APPLICATION OF DOUBLE ZERO METAMATERIALS AS RADAR ABSORBING MATERIALS FOR THE REDUCTION OF RADAR CROSS SECTION

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Abstract—We introduce and investigate the applications of double zero (DZR) metamaterials (having the real parts of permittivity and permeability equal to zero) as radar absorbing materials (RAMs). We consider a perfectly electric conductor (PEC) plate covered by several layers of DZR metamaterial coatings under an oblique plane wave incidence of arbitrary polarization. Several analytical formulas are derived for the realization of zero reflection from such structures. The angle of reflection in the DZR metamaterials becomes complex, which leads to the dissociation of the constant amplitude and equiphase planes. Then several examples of the applications of DZR metamaterials (in nondispersive and dispersive conditions) as RAMs and zero reflection coatings are provided. The characteristics and parameters of the DZR metamaterial media are determined in each case. The method of least squares is used to optimize the DZR coatings for the minimization of reflected power, which uses the combination of genetic algorithm and conjugate gradient method (GA-CG) to benefit from their advantages and avert their short comings.

1. INTRODUCTION

We intend to introduce a class of materials called double zero (DZR) metamaterials [1–9], of which the permittivity and permeability are purely imaginary [namely $\mathrm{Re}(\varepsilon_r) = 0$ & $\mathrm{Re}(\mu_r) = 0$]. Similarly, we may consider epsilon zero (EZR) metamaterials, of which the permittivity is purely imaginary [namely $\mathrm{Re}(\varepsilon_r) = 0$ & $\mathrm{Re}(\mu_r) \neq 0$] and mu zero (MZR) metamaterials, of which the permeability is purely

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imaginary [namely $\mathrm{Re}(\varepsilon_r) \neq 0$ & $\mathrm{Re}(\mu_r) = 0$]. In an earlier paper, we investigated the propagation of radio waves in some DZR metamaterial structures [10, 11].

However, in this paper we intend to investigate the properties of wave propagation incident onto a perfectly electric conductor, coated by DZR metamaterials. Some uncommon phenomena will appear for DZR metamaterials, which have not been observed for common materials and metamaterials. Exact formulas will be derived for zero reflection from a DZR coated PEC plate for different wave polarizations at a particular angle of incidence [12–14]. The DZR metamaterials are first assumed dispersionless and then the computation is extended to dispersive media. We use the full-wave matrix method for the analysis of this structure [15–18] and present several examples of plane wave incidence on it [19–21]. Subsequently, we study the applicability of DZR metamaterials for the fabrication of radar absorbing materials (RAMs) for the reduction of radar cross section (RCS) of various objects [22–27], coating the interior walls and objects inside anechoic chambers, design of antennas with low side lobe levels and protection against electromagnetic interference in high speed circuits. RAMs may be designed for operation at a single frequency or in a narrow frequency band width, which may be straightforward. However, for the realization of wide band RAMs, multilayer structures composed of lossy DPS common materials and (DNG, ENG and MNG) metamaterials have been used [28, 29].

In this paper, we investigate the usage of DZR metamaterials for the fabrication of RAMs. First, we assume them as dispersionless metamaterials to study their general behavior and then we consider their dispersive properties to study their physical realizability for wide band operation [30–33].

2. REALIZATION OF ZERO REFLECTION FROM A PEC PLATE COATED BY DZR

Consider a perfectly electric conductor (PEC) plate coated by a layer of double zero (DZR) metamaterials of thickness d as drawn in Fig. 1. A plane wave of TM polarization is obliquely incident at an angle of incidence θ_0 onto the structure.

The forward and backward traveling waves (with amplitudes A_l and B_l respectively) in layer l are [15, 17, 29]

$$\text{region } (l): \begin{cases} H_{ly} = \left(A_l e^{-jk_l \cos\theta_l z} + B_l e^{+jk_l \cos\theta_l z} \right) e^{-jk_{lx}x} \\ E_{lx} = \eta_l \cos\theta_l \left(A_l e^{-jk_l \cos\theta_l z} - B_l e^{+jk_l \cos\theta_l z} \right) e^{-jk_{lx}x} \end{cases} \quad (1)$$

