A DETERMINISTIC APPROACH TO THE SYNTHESIS OF PENCIL BEAMS THROUGH PLANAR THINNED ARRAYS

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Abstract—This paper presents a simple and innovative deterministic approach to the synthesis of uniformly excited thinned arrays able to fulfill constraints concerning both the sidelobe level and the value of the radiated far field (and/or of the directivity) in a set of given directions. Starting from a reference regular (periodic or even aperiodic) lattice and from an optimal continuous reference source fulfilling at best the required specifications, the proposed approach finds out both the number and the location of the isophoric (i.e., equi-amplitude) radiating elements to withdraw in a fast and effective fashion. In fact, it is based on a deterministic best-fitting procedure which takes inspiration from existing density taper techniques. Examples are provided with reference to the synthesis of large circular arrays and confirm the interest of the proposed procedure.

1. INTRODUCTION

In a number of applications, e.g., the synthesis of Direct Radiating Arrays (DRA) for transmission from satellites [1–4], one is interested in achieving a directive behavior of the overall array while using a reduced number of elements. Moreover, in order to save efficiency of amplifiers,

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different radiating elements (or different clusters of elements) should be fed with the same amplitude. As a consequence, a renewed interest has recently arisen in the synthesis of **sparse arrays** [4] (i.e., arrays whose uniformly excited elements are properly located onto a non-regular grid) of **clustered arrays** [1] (wherein the single elements are gathered into a number of equi-fed clusters) and, last but not least, in the synthesis of **thinned arrays** [2, 5–8].

In these architectural solutions, starting from a 'filled' array, a number of elements is withdrawn (or removed) in order to realize a suitable tapering on the aperture and hence the desired far field (and/or directivity) pattern, including given constraints on the SideLobe Level (SLL). Such a procedure allows to get nearly the same beamwidth of a filled array of equal size, while reducing the cost and weight of the structure. With respect to alternative architectural solutions, thinned arrays present the advantage of easiness of realization, as different elements usually lie on a regular grid, operate with equal amplitude, and are directly connected to the amplifiers.

While a number of recent approaches exploit global optimization procedures [3, 5–9], with the inherent computational complexity, a very simple deterministic and non-iterative procedure is proposed herein for the case of planar arrays having a circular shape.

The next section of the paper presents the basic idea and corresponding deterministic procedure, while Section 3 reports a representative set of results, including both comparisons with a recently published synthesis approach and numerical experiments concerning the synthesis of large thinned arrays for transmission from geostationary satellites.

2. THE BASIC IDEA AND THE CORRESPONDING SYNTHESIS APPROACH

In order to introduce the proposed approach, it proves convenient to recall herein a procedure due to Doyle [10] and in Skolnik [11] for the synthesis of linear sparse arrays. In particular, in [11], one starts from an ideal reference continuous source $i(x)$ (e.g., the classical Taylor distribution [12]) over an interval $(-a, a)$, radiating a desired pattern $F(u)$ which can be written as:

$$F(u) = \int_{-a}^{a} i(x) e^{j\beta ux} dx \qquad (1)$$

wherein β is the propagation constant and $u = \sin(\theta)$ (being θ the angular variable as measured with respect to the broadside direction).

Then, the cumulative current distribution [11] $I(x)$, i.e.,

$$I(x) = \int_{-a}^{x} i(x')dx' \qquad (2)$$

is computed. The same quantity can be defined for the actual source, i.e., the sparse array (which will give rise to a staircase cumulative distribution).

One can easily show [13, 14] that the (sparse) array fitting at best (through its cumulative distribution) the ideal cumulative distribution is also the one minimizing the functional:

$$R(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} |\, F(u) - F_A(u)\,|^2 \frac{1}{u^2} du \qquad (3)$$

wherein x_i represents the location of the i-th radiating element of the array, while $F_A(u)$ is the pattern radiated from the equi-amplitude excited N-elements array at hand, which, assuming $F(0) = F_A(0) = 1$, can be expressed as:

$$F_A(u) = \frac{1}{N} \sum_{n=1}^{N} e^{j\beta u x_n} \qquad (4)$$

in other words, the sparse arrays best fitting the given cumulative distribution is also the one which best fits the desired far field pattern in a L^2 weighted sense (with weight $1/u^2$, see [13, 14] for a more in depth discussion of such a point). A simple way to justify this results is to recall from the *Fourier Transform Theory* that an integration in one domain corresponds to a $1/ju$ factor (with $j = \sqrt{-1}$) in the other domain. As a consequence, requiring similarity between the two cumulative current distributions means to require similarity (in a $1/u$ sense) between the transforms of the currents (i.e., between the spectra).

