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A HYBRID MM/COMPACT 2-D FDFD METHOD FOR RECTANGULAR RIDGED WAVEGUIDE DISCONTINUITIES

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Abstract—A hybrid mode-matching/compact 2-D finite-difference frequency-domain (MM/compact 2-D FDFD) method is proposed for the analysis of rectangular ridged waveguide discontinuities. In order to apply MM technique, mode spectrum of the ridged waveguide is determined by an improved compact 2-D FDFD method with only two transverse field components at the cutoff frequencies which lead to two independent sets of real symmetric eigenvalue problems for TE and TM modes. Solving these two separate eigenvalue equations, cutoff wave numbers and discrete mode field functions can be obtained respectively from eigenvalues and eigenvectors. Finally, the generalized scattering matrix (GSM) of the rectangular-ridged waveguide step discontinuity can be easily calculated through the transverse field matching procedure. The method is demonstrated at the examples of two waveguide structures, and results are shown to be in excellent agreement with those by the commercial CAD software HFSS.

1. INTRODUCTION

Ridged waveguides which have advantages of broad bandwidth, low cutoff frequency, and low wave impedance are widely used in many microwave and millimeter-wave application [1–5]. For the accurate analysis of ridged waveguide components, mode sequences and modal field distributions for MM technique must be known accurately [6–8]. Therefore, adequate hybrid methods are desirable, which combine the advantages of the flexibility of the space discretization methods with

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the efficiency of the MM method [9, 10]. A hybrid MM/finite element (FE) technique has been proposed for the waveguide discontinuity with complex cross sections, where modal expansions can be derived by a 2-D FE method [11–14]. There is no doubt that the 2-D FE method is effective and flexible. However, the algorithm is a little complicated, especially in the process of solving the coefficients [15]. Another novel compact 2-D FDFD has been proposed to analyze the dispersion characteristics of both lossless and lossy waveguide structures [15–18]. Apart from its simple formulation, the advantage of compact 2-D FDFD method is that we can obtain all the modes and modal field distribution of a uniform guided wave structure by solving an eigenvalue problem. However, no MM techniques combined with compact 2-D FDFD are published for the analysis of waveguide discontinuities. In practice, the justification for separating TE and TM modes for MM technique is required due to the existence of degenerate modes in application of the compact 2-D FDFD with either four or six field components involved.

In this paper, a new hybrid MM/compact 2D-FDFD method is presented for the analysis of rectangular ridged waveguide discontinuities. This technique combines the computational efficiency of modal analysis with the versatility and flexibility of the improved compact 2-D FDFD approach. The real symmetric eigenvalue problem of ridged waveguides has been constructed by the compact 2-D FDFD with only two field components at the cutoff frequencies. Once the cutoff wave numbers and discrete mode field functions are obtained, the general scattering matrix of the step discontinuity can be easily calculated through the transverse field-matching procedure. Finally, the GSM of the whole waveguide structure can be found by cascading each junction modules.

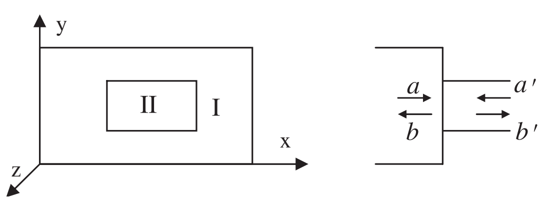

Figure 1. Waveguide step discontinuity with incident and scattered waves.