where in the left half space $l = 0$ and the permittivity and permeability
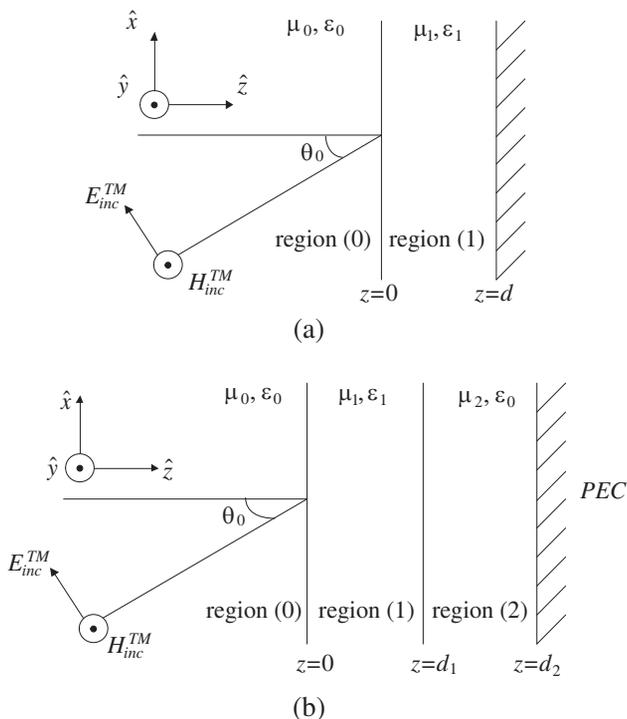
Figure 1. A perfect electric conducting (PEC) plate covered by DZR metamaterial layers.

are

$$\begin{cases} \varepsilon_l = \varepsilon_l' - j\varepsilon_l'' \\ \mu_l = \mu_l' - j\mu_l'' \end{cases} \Rightarrow \begin{cases} k_l = \omega\sqrt{\mu_0\varepsilon_0\mu_l\varepsilon_l} = \pm k' \pm jk'' \\ \eta_l = \sqrt{\frac{\mu_0\mu_l}{\varepsilon_0\varepsilon_l}} = \pm\eta' \pm j\eta'' \end{cases} \tag{2}$$

The incident wave amplitude is assumed equal to unity ($A_0 = 1$), the reflection coefficient is R. The permittivity, permeability, wave number and intrinsic impedance of the layer l are ε_l, μ_l, k_l and η_l, respectively. In DZR metamaterials, we have [10]

$$\begin{cases} \varepsilon = 0 - j\varepsilon'' \\ \mu = 0 - j\mu'' \end{cases} \Rightarrow \begin{cases} k = \omega\sqrt{-\mu''\varepsilon''} = 0 - jk'' \ (k' = 0) \\ \eta = \sqrt{\frac{\mu''}{\varepsilon''}} = +\eta' + j0 \ (\eta'' = 0) \end{cases} \tag{3}$$

where the parameters ε'', μ'', k'' and η' are assumed positive. The boundary conditions as the continuity of tangential electric and magnetic fields (at $z = 0$) and vanishing of the tangential electric field on PEC plane (at $z = d$) lead to the following matrix equation

for wave amplitudes

$$
\begin{bmatrix} +1 & -1 & -1 \\ \frac{\eta_0 \cos\theta_0}{\eta_1 \cos\theta_1} & +1 & -1 \\ 0 & e^{-jk_1\cos\theta_1 d} & -e^{+jk_1\cos\theta_1 d} \end{bmatrix} \begin{bmatrix} R \\ A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{\eta_0 \cos\theta_0}{\eta_1 \cos\theta_1} \\ 0 \end{bmatrix} \quad (4)
$$

The vanishing of the reflection coefficient $R = 0$ leads to the following equation

$$
\eta_1 \cos\theta_1 \left(1 - e^{-j2k_1\cos(\theta_1)d}\right) = \eta_0 \cos\theta_0 \left(1 + e^{-j2k_1\cos(\theta_1)d}\right) \quad (5)
$$

The same procedure may be carried out for the oblique incidence of the TE polarized plane wave

$$
\text{region } (l): \begin{cases} H_{lx} = \left(A_l e^{-jk_l\cos\theta_l z} + B_l e^{+jk_l\cos\theta_l z}\right) e^{-jk_{lx}x} \\ E_{ly} = \eta_l \sec\theta_l \left(-A_l e^{-jk_l\cos\theta_l z} + B_l e^{+jk_l\cos\theta_l z}\right) e^{-jk_{lx}x} \end{cases} \quad (6)
$$

which leads to the following formula for zero reflection

$$
\eta_1 \cos\theta_0 \left(1 - e^{-j2k_1\cos(\theta_1)d}\right) = \eta_0 \cos\theta_1 \left(1 + e^{-j2k_1\cos(\theta_1)d}\right) \quad (7)
$$

