COMPLEX MODE IN RECTANGULAR WAVEGUIDE FILLED WITH LONGITUDINALLY MAGNETIZED FERRITE SLAB

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Abstract—In microwaves, ferrites are characterized by a tensorial permeability which represents their anisotropy under a constant magnetic field. We present, in this article, a rigorous study of the formulation of the transverse operator method (TOM) with an extension to the case of the guides of rectangular waves partially charged with longitudinally magnetized ferrite. We show the existence of the complex modes in these types of structures with ferrite. A good agreement of the constant of propagation with the literature is obtained.

1. INTRODUCTION

Various techniques have been used to analyze guides of waves charged with dielectric [1–3]. However, the analytical study of the electromagnetic wave propagation in guides partially charged with longitudinally magnetized ferrite presents an appreciable lack and finds difficulties due to the anisotropy of the medium and the coupling of

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the electromagnetic fields in the equation of propagation [4–10]. In this article, we present a thorough study of the TOM applied to the case of the anisotropic mediums by considering a tensorial representation of the permeability of ferrite. We will apply this method to the case of the dielectric guides by demagnetizing ferrite, then to the case of the guide charged with longitudinally magnetized ferrite by following Galerkin's method. We obtain a good agreement with the literature for the fundamental mode. We will present the modes of a higher order and exploit the complex modes in this last guide with ferrite slab. It is an original result which has not been published in literature. With TOM, the stability of convergence of the constant of propagation is obtained through 15 modes.

2. ANALYSIS

The permeability of longitudinally magnetized ferrite is expressed by the tensor of Polder

$$\bar{\bar{\mu}} = \mu_0 \cdot \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} = \bar{\bar{\mu}}_{rf} \cdot \mu_0 \tag{1}$$

where μ, κ and μ_{rz} are real quantities.

For a partial magnetization of ferrite, Green [11] and Schloemann [12] give the empirical expressions of μ, κ and μ_{rz}.

$$\mu = \mu_d + (1 - \mu_d)(4\pi M/4\pi M_S)^{3/2} \tag{2}$$

$$\kappa = \frac{\gamma(4\pi M)}{\omega} \tag{3}$$

$$\mu_{rz} = \mu_d^{\left(1-\left(\frac{4\pi \cdot M}{4\pi \cdot M_S}\right)^{5/2}\right)} \tag{4}$$

where

$$\mu_d = \frac{1}{3}\left\{1 + 2\sqrt{1 - \left(\frac{\gamma(4\pi M_S)}{\omega}\right)^2}\right\} \tag{5}$$

ω is the work pulsation, γ is the gyromagnetic constant, $4\pi M_S$ is the magnetization at saturation and $4\pi M$ is the magnetization which is lower than saturation.

When the magnetization is null $\kappa = 0$ and $\mu = \mu_{rz} = \mu_d \simeq 1$, the ferrite then becomes dielectric isotope.

Our study focuses on the dispersion in the rectangular metal guides partially filled with dielectric or ferrite which makes all the height of the guide represented by Fig. 1. The planar discontinuity of

the permittivity and the permeability is described by the functions of Heaviside which intervene the distributions of Dirac and their derivatives after the application of the transverse operators on the transverse section of the guide.

By considering the propagation according to direction OZ and by eliminating the longitudinal components from the electromagnetic field, we can get a new formulation in terms of two transverse magnetic components [10]

$$\hat{L} H_t = k_z^2 H_t \tag{6}$$

Where $\hat{L}$ is the transverse operator defined by

$$
\begin{aligned}
\hat{L} &= k_0^2 \varepsilon_r \bar{\bar{\mu}}_{tt} - \varepsilon_r \partial_t \frac{1}{\varepsilon_r} \partial_t^+ - \eta_0 \partial_t \frac{1}{\mu_{rz}} \partial_t^+ \eta_0 \bar{\bar{\mu}}_{tt} \\
&= k_0^2 \varepsilon_r \bar{\bar{\mu}}_{tt} - \partial_t \partial_t^+ + \frac{\bar{\bar{\mu}}_{tt}}{\mu_{rz}} \partial_t \partial_t^+ + \bar{\bar{D}}_\varepsilon + \bar{\bar{D}}_\mu
\end{aligned}
\tag{7}
$$

