Progress In Electromagnetics Research Letters, Vol. 13, 121–129, 2010

FAST AND ACCURATE RADAR CROSS SECTION COMPUTATION USING CHEBYSHEV APPROXIMATION IN BOTH BROAD FREQUENCY BAND AND ANGULAR DOMAINS SIMULTANEOUSLY

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Abstract—To predict the three-dimensional radar cross section (RCS) pattern of an arbitrary shaped perfectly electric conductor objects in both a broad frequency band and angular domains simultaneously, the method of moments (MoM) combined with the Chebyshev polynomial approximation is presented. The induced current is expanded by a bivariate Chebyshev series. Using this function, the induced current can be obtained at any frequency and angle within the desired frequency band and angular domains. Numerical results show that the proposed method is found to be superior in terms of the CPU time to obtain the three-dimensional RCS pattern compared with the direct solution by MoM repeating the calculations at each frequency and angle. Good agreement between the presented method and the direct MoM is observed.

1. INTRODUCTION

The method of moments (MoM) has already been successfully used in various electromagnetic scattering problems [1–6]. In the MoM, the electric field integral equation (EFIE) is reduced to a matrix equation solved for the induced currents The radar cross section (RCS) is computed from the knowledge of the induced currents The generation of the matrix equation and its solution are the two major computationally intensive operations in MoM, especially for frequency-sweeping cases. For the monostatic RCS computation using the MoM, the MoM is still time-consuming since each frequency and different incident angle require a repeated solution of the induced

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currents If the RCS is highly frequency dependent, one needs to do the calculation at a finer increment of frequency to get an accurate result. This can be computationally intensive. For years there is a strong desire to find approximate solution techniques that can efficiently simulate a frequency response or predict monostatic RCS in the angular domains. The asymptotic waveform evaluation (AWE) technique is prevalent method, which provides a reduced-order model and has already been successfully applied to frequency extrapolation or angle extrapolation [7–11].

In the AWE technique, the induced current is expanded in the Taylor series around a frequency or an angle, and the Padé approximation is used to improve the accuracy. As compared with using MoM at each frequency, the AWE method is found to be superior in terms of the CPU time to obtain a frequency response. The accuracy of the Taylor series is limited by the radius of convergence, and the high-order derivatives of the dense impedance matrix must be stored to compute the coefficients, which will greatly increase the memory needed. The incorporation of the procedures to obtain frequency derivative information into the existing full-wave electromagnetic codes is not an easy task, and usually requires a large amount of additional programming work. The Chebyshev approximation theory is widely applied to antenna designing and electromagnetic scattering problems [12–14]. It is well known that the Chebyshev polynomial is as good as the best polynomial approximations. Compared with the AWE technique, the major advantage of the Chebyshev polynomial approximation is that it can avoid computing derivatives of the dense impedance matrix and excitation matrix, and it is accurate in much broader frequency band. It can be easily implemented into an existing MoM computer code.

The RCS is a function of both frequency and incident angle. The RCS contains both frequency and angle information simultaneously. In many practical applications, it is desirable to predict the three-dimensional RCS pattern of an arbitrary shaped object in both a frequency band and angular domains simultaneously. In [15], a simultaneous extrapolation technique based on AWE technique is proposed to predict RCS in both angular and frequency domains. However, this method trades reduced CPU time for increased memory. The expected effect frequency band is limited by the inherent property of the Taylor series, and the memory needed is greatly increased on account of the high-order derivatives of the dense impedance matrix and excitation matrix with respect to k and θ simultaneously.

In this paper, we expand the Chebyshev polynomial approximation into multidimensional extrapolation technique and present a new implementation of the Chebyshev polynomial approximation combined with MoM for the three-dimensional RCS computation in both a broad frequency band and angular domains simultaneously.