For TM polarization, Eq. (5) may be separated into two relations by equating the real and imaginary parts

$$
\begin{cases} \eta' - \eta'r\cos\alpha - \eta''r\sin\alpha \equiv s + sr\cos\alpha \\ \eta'' - \eta''r\cos\alpha + \eta'r\sin\alpha \equiv -sr\sin\alpha \end{cases} \quad (8)
$$

where

$$
r = e^{-2k''\cos(\theta_1)d}, \quad \alpha = 2k'\cos(\theta_1)d,
$$

$$
s = \frac{\eta_0 \cos\theta_0}{\cos\theta_1}, \qquad \eta_1 = \eta' + j\eta'', \quad k_1 = k' + jk'' \quad (9)
$$

These equations may be solved for η'' and η'

$$
\begin{cases} \eta' = \frac{s(1-r^2)}{1+r^2-2r\cos\alpha} \\ \eta'' = \frac{-2\,s\,r\sin\alpha}{1+r^2-2r\cos\alpha} \end{cases} \quad (10)
$$

or for $r\sin\alpha$ and $r\cos\alpha$

$$
\begin{cases} r\cos\alpha = \frac{\eta'^2 - s^2 + \eta''^2}{(\eta'+s)^2 + \eta''^2} \\ r\sin\alpha = \frac{-2\,s\eta''}{(\eta'+s)^2 + \eta''^2} \end{cases} \quad (11)
$$

Now, considering the conditions for DZR metamaterials, namely $k' = 0$ and $\eta'' = 0$ in Eq. (3), we have from Eq. (10) or Eq. (11)

$$
\alpha = 0 \ \& \ r = \frac{\eta' - s}{\eta' + s} \quad (12)
$$

Since $s \neq 0$ and $r \neq 0$ according to Eq. (9) and also $\alpha = 0$ leads to $k' = 0$. Consequently, the angle of zero reflection for a PEC plane coated by a DZR materials for the TM polarization may be obtained by Eqs. (12) and (9) and the Snell's law

$$k_0 \sin \theta_0 = -jk'' \sin \theta_1 \qquad (13)$$

leading to

$$
\begin{aligned}
\cos \theta_0 &= \left(1 + \frac{k''^2}{k_0^2} - \frac{k''^2}{k_0^2} \cos^2 \theta_1 \right)^{1/2} \\
\cos \theta_1 &= \left(1 + \frac{k_0^2}{k''^2} - \frac{k_0^2}{k''^2} \cos^2 \theta_0 \right)^{1/2}
\end{aligned}
\qquad (14)
$$

The angle of zero reflection for the TE polarization may be obtained by Eq. (12), but the parameter s should be changed to

$$s = \frac{\eta_0 \cos \theta_1}{\cos \theta_0} \qquad (15)$$

Now we consider the conditions of zero reflection of a PEC plate coated by several types of metamaterials.

  a) Consider a lossless DPS or DNG coating having $\eta'' = 0$ and $k'' = 0$. However, the condition of zero reflection for such a structure is $k' = 0$ from Eqs. (9) and (11). Consequently $k = k' - jk'' = 0$, and the conditions of wave propagation in the coating and no reflection from it do not exist.

  b) Consider a lossless ENG or MNG coating having $\eta' = 0$ and $k' = 0$. In this case, $r = 1$ according to Eq. (10) and consequently $k'' = 0$ from Eq. (9). Therefore, again the conditions of zero reflection do not appear in these cases.

  c) If the coating is made of lossy metamaterials or common materials, the conditions of zero reflection may be specified by Eq. (10) or (11). However, in lossless cases, zero reflection may not happen. For example, for the case $\angle \mu = \angle \varepsilon$, then $\eta'' = 0$ and $k' = 0$. That is $\angle k = -90°$, leading to $\angle \mu = \angle \varepsilon = -90°$. Consequently, $\mathrm{Re}(\varepsilon_r) = 0$ & $\mathrm{Re}(\mu_r) = 0$, which indicate that the metamaterial is lossy DZR.