The above idea can be profitably extended to the synthesis of planar thinned arrays, by fitting the corresponding 'actual' to an 'ideal' cumulative aperture distribution. If the required ideal pattern is a circularly symmetric pencil beam, this function will be defined as:

$$I(\rho) = \int_{0}^{\rho} i(r)rdr \qquad (5)$$

where $i(r)$ is the ideal rotationally symmetric source. In fact, a proper fitting of the cumulative distributions corresponds to an L^2 weighted fitting of the corresponding plane wave spectra [15]. This result can be profitably exploited to devise a very simple deterministic procedure for the synthesis of thinned arrays.

In the overall procedure, the first step lies in the choice of a reference grid. Although the proposed approach can be applied to any kind of regular grid, the usual square and hexagonal grids will be considered in the following, and, in order to preserve some symmetry in the final layout, we assume that one node of the grid is exactly at the center of the circular region at hand. Then, the following procedure can be devised:

First, establish an optimal continuous non-negative reference source best fulfilling the required directivity specifications (to this end, the approach of [16] could be profitably exploited). Then, compute the corresponding cumulative current distribution according to (5);

Second, set 'on' or 'off' the central element depending on whether the continuous reference current is greater than or equal to zero; set $r' = 0$ and $I_A(r')$ equal to A (if the central element is set 'on') or zero (if the central element is set 'off'), being A the (common) amplitude of all the 'on' elements of the array;

Third, set $r' = r' + \Delta r$ and consider the actual radial cumulative current distribution $I_A(r')$, (i.e., the overall number of 'on' elements located into the circle of radius r', multiplied by A). Then, if $I_A(r')$ is lower than the ideal cumulative current distribution computed in the first step, set 'on' to all the antennas contained in the annular region of space between r' and $r' + \Delta r$; otherwise, set them 'off' (i.e., withdraw them);

Finally, repeat the third step until the boundary of the circular region one has at his disposal is reached.

It is interesting to note that the above procedure can be applied also to aperiodic grids (such as those in [1]). This solution should provide an increased robustness with respect to grating lobes, but it would be more difficult to be manufactured as compared with periodic grids.

In order to compare the radiation performances of given thinned arrays, a Thinning Factor TF is usually defined as:

$$TF = \frac{n_{RA} - n_{TA}}{n_{RA}} \tag{6}$$

where n_{RA} and n_{TA} are the number of elements of the reference (filled) array and of the thinned array respectively.

Moreover, in the overall procedure, the two parameters A and Δr come into play. The first one is related to the degree of thinning one wants to achieve. Note that, in order to get the right fitting of the cumulative distributions on the border of the structure, the equality:

$$n_{TA}A = I(a) \tag{7}$$

must be at least approximately true (being a the radius of the circular domain at hand). Values of A lower than the ones determined from (7)

will allow a less effective thinning. Moreover, a lower bound for A is anyway determined from:

$$n_{RA}A \geq I(a) \qquad (8)$$

As far as the choice of Δr is concerned, note that small values of Δr, allowing one to manage very few elements at a time (and therefore to get a fine control of the cumulative distribution of the actual array), are recommended in order to get a good fitting of the ideal cumulative distribution. On the other hand, as long as rectangular or hexagonal grids are used, very small values of Δr may induce an angular periodicity of the pattern with respect to the azimuthal variable.

Some finer understanding of what is going on can be gained by considering the field radiated by each annular ring in case of a vanishingly small value of Δr. In particular, by virtue of the regularity of the underlying grid and of its symmetry with respect to the center, any annular ring will necessarily contain a number of elements which are a multiple of four in case of a square grid and of six in case of hexagonal grids. Also, these elements will be uniformly spaced in angle, so that the corresponding far field can be computed as:

$$AF(\theta, \phi) = NI_0 \sum_{m=-\infty}^{\infty} J_{mN}(\beta\tau\sin(\theta))e^{mN(\frac{\pi}{2}-\phi+\phi_S)} \qquad (9)$$

wherein N is the number of feeds in the annular ring, τ the radius of the circle, I_0 a constant (which indicates the common excitation coefficient), ϕ_S the angular displacement of the feeds on the circle, and J_{mN} the Bessel Function of the first kind and order mN. Then, by virtue of the properties of the Bessel Functions, the far field will exhibit a circular symmetry for sufficiently small θ angles, whereas such a symmetry will be lost for increasing values of θ. Also, as the displacement angle is of the kind π/mk (with k an integer and $m = 4$ or 6), the azimuthal harmonics entering (9) will not cancel out, and therefore the final pattern will have the same azimuthal harmonics as in (9).