2. FORMULATION

Consider an arbitrarily shaped perfectly conducting body. For RCS calculations, a plane wave is assumed to be incident at an angle (θ, φ). At the surface of the PEC body, the total tangential electric field is zero. The total tangential field in terms of the scattered and incident fields on the PEC body is, therefore, written as

$$\vec{E}_{scat} + \vec{E}_{inc} = 0 \tag{1}$$

The electric field integral equation (EFIE) can be discretized into the following linear equations by using the method of moments. The induced current density can then be expanded using the Rao-Wilton-Glisson (RWG) basis functions [16]. Applying Galerkin's method to EFIE results in

$$[Z_{mn}(k)] \cdot [I_n(k, \theta, \varphi)] = [V_m(k, \theta, \varphi)] \tag{2}$$

where

$$\begin{aligned} Z_{mn}(k) &= \frac{j\eta_0}{4\pi}\left[\iint f_m \cdot \iint f_n k \frac{e^{-jkR}}{R} ds' ds \right. \\ &\quad \left. - \iint (\nabla \cdot f_m) \iint (\nabla' \cdot f_n) \frac{e^{-jkR}}{kR} ds' ds \right] \qquad (3) \\ V_m(k, \theta, \varphi) &= \iint f_m \cdot \vec{E}^{inc}(k, \theta, \varphi) ds \qquad (4) \end{aligned}$$

I_n refer to the unknown coefficients of the current density expansion, $[Z_{mn}]$ is the impedance matrix, $[V_m]$ is the excitation vector. k is the free-space wave number. The direct solution by MoM is still time-consuming since each frequency and different incident angle require a repeated solution of the induced current. Here the presented multidimensional extrapolation technique is applied to the three-dimensional RCS computation to eliminate repetition computations. For a desired frequency band and angular domains, frequency points and incident angle points corresponding to the Chebyshev nodes are found by transformation of coordinates, the induced current is expanded by the bivariate Chebyshev series. Using this polynomial function, the induced current can be obtained at any frequency and angle within the whole frequency band and angular domains. The monostatic RCS simultaneous versus frequency and angle use only a few frequency points and incident angle points.

For a desired frequency band $k \in [k_a, k_b]$ and angular domains $\theta \in [\theta_a, \theta_b]$, the coordinate transform is used as

$$\begin{cases} k = \frac{1}{2}\left[\tilde{k}\left(k_b - k_a\right) + \left(k_b + k_a\right)\right] \\ \theta = \frac{1}{2}\left[\tilde{\theta}\left(\theta_b - \theta_a\right) + \left(\theta_b + \theta_a\right)\right] \end{cases} \tag{5}$$

such that $\tilde{k} \in [-1, 1]$ and $\tilde{\theta} \in [-1, 1]$. The Chebyshev approximation for $I_n(k, \theta)$ (with φ constant and $n = 1, 2, \ldots, N$) is given by

$$\begin{aligned} I_n(k, \theta) &= I_n\left(\frac{\left[\tilde{k}\left(k_b - k_a\right) + \left(k_b + k_a\right)\right]}{2}, \frac{\left[\tilde{\theta}\left(\theta_b - \theta_a\right) + \left(\theta_b + \theta_a\right)\right]}{2}\right) \\ &\approx \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} c_n^{i,j} T_i\left(\tilde{k}\right) T_j\left(\tilde{\theta}\right) \end{aligned} \tag{6}$$

where N_1 and N_2 are the number of Chebyshev nodes, $c_n^{i,j}$ denote the unknown coefficients. The fundamental recurrence relation of the Chebyshev polynomial $T_n(x)$ is

$$T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x) \tag{7}$$

which together with the initial conditions $T_0(x) = 1$, $T_1(x) = x$. The unknown coefficients $c_n^{i,j}$ in (6) are given by

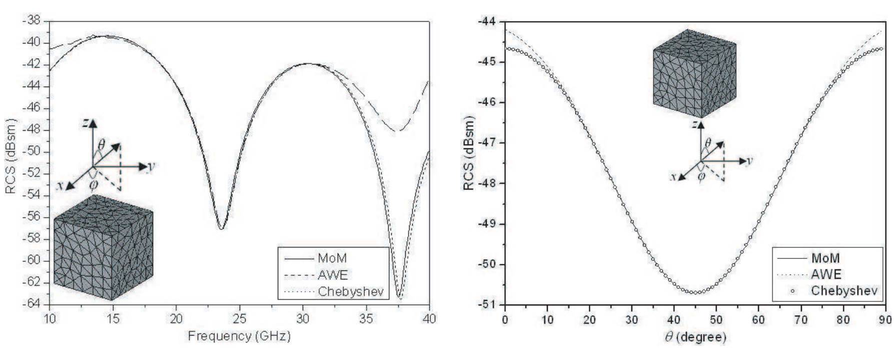
$$c_n^{i,j} = \frac{d_i d_j}{(N_1 + 1)(N_2 + 1)} \sum_{p=0}^{N_1} \sum_{q=0}^{N_2} I_n(k_p, \theta_q) T_i\left(\tilde{k}_p\right) T_j\left(\tilde{\theta}_q\right) \tag{8}$$