3. THE PHYSICAL INTERPRETATION OF COMPLEX ANGLES OF REFRACTION IN DZR METAMATERIALS

Consider a plane wave traveling in a DZR metamaterial medium in the direction $\hat{n}_1 = \hat{u}_x \sin \theta_1 + \hat{u}_z \cos \theta_1$ with $\bar{r} = \hat{u}_x x + \hat{u}_z z$ and

$k = k' - jk'' = 0 - jk''$

$$\exp(\pm jk_1\hat{n}_1 \cdot \bar{r}) = \exp(\pm jk_1x\sin\theta_1 \pm jk_1z\cos\theta_1)$$
$$= \exp(\pm pz \pm j[xk_0\sin\theta_0 + qz]) \qquad (16)$$

Using the Snell's law ($k_0\sin\theta_0 = k_1\sin\theta_1$) and defining the following parameters

$$\cos\theta_1 = \rho\,e^{j\delta}$$
$$p = k_1''\rho\cos\delta \qquad (17)$$
$$q = k_1''\rho\sin\delta$$

Observe that the equiamplitude planes are $z = constant$ and the equiphase planes are $xk_0\sin\theta_0 + qz = constant$, which do not coincide. We may define a real angle of refraction as

$$\Psi = \tan^{-1}\left(\frac{k_0\sin\theta_0}{q}\right) \qquad (18)$$

along which direction the constant phase planes travel, namely

$$\hat{n}_\Psi = \hat{x}\sin\Psi + \hat{z}\cos\Psi \qquad (19)$$

Then

$$\exp(\pm jk_1\hat{n} \cdot \bar{r}) = \exp\left(\pm pz \pm j\sqrt{k_0^2\sin^2\theta_0 + q^2}\,\hat{n}_\Psi \cdot \bar{r}\right) \qquad (20)$$

4. NUMERICAL EXAMPLES

We consider several examples of zero reflection from a PEC plane coated by DZR metamaterial. The computations are based on Eqs. (12) and (14).
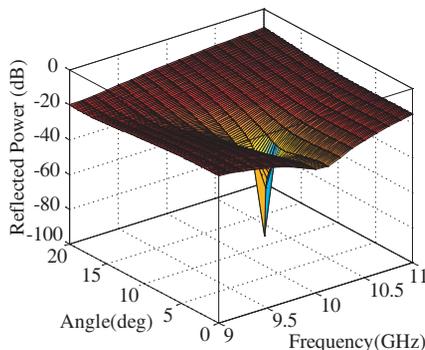
4.1. Example 1 (Nondispersive DZR Coating)

Consider a TE plane wave of frequency 12 GHz incident on a PEC plane coated with a layer of DZR metamaterial with thickness $d = 2$ cm and nondispersive characteristics $\varepsilon = -j0.1$ and $\mu = -j0.3$. The incident angle of no reflection is computed equal to $\theta_0 = 10.45°$, by Eqs. (12) and (14). The reflected power is computed by Eq. (4) for various values of incident angles and drawn in Fig. 2, which shows zero reflection at the same angle $\theta_0 = 10.45°$.

The same calculations are repeated for frequencies $f = 10$ and 20 GHz, which give the angles of zero reflection equal to $\theta_0 = 6.85°$ and $13.33°$, respectively. Again they are verified by the computation of reflected power. However, the frequency response of reflection coefficient is narrow band.
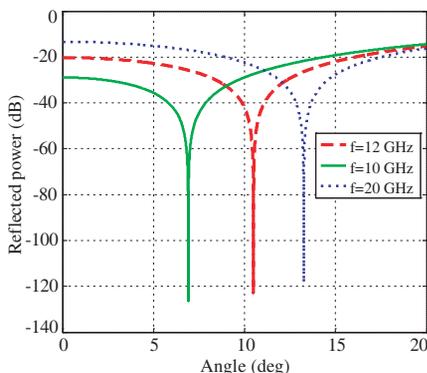
Figure 2. Reflected power (in dB) from the structure in Fig. 1(a) with characteristic given in example 1 versus the angle of incidence at frequencies $f = 12$, 10, 20 GHz for TE polarization.

Figure 3. 3-D diagram of reflected power (in dB) from the structure in Fig. 1(a) with characteristics given in example 2 versus angle of incidence and frequency for TE polarization.