Such a circumstance, which is generally undesired, may be instead useful in some applications, such as the one presented in the second part of Section 3.

3. NUMERICAL EXPERIMENTS

This section aims at supporting the given theory and testing the effectiveness and the actual interest of the proposed approach. The

use and the kind of outcomes of the procedure are exemplified in two different sets of numerical experiments.

In particular, the first part of the analysis is devoted to a comparison with a recently published synthesis procedure [7], while in the second part an actual design problem (i.e., the synthesis of thinned arrays for transmission from geostationary satellites) is dealt with.

3.1. Comparison with a Recently Published Synthesis Procedure

In order to compare the proposed approach with the one given in [7], the above thinning procedure has been applied to the same reference arrays as in [7], so that arrays lying on a square grid with an inter-element spacing of half a wavelength and contained within circles of diameter 25λ, 33λ, 66λ, and 100λ (being λ the wavelength in free space) have been considered. In all cases, an isotropic element pattern has been used, and a $TF \cong 60\%$ (i.e., a Fill Factor $FF \cong 40\%$) has been enforced by acting on the A value.

The results achieved in the different cases are synthetically reported in Table 1 in terms of directivity, half-power beamwidth (HPBW), and SLL normalized to the maximum directivity value.

When comparing these results to the ones in [7], it can be noticed that similar overall performances can be achieved with a very reduced computational burden. In fact, the performances of the proposed approach are better than the ones in [7] in all performance parameters in the case wherein the reference grid has a diameter of 100λ, while the small degradation in terms of SLL one noticed in all other case is compensated by the smaller HPBW (according to a well known trade-off). The runtime for the overall synthesis procedure (but for directivity evaluation) was less than 3 seconds per considered case (by using a MATLAB R2008b code on a PC running 32-bit Windows Vista,

Table 1. Properties and performances (maximum directivity, HPBW, and SLL) of the thinned arrays achieved by applying the proposed procedure to four different reference grids.

Diameter (in terms of λ)	Fill Factor	# of turned ON elements	Maximum SLL [dB]	HPBW [deg]	Directivity [dBi]
25	42%	824	-23.5	3.05	33.7
33.33	41.5%	1461	-25.4	2.27	36.2
66.67	38%	5352	-30	1.15	41.6
100	40%	12580	-33.9	0.76	45.3

equipped with a 2.53 GHz Processor and 4 GB of RAM memory), which is much lower than the computational time (9 minutes) required in [7].

A rather obvious idea amounts to extract the very best from the different approaches, so that the present one could be used as a smart initial point for [7] (thus avoiding the need for a *multi-start strategy*), or, which is the same, [7] can be used to refine the present results.

For the sake of completeness, a sample result (for the case wherein the diameter of the reference grid is equal to 100λ) is reported in Fig. 1 and Fig. 2, wherein the layout and corresponding directivity pattern are respectively shown. It is interesting to note that, differently from [7], a rather symmetrical layout is achieved.
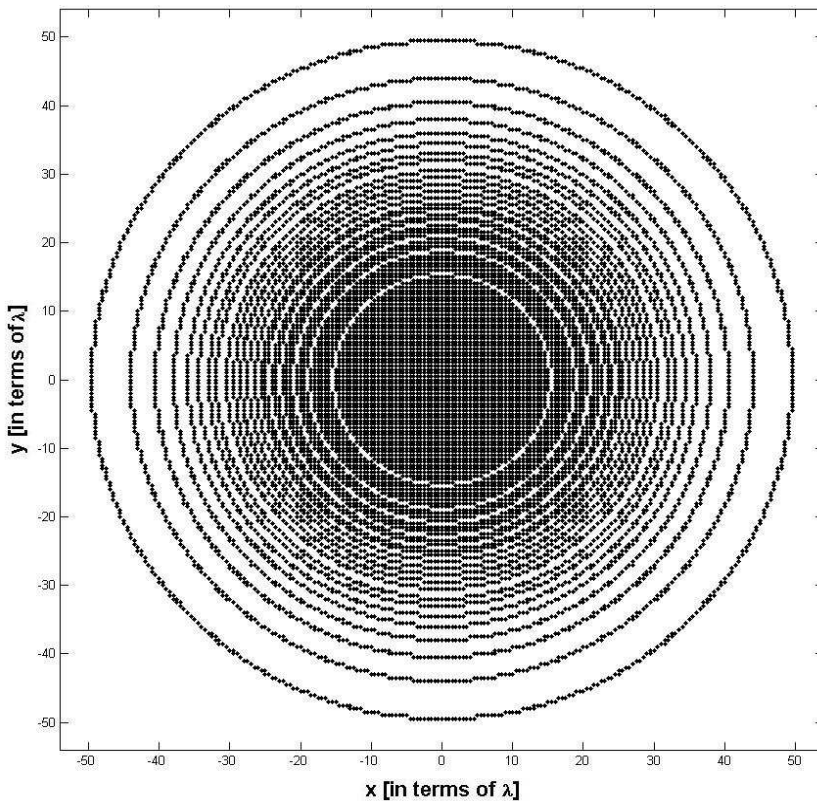