with $d_0 = 1$ and $d_i = d_j = 2$ for $i = 1, \ldots, N_1$, $j = 1, \ldots, N_2$. $\tilde{k}_p$ and $\tilde{\theta}_q$ are the Chebyshev zeros for $T_{N_1+1}\left(\tilde{k}\right)$ and $T_{N_2+1}\left(\tilde{\theta}\right)$, respectively. Using (8), the induced currents distribution can be obtained at any frequency and angle within the whole frequency band and angular domains.

3. NUMERICAL RESULTS

To validate the analysis presented in the previous section, two numerical examples are considered. The numerical data obtained using the presented method is compared with the results calculated at each frequency and different incident angle using MoM. The numerical results are also compared with the data obtained using AWE technique with the Padé approximation.

The monostatic RCS frequency and angular response of a PEC cube ($0.5\,\text{cm} \times 0.5\,\text{cm} \times 0.5\,\text{cm}$) is calculated by simultaneous extrapolation technique based on AWE technique in [15] with an φ polarized plane wave incident at $\varphi = 0°$. The cube is discretized with 892 triangular elements resulting into 1338 unknown current coefficients. The AWE frequency and angular response with 0.1 GHz and 1° increments is calculated using the Padé approximation with the expansion point at $f_0 = 25\,\text{GHz}$ and $\theta_0 = 45°$. In the AWE, the expansion point is chosen to be the center point of the frequency band and incident angular band of interest. This choice of expansion point gives the maximum bandwidth as AWE is equally valid on both sides of the expansion point. The monostatic RCS frequency and angular response is plotted in Fig. 1 along with the calculations using 8th order bivariate Chebyshev series expansion. It can be observed that the Chebyshev approximation produces more accurate results than the AWE technique with the Padé approximation in the whole frequency band and angular domains. This comparison clearly shows how monostatic RCS can be obtained by the presented method in much broader frequency band and angular domains simultaneously. Fig. 2 shows the three-dimensional RCS pattern of the cube simultaneous versus frequency and angle obtained by the presented method. The proposed method took 58 min CPU time to obtain the monostatic RCS of the cube in both the frequency band and angular domains simultaneously whereas the direct MoM took 931 min. This comparison clearly shows how the monostatic RCS simultaneous versus frequency and angle can be obtained much faster by means of the proposed method.



(a) RCS versus frequency (b) RCS versus angle

Figure 1. Monostatic RCS of the cube at $\varphi = 0°$ for MoM, AWE, Chebyshev approximation.

