear materials, such as semiconductor-doped glasses, have non-Kerr nonlinearity. Therefore, it is worthwhile investigating some cases where the increase in index is proportional to the field raised to the power different from two. When dealing with very broad optical

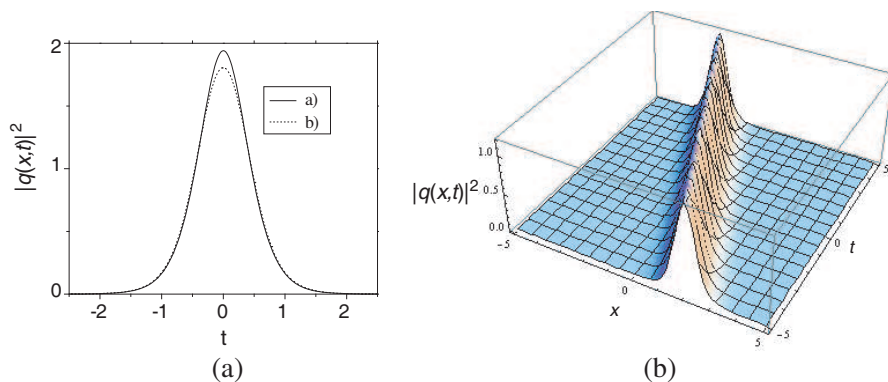


Figure 1. (a) Optical soliton with Kerr law nonlinearity a) numerical simulation of Equation (18), b) from He's variational approach, where the perturbation parameters are $t = 5$, $\gamma = 0.014$, $\sigma = 0.012$ and $\psi = 0.001$. (b) Numerical simulation of the optical soliton with Kerr law nonlinearity and parameters: $a = 0.2$, $b = 1$.

spectra (ultrashort optical pulses), dispersion up to fourth or even fifth and sixth order must be taken into account [2, 16, 17, 35, 36].

For the case of power law nonlinearity, where $F(u) = u^n$, the perturbed NLSE is given by

$$iq_t + aq_{xx} + b|q|^{2n}q = -i\gamma q_{xxx} + \sigma q_{xxxx} + \psi q_{xxxxx} \tag{30}$$

In (30), the parameter n dictates the power law parameter. The special case with $n = 1$ reduces to Kerr law nonlinearity. For power law nonlinearity, it is necessary to have $0 < n < 2$ to prevent wave collapse [1, 4, 5] and, in particular, $n \neq 2$ to avoid self-focusing singularity [4]. Thus, (7) reduces to

$$P_1g - bg^{2n+1} - P_2g'' + P_3g^{(iv)} + \psi g^{(vi)} = 0 \tag{31}$$

In this case, therefore, the stationary integral (17) is given by

$$J = \int_{-\infty}^{\infty} \left[P_1g^2 - \frac{b}{n+1}g^{2n+2} - P_2\left(\frac{dg}{ds}\right)^2 - P_3\left(\frac{d^2g}{ds^2}\right)^2 + \psi\left(\frac{d^3g}{ds^3}\right)^2 \right] ds \tag{32}$$

For power law nonlinearity, the hypothesis

$$g(s) = \frac{A}{\cosh^{\frac{1}{n}}(Bs)} \tag{33}$$

simplifies J to

$$J = \left[P_1\frac{A^2}{B} - \frac{2b}{(n+1)(n+2)}\frac{A^{2n+2}}{B} - P_2\frac{A^2B}{n(n+2)} + P_3G_1(n)A^2B^3 + \psi G_2(n)A^2B^5 \right] \frac{\Gamma(\frac{1}{n})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{n} + \frac{1}{2})} \tag{34}$$

where

$$G_1(n) = \frac{4n+3}{n^2(n+2)(3n+2)} \tag{35}$$

and

$$G_2(n) = \frac{1}{n^6(n+2)(3n+2)(5n+2)} \left\{ n(3n+2)(5n+2) + 4(n+1)^3(2n+1)^2(5n+2) - 8(n+1)^3(2n+1)^3 - 4(n+1)(2n+1)(3n+2)(5n+2) + 8(n+1)^2(2n+1)(5n+2) \right\} \tag{36}$$