Figure 1. Thinned array composed by 12580 elements located over a circle of radius 50λ.
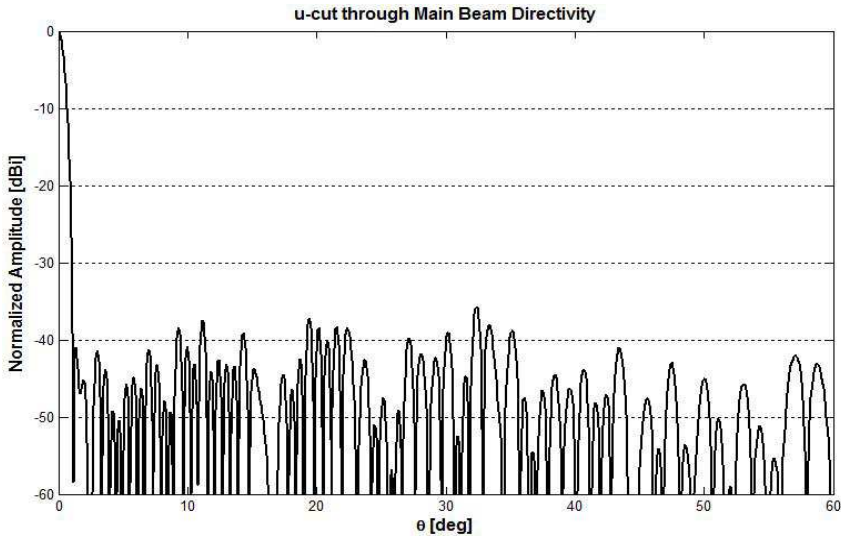
Figure 2. *U*-cut through Main Beam Directivity achieved by the thinned array shown in Fig. 1.

3.2. Synthesis of Isophoric DRA for Satellite Communications

In order to test the outcomes of the proposed approach in an actual design problem, we thought it worthwhile to try to synthesize a radiation pattern fulfilling the specifications of the ESA (European Space Agency) tender (see [1] and [17]).

Roughly speaking, these specifications require to design a multibeam DRA for transmission from geostationary satellites having a maximum radius of 60λ and achieving (in each of the 19 different spots required for Europe covering) a gain of at least 43.8 dBi at the EOC (i.e., *Edge Of Coverage*, which is positioned $0.325°$ from the maximum). Separation amongst neighboring beams is ensured by either sub-band or polarization diversity, according to a 'four colours' scheme, while separation amongst iso-colour beams is achieved by means of suitable constraints on the behaviour of sidelobes. In particular, a sufficient condition in order to get such a separation is that of having sidelobes 20 dB lower than the EOC gain in all the directions out of the $0.795°$ cone from maximum and up to $\theta = 16°$ (which denotes the border of the Earth cone as seen from geostationary satellites), and sidelobes 10 dB lower than the EOC gain in the remaining directions.

In order to achieve these performances, a triangular equilateral grid was selected for the reference array, wherein, because of the specific application, a spacing (much) larger than a wavelength amongst neighboring elements has been chosen. In particular, consistent with the existing literature [1–3], a spacing of 3.8λ (centre-to-centre) has been chosen in order to ensure that grating lobes will fall outside of the Earth. Note such a circumstance allows to exploit radiating elements which are per se directive, thus usefully contributing to the overall link budget. In fact, according to the criterion used in [18], a pattern of the kind $\cos^{33}(\theta)$ has been used as element factor.