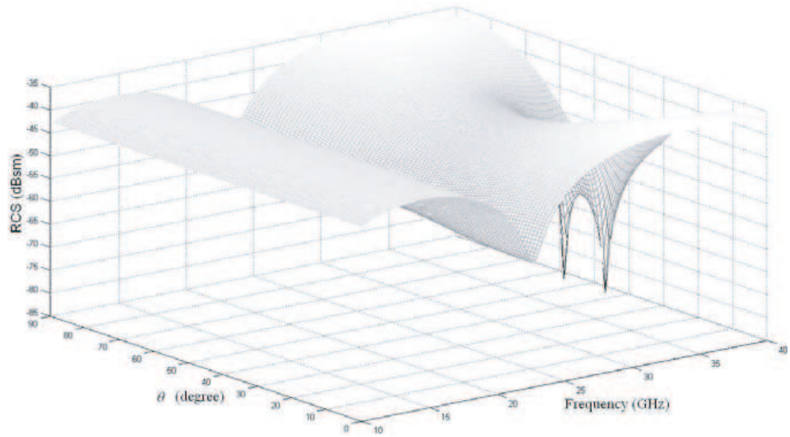
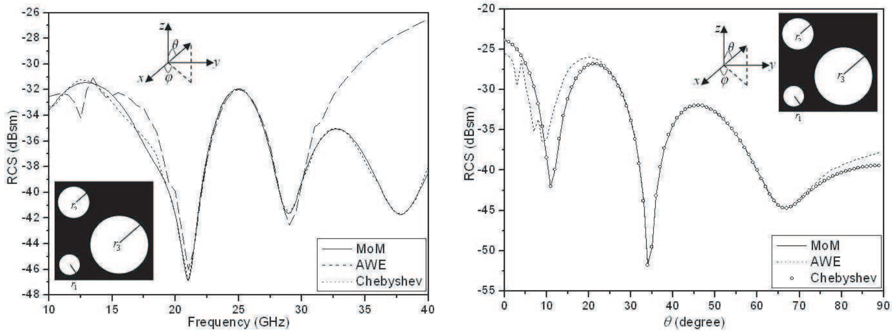


Figure 2. Monostatic RCS pattern of the cube simultaneous versus frequency and angle.

To better demonstrate the utility of the presented method, a metallic plate (2 cm × 2 cm) with three circular holes ($r_1 = 0.15$ cm, $r_2 = 0.25$ cm, $r_3 = 0.7$ cm) is considered. The holes were introduced to increase the complexity of the monostatic RCS pattern. The target is discretized with 692 triangular elements which result into 951 basis functions. Fig. 3 shows the monostatic RCS frequency and angular response of a metallic plate with three circular holes at $\varphi = 0°$ obtained by MoM, AWE technique, Chebyshev approximation. For the AWE, Padé approximation is employed within the whole frequency band and angular domains. One points ($f_0 = 25$ GHz, $\theta_0 = 45°$) is then used as expansion point. The AWE obtains accurate results only in narrow frequency band and angular domains, whereas Chebyshev approximation ($N_1 = 8$, $N_2 = 8$) produces an accurate solution in the whole frequency band and angular domains. This comparison clearly shows how monostatic RCS can be obtained by the presented method in much broader frequency band and angular domains simultaneously. It can be observed from Fig. 1 and Fig. 3 that the results obtained by the Chebyshev approximation are in good agreement with the direct solution by MoM; hence the curves representing the Chebyshev approximation and MoM are almost indistinguishable. Fig. 4 shows the three-dimensional RCS pattern of the metallic plate with three circular holes simultaneous versus frequency and angle. The CPU time required for MoM and Chebyshev approximation are 620 min and 48 min, respectively. The presented technique is superior in terms of the CPU time to predict three-dimensional monostatic RCS pattern.



(a) RCS versus frequency (b) RCS versus angle

Figure 3. Monostatic RCS of the metallic plate with three holes at $\varphi = 0°$ for MoM, AWE, Chebyshev approximation.

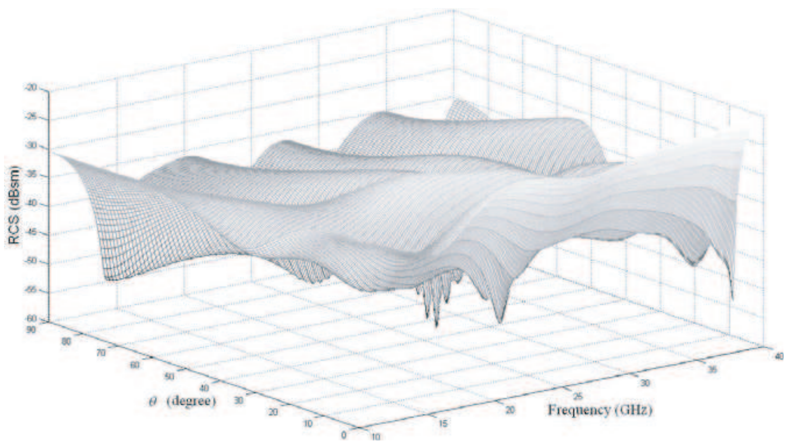


Figure 4. Monostatic RCS pattern of the metallic plate with three holes simultaneous versus frequency and angle.