4.2. Example 2 (Dispersive DZR Coating)

Now, we include the Drude and Lorentz dispersion models [10, 29, 34] in the computation

$$\begin{cases} \varepsilon_r = 1 - \dfrac{f_{ep}^2}{f^2 + \gamma_e^2} - j\dfrac{f_{ep}^2 \gamma_e}{f^3 + f\gamma_e^2} & \text{Drude model} \\ \mu_r = 1 - \dfrac{(f_{mp}^2 - f_{mo}^2)(f^2 - f_{mo}^2)}{(f^2 - f_{mo}^2)^2 + f^2\gamma_m^2} - j\dfrac{(f_{mp}^2 - f_{mo}^2)f\gamma_m}{(f^2 - f_{mo}^2)^2 + f^2\gamma_m^2} & \text{Lorentz model} \end{cases} \quad (21)$$

We use the method of least squares to determine the parameters f_{ep}, γ_e, f_{mp}, f_{mo} and γ_m to achieve the characteristics $\varepsilon = -j0.1$ and $\mu = -j0.3$ at the single frequency $f = 10$ GHz. They are

$$f_{ep} = 10.05, \quad \gamma_e = 1, \quad f_{mp} = 10.414,$$
$$f_{mo} = 2.482, \quad \gamma_m = 2.815 \quad \text{(all in GHz)} \quad (22)$$

For an incident TE polarized plane wave, the angle of zero reflection is $\theta_0 = 6.85°$ at $f = 10$ GHz. The 3-D diagram of reflected power versus frequency (9–11 GHz) and angle of incidence (0–20°) is drawn in Fig. 3.

4.3. Example 3 (Nondispersive and Dispersive DZR Coating)

We consider a case where several parameters are assumed and we obtain the characteristics of the DZR coating for zero reflection. For

example, assume $d = 1$ cm, $\theta_0 = 0°$ and $f = 10$ GHz and by Eq. (12) we determine $\varepsilon = -j0.5$ and $\mu = -j0.7$.

The reflected power is drawn in Fig. 4, for these parameters, which is zero at $f = 10$ GHz. Next, we determine the parameters of the Drude and Lorentz dispersion relations to obtain the aforementioned values for ε, μ, f by the method of least squares

$$f_{ep} = 10.917, \quad \gamma_e = 4.379, \quad f_{mp} = 11.547,$$
$$f_{mo} = 5, \qquad \gamma_m = 5 \quad \text{(all in GHz)} \tag{23}$$

The reflected power for this case is computed for 8–12 GHz and drawn in Fig. 4 for comparison with the nondispersive case. The dispersive frequency response of reflected power is narrower than the nondispersive case, as expected.

To gain a better feel for the optimum values of permittivity and permeability (for the parameters in Eq. (23)), the real and imaginary parts of ε, μ are drawn versus frequency in Fig. 5. Observe that this metamaterial may be categorized as DZR at $f = 10$ GHz.

The DZR metamaterials characterized by the above values for ε, μ could be realizable, if the Kramers-Kronig's relations are satisfied:

$$\frac{\partial(f\mu)}{\partial f} > 0, \quad \frac{\partial(f\varepsilon)}{\partial f} > 0 \tag{24}$$

We have used the Drude and Lorentz dispersion relations which satisfy these relation. To check their satisfactions, we draw the
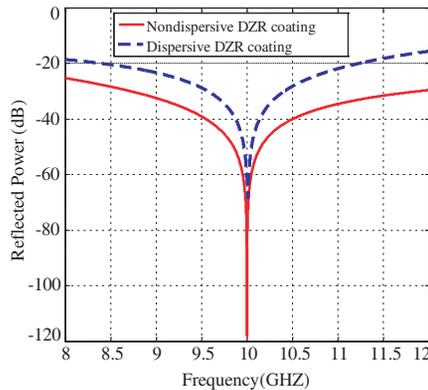
Figure 4. Reflected power (in dB) from the structure in Fig. 1(a) with characteristics given in example 3 versus frequency for normal incidence ($\theta_0 = 0°$) for two cases: (a) Nondispersive DZR coating with characteristics in example (3), (b) dispersive DZR coating with parameters given in Eq. (23).
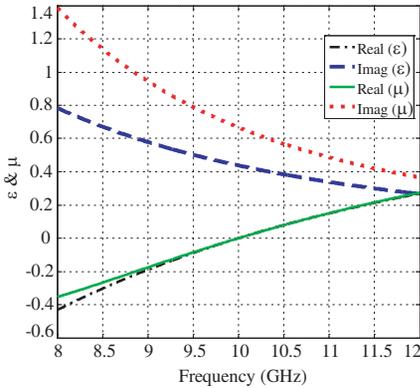
Figure 5. The real and imaginary parts of ε, μ versus frequency.
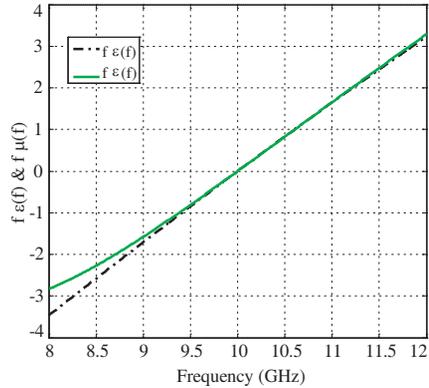
Figure 6. Variation of $f\varepsilon(f)$, & $f\mu(f)$ versus frequency for the Drude and Lorentz dispersion relations in Eq. (21) with parameters in Eq. (23).