Then, a convenient continuous reference source was synthesized according to the approach in [16], and different synthesis steps were executed. Table 2 provides a summary of the obtained results, which suggests that the layout achieved by using a $TF = 42.9\%$ represents an interesting overall design solution for the problem at hand. The resulting thinned array is composed by 529 radiating elements and shown in Fig. 3, while the corresponding cumulative distribution is reported in Fig. 4 (red curve), and the directivity performances are shown in Fig. 5 and Fig. 6.

The achieved layout fulfills the constraints on HPBW ($0.576°$), EOC directivity ($44.5\,\mathrm{dBi}$), and SLL outside the Earth disc ($-10.5\,\mathrm{dB}$ from EOC directivity). Moreover, it gives back a SLL $= -18.4\,\mathrm{dB}$ form EOC directivity inside the Earth disc (i.e., a SLL very close to the $-20\,\mathrm{dB}$ threshold, which would ensure satisfaction of the separation constraints amongst neighboring iso-colour beams). On the other hand, such a condition is just a sufficient one (it is not strictly necessary), and a further useful circumstance comes into play. In fact, both the widely diffused 'four colours' scheme and adopted equilateral triangular cell enforce a hexagonal symmetry (of the spot locations and

Table 2. Directivity and SLL as functions of the degree of thinning imposed in the numerical experiment.

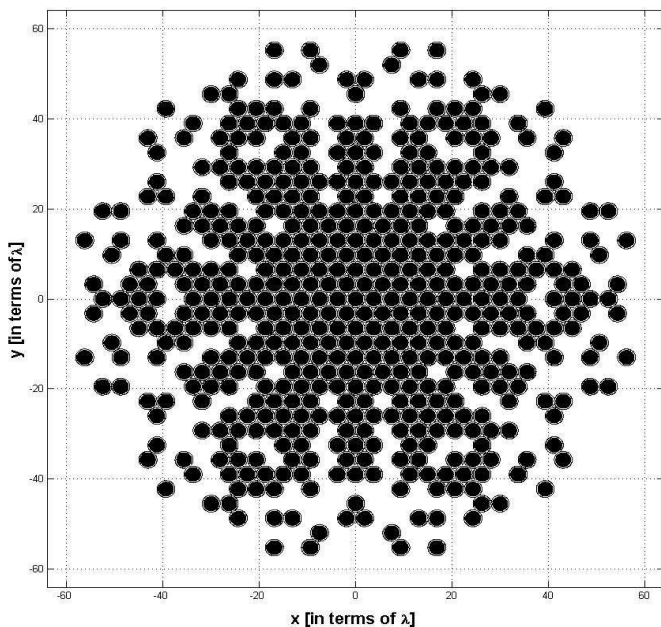| Thinning Factor	# of turned ON Elements	Directivity [dBi]	Maximum SLL on Earth [dBi]
34.5%	607	49	−22.7
42.9%	529	48.3	−22.2
53.3%	433	47.5	−21.8
61.7%	355	46.7	−16.4
73.4%	247	45.1	−14.7
83.1%	157	43.2	−12.8

Figure 3. Thinned array composed by 529 elements located over a circle of radius 60λ.
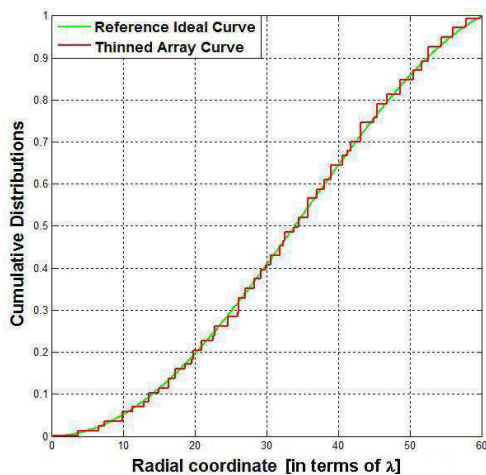