2. THEORY

According to the principle of MM technique [10, 11], the transverse electric $\vec{E}_t$ and magnetic $\vec{H}_t$ in waveguide section are represented by mode field functions in the following way:

$$\begin{aligned}\vec{E}_t &= \sum_{m=1}^{\infty} \vec{e}_m^h \sqrt{Z_m^h}\left(a_m^h + b_m^h\right) + \sum_{m=1}^{\infty} \vec{e}_m^e \sqrt{Z_m^e}\left(a_m^e + b_m^e\right) \\ \vec{H}_t &= \sum_{m=1}^{\infty} \vec{h}_m^h \sqrt{Y_m^h}\left(a_m^h - b_m^h\right) + \sum_{m=1}^{\infty} \vec{h}_m^e \sqrt{Y_m^e}\left(a_m^e - b_m^e\right)\end{aligned} \tag{1}$$

where $Z=1/Y$ are the characteristic impedances equivalent to modal wave impedances in this paper. $\vec{e}$ and $\vec{h}$ are the mode field functions

$$\vec{e} = \begin{cases} \vec{z} \times \nabla_t \varphi & TE \\ -\nabla_t \varphi & TM \end{cases} \qquad \vec{h} = \vec{z} \times \vec{e} \tag{2}$$

with $\vec{z}$ being the unit vector in the z-direction, and the potential φ are solution of the 2-D Helmholz equation. Choosing the modal wave impedances as the characteristic impedances, we have the modal orthogonality and normalization condition

$$\begin{aligned}\iint_S \vec{e}_m \cdot \vec{e}_m &= 1 \\ \iint_S \vec{e}_m \cdot \vec{e}_n &= 0\end{aligned} \tag{3}$$

Matching the tangential E_t and H_t along the transverse surface of the general step discontinuity (Fig. 1), which is assumed to be located at $z = 0$, yields the relation between the incident and scattered modal wave amplitude coefficients as follows:

$$\begin{aligned}\{[a] + [b]\} &= [W]\{[a'] + [b']\} \\ -[W]^T\{[a] - [b]\} &= \{[a'] - [b']\}\end{aligned} \tag{4}$$

where the matrix $[W]$ is expressed as

$$[W] = \mathrm{diag}\left[\sqrt{Y}\right][M]\mathrm{diag}\left[\sqrt{Z'}\right] \tag{5}$$

$M[i,j]$ is the element of the frequency-independent coupling matrix

$$M[i,j] = \iint_{\mathrm{II}} \vec{e}_i \cdot \vec{e}_j' ds' \tag{6}$$

From Equations (4)–(6), the GSM of the complete step discontinuity can be deduced by simple matrix algebra

$$\begin{aligned}
[S_{11}] &= -\left([I]+[W]\cdot[W]^T\right)^{-1}\left([I]-[W]\cdot[W]^T\right)\\
[S_{21}] &= [W]^T\cdot([I]-[S_{11}])\\
[S_{22}] &= \left([I]+[W]^T\cdot[W]\right)^{-1}\cdot\left([I]-[W]^T\cdot[W]\right)\\
[S_{12}] &= [W]\cdot([I]+[S_{22}])
\end{aligned} \tag{7}$$

In order to calculate the GSM of the waveguide step discontinuity with the above procedure, cutoff wave numbers and mode field functions of the general waveguide must be calculated firstly. In this paper, an improved compact 2-D FDFD method with only two field components at cutoff frequencies is presented for the modal analysis of the ridged waveguide. Substituting the propagation constant $\gamma = 0$ into formulas [8–11] in [16], it is noticed that the field components involved in the eigenvalue problem can be divided into two independent groups: E_x, E_y for TE modes and H_x, H_y for TM modes:

TE modes:

$$\begin{aligned}
k_c^2 E_x(i,j) &= \frac{1}{d_x d_y}[E_y(i,j-1)-E_y(i+1,j-1)-E_y(i,j)+E_y(i+1,j)]\\
&\quad -\frac{1}{d_y^2}E_x(i,j-1)+\frac{2}{d_y^2}E_x(i,j)-\frac{1}{d_y^2}E_x(i,j+1)
\end{aligned} \tag{8}$$

$$\begin{aligned}
k_c^2 E_y(i,j) &= \frac{1}{d_x d_y}[E_x(i-1,j)-E_x(i,j)-E_x(i-1,j+1)+E_x(i,j+1)]\\
&\quad -\frac{1}{d_x^2}E_y(i-1,j)+\frac{2}{d_x^2}E_y(i,j)-\frac{1}{d_x^2}E_y(i+1,j)
\end{aligned} \tag{9}$$