functions $f\varepsilon_i(f)$, $f\mu_i(f)$ versus frequency in Fig. 6 and observe that they are increasing functions of f. Consequently, the above DZR metamaterials satisfy the causality and Kramers-Kronig's conditions and are physically realizable.

Different structures may be designed for DZR metamaterials, similar to those proposed in [14, 30, 33].

5. DESIGN AND OPTIMIZATION OF DZR MULTILAYER COATINGS FOR THE REDUCTION OF REFLECTED POWER

We intend to use DZR metamaterials for the design of multilayer structures to minimize the reflected power from a PEC plane in a specified frequency band width and a range of incident angles. Consider a PEC plate covered by two layers of DZR metamaterials under oblique incidence of a TM plane wave as shown in Fig. 1(b). We construct an error function for the minimization of the reflected power

$$error = \sum_{i=0}^{n_f} \sum_{j=0}^{n_\theta} W_{ij} R_{ij} \qquad (25)$$

where R_{ij} is the reflected power at angle θ_i and frequency f_j and W_{ij} is a weighting function. The error is a complicated function of ε_1'', μ_1'', ε_2'', μ_2'', d_1 and d_2. The combination of Genetic Algorithm

(GA) [35] and Conjugate Gradient (CG) method is used for the minimization of the error function to benefit from the global extremum seeking property of GA which is not sensitive to the initial values of variables and the rapid convergence of CG to a local minimum point.

5.1. Optimum Design of a Two Layer DZR RAM Coating in the X Band at a Single Incident Angle

Consider a TM plane wave obliquely incident on the structure in Fig. 1 (b) at an angle of $\theta_0 = 45°$ at a frequency in the bandwidth $\Delta f = [8\text{–}12]$ GHz. The error function in Eq. (25) is minimized by the combination of GA and CG, which gives the following parameters

$$\begin{array}{llll} \text{First layer}: & d_1 = 3.63\,\text{mm}, & \varepsilon_1 = 0 - j2.25, & \mu_1 = 0 - j0.81 \\ \text{Second layer}: & d_2 = 0.44\,\text{mm}, & \varepsilon_2 = 0 - j0.05, & \mu_2 = 0 - j1.29 \end{array} \quad (26)$$

The reflected power as a function of frequency for the nondispersive case is drawn in Fig. 7. Observe that the two layer DZR RAM drastically reduces the reflected power, which has effectively a better performance than a single DZR layer of higher thickness. Then, we use the Drude and Lorentz dispersion relations in Eq. (21) in the design procedure. We use the method of least squares [36] to determine the dispersion parameters f_{ep}, γ_e, f_{mp}, f_{mo}, γ_m to achieve the specifications in Eq. (26). The frequency bandwidth is divided into n intervals and the error function is minimized to determine the
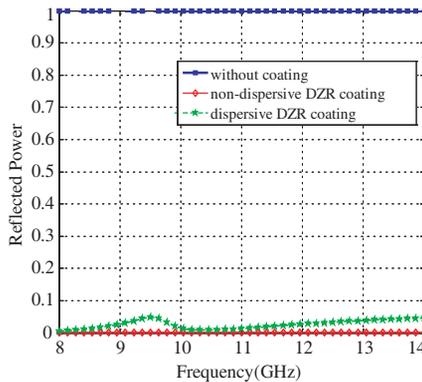
Figure 7. Reflected power from a PEC plane versus frequency (a) without coating (b) with two layers of nondispersive DZR metamaterial coatings with characteristics given in Eq. (26), (c) with two layers of dispersive DZR metamaterials with characteristics given in Eq. (27).