Figure 4. Cumulative distributions: (green curve) reference 'ideal' function; (red curve) synthesized function (corresponding to the thinned array shown in Fig. 3).
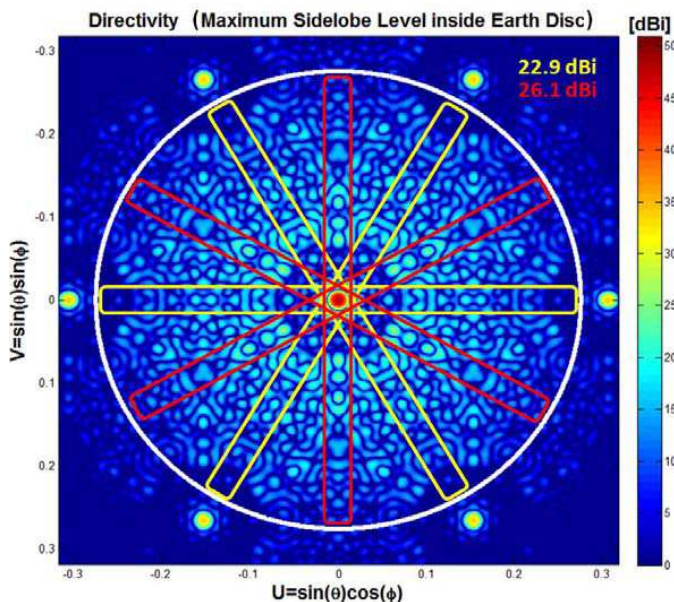
Figure 5. Directivity pattern (in the spectral plane U-V) provided by the thinned array shown in Fig. 3. Due to the hexagonal symmetry of the reference grid, the highest level of sidelobes (26.1 dBi) is achieved only in six directions on the spectral plane U-V (as evidenced by the red cuts), while a significantly (at least 3.2 dB) lower value is exhibited in the directions located mid-way amongst them (as evidenced by the yellow cuts). The white circle represents the Earth disc as seen from geostationary satellites.

of the radiation pattern respectively). Then, a smart interlocking of the antenna grid and of the iso-colour beams will allow to achieve the required separation amongst the iso-colour beams.

In particular, it will suffice to locate the six directions presenting the highest sidelobes of the radiation pattern in those directions midway amongst the six nearest iso-colour beams (see Fig. 5, wherein the Earth disc as seen from geostationary satellites is also evidenced, for a better understanding). In this way, the six nearest iso-colour beams will be located along the six directions corresponding to the lowest sidelobes of the central spot pattern (see Fig. 5).

By doing so, the SLL amongst iso-colour beams becomes equal to -21.6 dB (from EOC directivity), which fully satisfies the overall requirements.
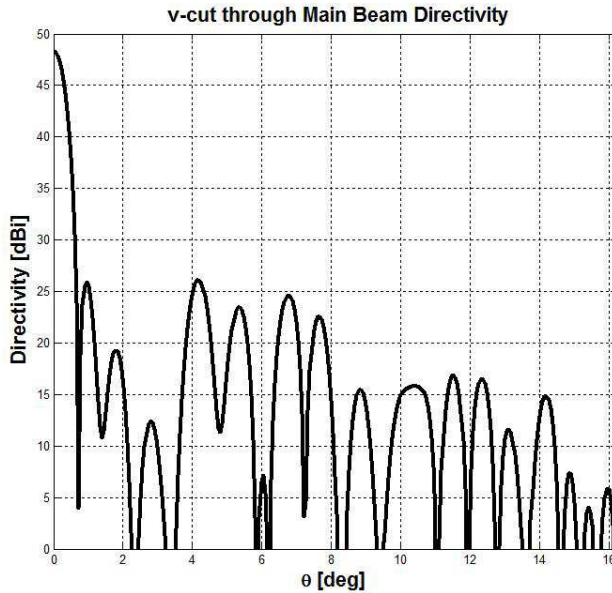
Figure 6. *V*-cut through main beam of the directivity shown in Fig. 5.

4. CONCLUSIONS

An effective strategy for the optimal synthesis of pencil beams via planar thinned arrays has been presented and assessed. Starting from a reference aperture field and a filled array over a regular (periodic or even aperiodic) grid, the proposed approach allows one to state (without exploiting global optimization techniques) the elements to withdraw in such a way to enforce constraints concerning both the sidelobe level and the value of the directivity in a set of given directions. Notably, the computational effectiveness of the procedure can allow a fast trade off amongst alternative solutions using different grids or Thinning Factors.