TM modes:

$$\begin{aligned}
k_c^2 H_x(i,j) &= \frac{1}{d_x d_y}[H_y(i-1,j)-H_y(i,j)-H_y(i-1,j+1)+H_y(i,j+1)]\\
&\quad -\frac{1}{d_y^2}H_x(i,j-1)+\frac{2}{d_y^2}H_x(i,j)-\frac{1}{d_y^2}H_x(i,j+1)
\end{aligned} \tag{10}$$

$$\begin{aligned}
k_c^2 H_y(i,j) &= \frac{1}{d_x d_y}[H_x(i,j-1)-H_x(i+1,j-1)-H_x(i,j)+H_x(i+1,j)]\\
&\quad -\frac{1}{d_x^2}H_y(i-1,j)+\frac{2}{d_x^2}H_y(i,j)-\frac{1}{d_x^2}H_y(i+1,j)
\end{aligned} \tag{11}$$

where k_c is the cutoff wave number. Let d_x and d_y denote mesh sizes in the x and y directions. After implementing all the boundary conditions, both the eigenvalue Equations (8)–(9) for TE modes and the eigenvalue Equations (10)–(11) for TM modes can be finally concluded in the same form as

$$[A]\{X\} = k_c^2\{X\} \tag{12}$$

where $\{X\}=\{E_x, E_y\}^T$ and $\{X\}=\{H_x, H_y\}^T$ for TE and TM modes, respectively. It is obvious that the coefficient matrix $[A]$ is real, symmetric and has positive diagonal elements. The modal wave impedance Z can be determined by the solution frequency and eigenvalue k_c. Meanwhile, the discrete mode field function can be obtained from the normalization of the eigenvetor. Applying the modal normalization condition, the discrete mode field functions with uniform grid division satisfy the following equation.

$$\sum_{j=1}^{JJ-1}\sum_{i=1}^{II-1}\left[e_x(i,j)^2+e_y(i,j)^2\right]=\sum_{j=1}^{JJ-1}\sum_{i=1}^{II-1}\left\{\left[\frac{E_x(i,j)+E_x(i,j+1)}{2}\right]^2+\left[\frac{E_y(i,j)+E_y(i+1,j)}{2}\right]^2\right\}=1 \tag{13}$$

$$\sum_{j=1}^{JJ-1}\sum_{i=1}^{II-1}\left[h_x(i,j)^2+h_y(i,j)^2\right]=\sum_{j=1}^{JJ-1}\sum_{i=1}^{II-1}\left\{\left[\frac{H_x(i,j)+H_x(i+1,j)}{2}\right]^2+\left[\frac{H_y(i,j)+H_y(i,j+1)}{2}\right]^2\right\}=1 \tag{14}$$

Based on the above eigenmodal analysis of the ridged waveguide and the analytical solution to the rectangular waveguide eigenvalue problem, the GSM of the rectangular-ridged step discontinuity can be finally calculated from Equations (5)–(7).

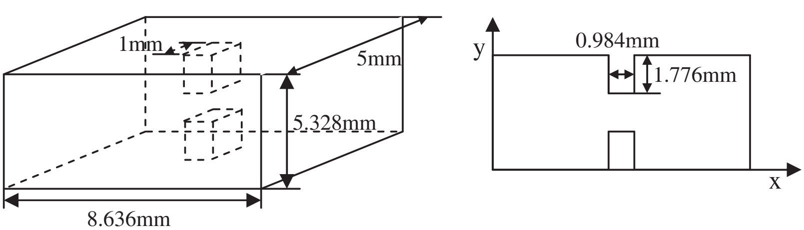

Figure 2. Double-ridged iris in rectangular waveguide.