optimum values of parameters

First layer : $\quad f_{ep} = 14.09, \ \gamma_e = 8, \ f_{mp} = 12, \ f_{mo} = 0.5, \ \gamma_m = 8$

Second layer : $f_{ep} = 9.61, \ \gamma_e = 0.43,$ $\qquad\qquad\qquad\qquad$ (27)

$\qquad\qquad\quad f_{mp} = 8.17, \ f_{mo} = 5.78, \ \gamma_m = 8$

The reflected power for the dispersive case is also drawn in Fig. 7 as a function of frequency. The reflected power for the PEC plane without coating (which is equal to one) is also drawn in Fig. 7 for comparison. It is obvious that the reflected power for the dispersive case is generally greater than that due to the nondispersive case.

5.2. Optimum Design of a Double Layer DZR RAM Coating in the X Band and a Wide Interval of Incident Angles

Consider the same situation as in the last example, but the interval of incidente angles is $0 \leq \theta_0 \leq 45°$. The error function in Eq. (25) is minimized to determine the appropriate parameters.

$\qquad$ First layer : $\ d_1 = 0.45\,\text{mm}, \ \varepsilon_1 = 0 - j5, \ \mu_1 = 0 - j0.5$

$\qquad$ Second layer : $\ d_2 = 1.36\,\text{mm}, \ \varepsilon_2 = 0 - j5, \ \mu_2 = 0 - j5$ $\quad$ (28)

The 3-D diagram of reflected power as a function of frequency and angle of incidence is drawn in Fig. 8. Observe that even the small thickness of the two layers (less than $2\,\text{mm}$) has a good performance of low reflection in a wide frequency bandwidth and wide angles of incidence.
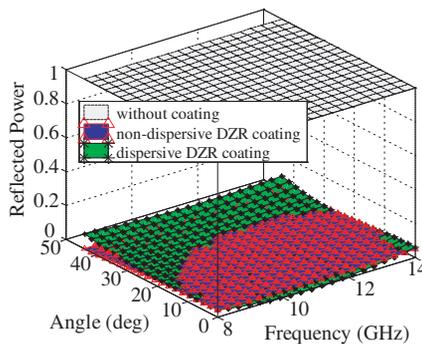
Figure 8. 3-D diagram of reflected power from a PEC plane versus frequency and angle of incidence under oblique incidence of a TM plane wave for three cases: (a) without coating. (b) With two layers of nondispersive DZR metamaterial coatings with characteristics given in Eq. (28). (c) With two layers of dispersive DZR metamaterial coatings with characteristics given in Eq. (29).

Next, we use the Drude and Lorentz dispersion models and use the method of least squares to determine their parameters to achieve the characteristics given in Eq. (28)

$$\text{First layer}: \ f_{ep} = 14.1, \ \gamma_e = 8, \ f_{mp} = 10.86,$$
$$f_{mo} = 0.5, \ \gamma_m = 5.26 \quad (\text{all in GHz})$$
$$\text{Second layer}: \ f_{ep} = 14.1, \ \gamma_e = 8, \ f_{mp} = 21.38,$$
$$f_{mo} = 9.23, \ \gamma_m = 8 \quad (\text{all in GHz})$$

(29)

The 3-D diagram of reflected power in this case is also drawn in Fig. 8, which may be compared with the above nondispersive case and also a PEC plane without DZR coatings.

6. CONCLUSION

In this paper, we introduced double zero (DZR) metamaterials (with zero real parts of permittivity and permeability), which are essentially lossy media with frequency dispersive characteristics possessing some known dispersive model.

We then investigated their applications for radar absorbing metamaterials (RAMs) for the reduction of reflected power. Several examples were considered, which included multilayer coatings of perfectly electric conductor (PEC) plates by dispersive and nondispersive DZR metamaterials under plane wave incidence of general polarization. Several exact formulas were derived for no reflection from such structures. The method of least squares (MLS) was used to design and optimize such multilayer structures by determining characteristics of the layer coatings. Consequently, it is shown that DZR metamaterials are suitable for application as RAMs.

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