Several numerical experiments have shown the interest of the strategy. In particular, comparisons have been discussed with respect to a recently published synthesis procedure, and the capability of the proposed technique to synthesize an interesting solution for multibeam Direct Radiating Arrays for transmission from geostationary satellites has also been shown.

REFERENCES

1. Morabito, A. F., T. Isernia, M. Labate, M. D'Urso, and O. M. Bucci, "Direct radiating arrays for satellite communications via aperiodic tilings," *Progress In Electromagnetics Research*, PIER 93, 107–124, 2009.

2. Toso, G., C. Mangenot, and A. G. Roederer, "Sparse and thinned arrays for multiple beam satellite applications," *Proc. European Conf. on Antennas and Propagation, EUCAP 2007*, Edinburgh, UK, Nov. 11–16, 2007.

3. Caille, G., Y. Cailloce, C. Guiraud, D. Auroux, T. Touya, and M. Masmoudi, "Large multibeam array antennas with reduced number of active chains," *Proc. European Conf. on Antennas and Propagation, EUCAP 2007*, Edinburgh, UK, Nov. 11–16, 2007.

4. Bucci, O. M., T. Isernia, A. F. Morabito, S. Perna, and D. Pinchera, "Aperiodic arrays for space applications: An effective strategy for the overall design," *Proc. European Conf. on Antennas and Propagation, EUCAP 2009*, Berlin, Mar. 2007.

5. Haupt, R. L., "Thinned arrays using genetic algorithms," *IEEE Transactions on Antennas and Propagation*, Vol. 46, No. 3, 70–73, Jun. 2004.

6. Mahanti, G. K., N. Pathak, and P. Mahanti, "Synthesis of thinned linear antenna arrays with fixed sidelobe level using real-coded genetic algorithm," *Progress In Electromagnetics Research*, PIER 75, 319–328, 2007.

7. Keizer, W., "Large planar array thinning using iterative FFT techniques," *IEEE Transactions on Antennas and Propagation*, Vol. 57, No. 10, 3359–3362, Oct. 2009.

8. Razavi, A. and K. Forooraghi, "Thinned arrays using pattern search algorithms," *Progress In Electromagnetics Research*, PIER 78, 61–71, 2008.

9. Ares-Pena, F., A. Rodriguez-Gonzalez, E. Villanueva-Lopez, and S. R. Rengarajan, "Genetic algorithms in the design and optimization of antenna array patterns," *IEEE Transactions on Antennas and Propagation*, Vol. 47, No. 10, 506–510, 1999.

10. Doyle, W., "On approximating linear array factors," *RAND Corp Mem RM-3530-PR*, Feb. 1963.

11. Skolnik, M. I., "Nonuniform arrays," *Colin-Zucker Antenna Theory*, Chapter 6, McGraw-Hill, New York, 1969.

12. Taylor, T. T., "Design of circular apertures for narrow beamwidth and low sidelobes," *IEEE Transactions on Antennas and Propagation*, Vol. 8, No. 1, 17–22, Jan. 1960.

13. Isernia, T., O. M. Bucci, M. D'Urso, M. Toso, and P. Angeletti, "A new density taper approach to the synthesis of uniform amplitude sparse arrays," *30th ESA Antenna Workshop on Antennas for Earth Observation, Telecommunication and Navigation Space Missions*, ESTEC, Noordwijk, The Netherland, May 27–30, 2008.

14. Isernia, T., O. M. Bucci, M. D'Urso, M. Toso, and P. Angeletti, "Deterministic synthesis of uniform amplitude sparse arrays via new flexible density taper techniques," *IEEE Transactions on Antennas and Propagation*, 2009.

15. Willey, R., "Space tapering of linear and planar arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 10, No. 4, 369–377, July 1962.

16. Bucci, O. M., T. Isernia, and A. F. Morabito, "Optimal synthesis of directivity constrained pencil beams by means of circularly symmetric aperture fields," *IEEE Antennas and Wireless Propagation Letters*, Vol. 8, 1386–1389, 2009.

17. ESA/ESTEC, "Tender AO/1-5598/08/NL/ST," Jan. 2008, www.esa.int.

18. Fondevila-Gomez, J., J. A. Rodriguez-Gonzalez, E. Moreno, G. Toso, and F. Ares, "Sum pattern synthesis by using planar arrays with square lattices and circular boundaries," *Proc. European Conf. on Antennas and Propagation, EUCAP 2007*, Edinburgh, UK, Nov. 11–16, 2007.