3. NUMERICAL RESULTS

To verify the correction of this method, a double-ridged iris and a single-ridged waveguide filter have been analyzed. In the following analysis, we chose the total number of the ridged waveguide modes of 25 and the maximum mode index in x and y directions in the rectangular waveguide of 5 and 3. The example of the double-ridged iris in the rectangular waveguide is shown in Fig. 2. By the improved compact 2-D FDFD presented in this paper, the field distribution of the domain mode of the double-ridged waveguide can be observed in Fig. 3. A good agreement of the cutoff wave numbers of the double-ridged waveguide between the method presented in this paper and the commercial CAD software HFSS can be observed in Fig. 4, where the cutoff wave numbers of TE and TM modes respectively marked with circle points and square points by HFSS are compared with those marked with star points by the proposed method. In Figs. 5–6 the scattering parameters of the waveguide iris are compared with those by HFSS, and the good results prove the correctness of the method.

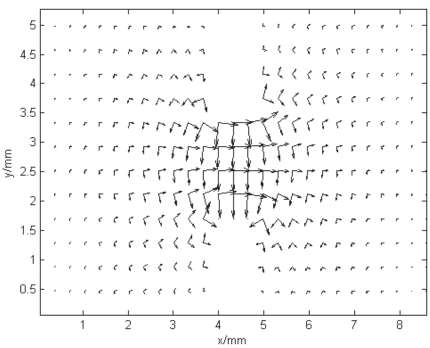

Figure 3. Field distribution of the domain mode.

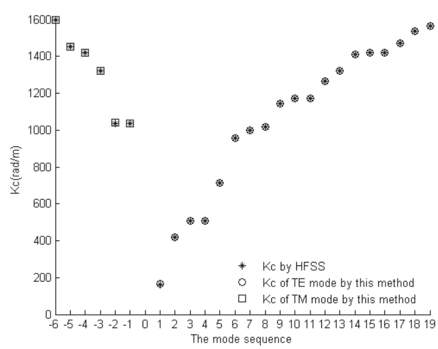

Figure 4. Cutoff wave numbers compared with HFSS.

The design example of a single-ridged waveguide filter is shown in Fig. 7. It is shown that the field distributions of the domain mode in the single-ridged waveguide coincide with the actual situation in Fig. 8. The cutoff wave numbers of the single-ridged waveguide agree well with those by the commercial CAD software HFSS in Fig. 9, where the cutoff wave numbers of TE and TM modes respectively marked with circle and square points by the HFSS are compared with those marked with star points by the proposed method. Once the GSM of the rectangular-ridged waveguide step discontinuity is calculated, the GSM of the whole filter can be found by cascading each straight

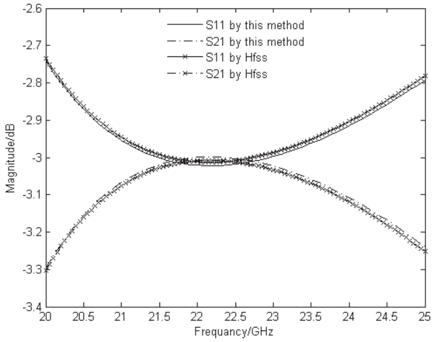

Figure 5. Magnitude compared with CAD software HFSS.

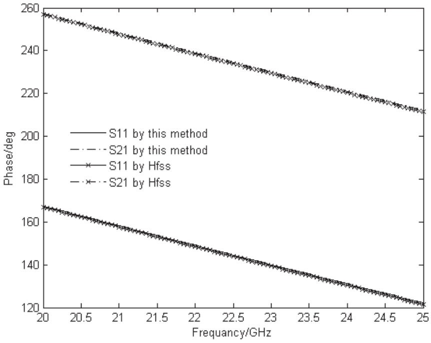

Figure 6. Phase compared with CAD software HFSS.