The relations (19) and (20) when applied to (34) yields the amplitude of the soliton as

$$A = \left[\frac{n(n+2)P_1 - P_2B^2 + n(n+2)G_1(n)P_3B^4 + n(n+2)G_2(n)\psi B^6}{2nb} \right]^{\frac{1}{2n}} \tag{37}$$

where the inverse width, that is obtained from the algebraic equation $n(5n + 6)G_2(n)\psi B^6 + n(3n + 4)G_1(n)P_3B^4 + P_2B^2 - n^2P_1 = 0$ (38) whose solution is given by

$$\begin{aligned}
 B = & \left[\left\{ -\frac{1}{27} \left(\frac{(3n + 4)G_1(n)P_3}{(5n + 6)G_2(n)\psi} \right)^3 + \frac{(3n + 4)G_1(n)P_2P_3}{6n(5n + 6)^2G_2^2(n)\psi^2} \right. \right. \\
 & \left. \left. + \frac{nP_1}{2(5n + 6)G_2(n)\psi} + \sqrt{D_2} \right\}^{\frac{1}{3}} \right. \\
 & \left. + \left\{ -\frac{1}{27} \left(\frac{(3n + 4)G_1(n)P_3}{(5n + 6)G_2(n)\psi} \right)^3 + \frac{(3n + 4)G_1(n)P_2P_3}{6n(5n + 6)^2G_2^2(n)\psi^2} \right. \right. \\
 & \left. \left. + \frac{nP_1}{2(5n + 6)G_2(n)\psi} - \sqrt{D_2} \right\}^{\frac{1}{3}} - \frac{(3n + 4)G_1(n)P_3}{3(5n + 6)G_2(n)\psi} \right]^{\frac{1}{2}} \quad (39)
 \end{aligned}$$

and the discriminant D_2 is given by

$$\begin{aligned}
 D_2 = & - \left[\frac{1}{9} \left\{ \frac{(3n + 4)G_1(n)P_3}{(5n + 6)G_2(n)\psi} \right\}^2 - \frac{P_2}{3n(5n + 6)G_2(n)\psi} \right]^3 \\
 & + \left[\frac{1}{27} \left\{ \frac{(3n + 4)G_1(n)P_3}{(5n + 6)G_2(n)\psi} \right\}^3 - \frac{(3n + 4)G_1(n)P_2P_3}{6n(5n + 6)^2G_2^2(n)\psi^2} - \frac{nP_1}{2(5n + 6)G_2(n)\psi} \right]^2 \quad (40)
 \end{aligned}$$

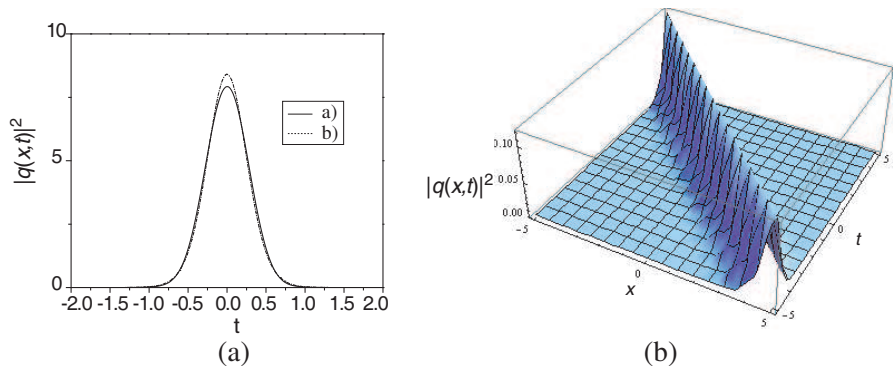


Figure 2. (a) Optical soliton with power law nonlinearity $n = 1/2$, a) numerical simulation of Eq. (18), b) from He’s variational approach, where the perturbation parameters are $t = 5$, $\gamma = 0.014$, $\sigma = 0.012$ and $\psi = 0.001$. (b) Numerical simulation of the optical soliton with power law nonlinearity where $n = 1/2$, $a = 0.5$, $b = 1$.

Subsequently, the soliton amplitude A is obtained from (37).

The following figure shows the numerical simulation of the optical soliton with power law nonlinearity where $n = 1/2$ and the perturbation parameters are chosen as $\gamma = 0.014$, $\sigma = 0.012$ and $\psi = 0.001$. Figure 2(a), shows the comparison between the solutions between the variationally obtained solution and asymptotically obtained solution.

4. CONCLUSION

In this paper, the HVP is used to carry out the integration of the NLSE with higher order dispersion terms. Both, Kerr law and power law nonlinearities are considered. After obtaining the 1-soliton solutions with these two kind of nonlinearities, the parameter domains are also identified for the soliton solution to exist. These results are from purely analytical standpoint and thus a closed form soliton solution has been obtained. The numerical simulations are also obtained.

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