4. CONCLUSIONS

The monostatic RCS pattern in both a broad frequency band and spatial domains simultaneously calculation by the presented Chebyshev approximation combined with MoM is presented. From the numerical examples presented in this paper, the presented method is found to be superior in terms of the CPU time to obtain monostatic RCS compared with the direct solution by MoM. Compared with the AWE technique, the presented technique can obtain accurate results over the entire frequency band and angular domains. The accuracy and extrapolation range of the Chebyshev approximation depend on several factors such as pattern shape and the order of the Chebyshev approximation. For pattern shape containing lobes and nulls, higher order bivariate Chebyshev series expansion are necessary.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China under Grant 60801042.

REFERENCES

1. Harrington, R. F., *Field Computation by Moment Methods*, IEEE Press, New York, 1993.
2. Zou, Y., Q. Liu, and J. Guo, "Fast analysis of body-of-revolution radomes with method of moments," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 13, 1803–1817, 2007.
3. Hatamzadeh-Varmazyar, S., "A moment method simulation of electromagnetic scattering from conducting bodies," *Progress In Electromagnetics Research*, PIER 81, 99–119, 2008.
4. Essid, C., M. B. B. Salah, K. Kochlef, A. Samet, and A. B. Kouki, "Spatial-spectral formulation of method of moment for rigorous analysis of microstrip structures," *Progress In Electromagnetics Research Letters*, Vol. 6, 17–26, 2009.
5. Danesfahani, R., S. Hatamzadeh-Varmazyar, E. Babolian, and Z. Masouri, "Applying shannon wavelet basis functions to the method of moments for evaluating the radar cross section of the conducting and resistive surfaces," *Progress In Electromagnetics Research B*, Vol. 8, 257–292, 2008.
6. Bogdanov, F. G., R. G. Jobava, A. L. Gheonjian, E. A. Yavolovskaya, N. G. Bondarenko, T. N. Injgia, and S. Frei, "Development and application of an enhanced MoM scheme with integrated generalized N-port networks," *Progress In Electromagnetics Research M*, Vol. 7, 135–148, 2009.
7. Cockrell, C. R. and F. B. Beck, "Asymptotic waveform evaluation (AWE) technique for frequency domain electromagnetic analysis," *NASA Tech. Memo 110292*, Nov. 1996.
8. Reddy, C. J. and M. D. Deshpande, "Application of AWE for RCS frequency response calculations using method of moments," *NASA Contractor Rep. 4758*, Oct. 1996.
9. Reddy, C. J., M. D. Deshpande, C. R. Cockrell, and F. B. Beck, "Fast RCS computation over a frequency band using method of moments in conjunction with asymptotic waveform evaluation

technique," *IEEE Transactions on Antennas and Propagation*, Vol. 46, No. 8, 1229–1233, 1998.

10. Erdemli, Y. E., J. Gong, C. J. Reddy, and J. L. Volakis, "Fast RCS pattern fill using AWE technique" *IEEE Transactions on Antennas and Propagation*, Vol. 46, No. 11, 1752–1753, 1998.
11. Erdemli, Y. E., J. Gong, C. J. Reddy, and J. L. Volakis, "AWE technique in frequency domain electromagnetics" *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 13, 359–378, 1999.
12. Mason, J. C. and D. C. Handscomb, *Chebyshev Polynomials*, CRC Press LLC, New York, 2000
13. Murty, V. N., "Best approximation with Chebyshev polynomials," *SIAM J. Numer. Anal.*, No. 8, 717–721, 1971.
14. Raedt, H. D., K. Michielsen, J. S. Kole, and M. T. Figge, "Solving the Maxwell equations by the Chebyshev method: A one-step finite-difierence time-domain algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 51, No. 12, 3155–3160, 2003.
15. Tong, C. M., W. Hong, and N. C. Yuan, "Simultaneous extrapolation of RCS in both angular and frequency domains based on AWE technique," *Microwave optical Technology Letters*, Vol. 32, No. 4, 290–293, 2002.
16. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, Vol. 30, No. 5, 409–418, May 1982.