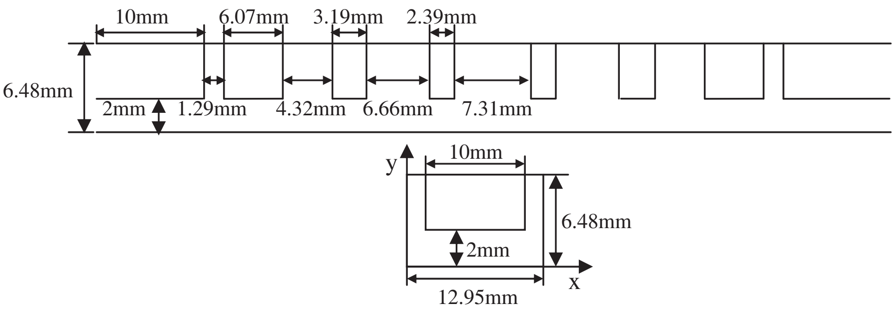

Figure 7. Rectangular single-ridged waveguide filter.

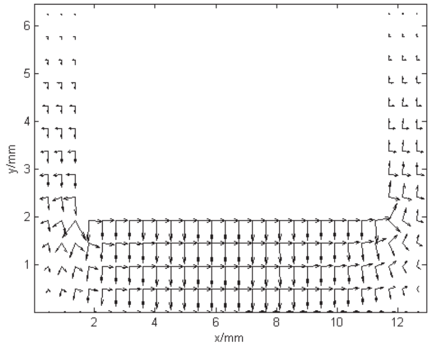

Figure 8. Field distribution of the domain mode.

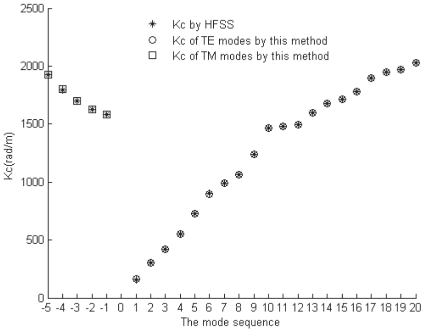

Figure 9. Cutoff wave numbers compared with HFSS.

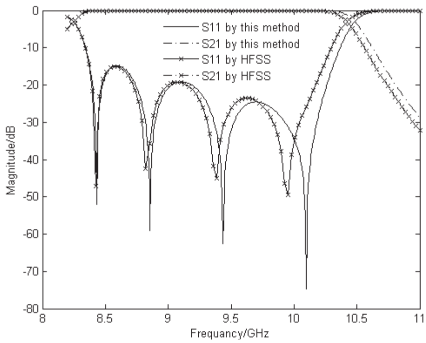

Figure 10. Magnitude compared with CAD software HFSS.

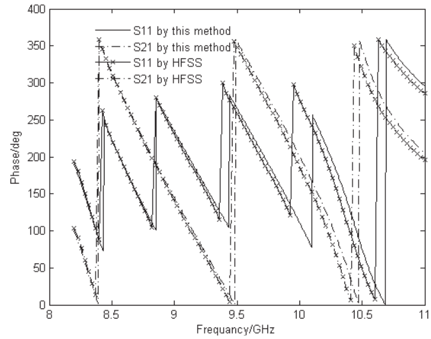

Figure 11. Phase compared with CAD software HFSS.

waveguide and step discontinuity. The scattering parameters of the filter are compared with those by HFSS, and the good results prove the correctness of the method in Figs. 10–11.

4. CONCLUSION

A new hybrid MM/compact 2D-FDFD method is presented for the analysis of ridged waveguide discontinuities. An improved compact 2-D FDFD method at cutoff frequencies is applied to the modal analysis of the ridged waveguide. Once the cutoff wave numbers and discrete mode field functions are obtained from the eigenvalue equations, the general scattering matrix of the step discontinuity can be easily calculated through the transverse field-matching procedure. Accurate results of scattering parameters have been observed.

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