Where

$$\bar{\bar{D}}_\varepsilon = -\varepsilon_r \partial_t \left(\frac{1}{\varepsilon_r} \right) \partial_t^+ \tag{8}$$
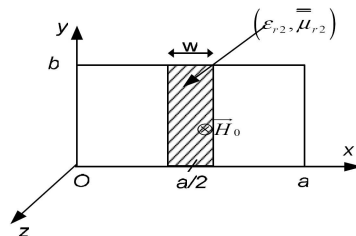
$$\bar{\bar{D}}_\mu = -\eta_0 \partial_t \left(\frac{1}{\mu_{rz}} \right) \partial_t^+ \eta_0 \bar{\bar{\mu}}_{tt} \tag{9}$$

$\bar{\bar{D}}_\varepsilon$ and $\bar{\bar{D}}_\mu$ are the matrixes of discontinuities on axis x of the permittivity and the permeability of the heterogeneous medium in the guide.

The expressions of the permittivity and the permeability can be written in the following form

$$\varepsilon_r(x) = 1 + (\varepsilon_{r2} - 1) \cdot U(X) \tag{10}$$

$$\mu_{rz}(x) = 1 + (\mu_{rz2} - 1) \cdot U(X) \tag{11}$$

Figure 1. Transverse section of the waveguides charged with ferrite.

Where

$$U(X) = U(x - (a/2 - w/2)) - U(x - (a/2 + w/2)) \quad (12)$$

$$\frac{\partial_x \varepsilon_r}{\varepsilon_r} = 2\frac{\varepsilon_{r2} - 1}{\varepsilon_{r2} + 1} \cdot \delta(X) \quad (13)$$

$$\delta(X) = \partial_x U(X) \quad (14)$$

$$\partial_x \left(\frac{1}{\mu_{rz}}\right) = -2\frac{\mu_{rz2} - 1}{(\mu_{rz2})^2 + 1} \cdot \delta(X) \quad (15)$$

For the structure of Fig. 1, the expressions of the transverse fields verifying the edge conditions can be expressed as follows.

$$H_x = \sum_{n,m=0}^{N} H_{x,nm} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \quad (16)$$

$$H_y = \sum_{n,m=0}^{N} H_{y,nm} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \quad (17)$$

The decomposition of the transverse field $\vec{H}_t$ in a complete base (base of empty guide) which verifies the conditions within the guide limits (see Equations (16) and (17), by following the method of Galerkin, allows us to obtain a system with the eigenvalues, which is written in the following form

$$T \cdot H_t = k_z^2 \cdot H_t \quad (18)$$

T is a square matrix of order $(2N \times 2N)$ with N: mode numbers; m and n are natural entireties verifying: $(m, n) \neq (0, 0)$. The eigenvalues and the proper vectors of T are respectively the square of the propagation constant k_z and the coefficients of development of the field of the guide.

3. RESULTS OF SIMULATIONS AND DISCUSSIONS

Let us consider a rectangular guide of waves of width a and height $b = a/2$ as shown in Fig. 1, partially and symmetrically filled with dielectric of permittivity $\varepsilon_{r2} = 2.45$. We compare our results of the constant of propagation of the fundamental mode by demagnetizing the ferrite (the ferrite becomes an isotope medium) with those of Ref. [13] which applies variation calculation to this same guide.

In the absence of the dielectric slab, we know that the fundamental mode is the mode TE-10, which has an electric field in the direction y varying in $\sin(\pi x/a)$ in x. If the constant of the dielectric slab is not very large, we must wait until the field is a slightly disturbed version of the mode TE-10 of the empty waveguide.
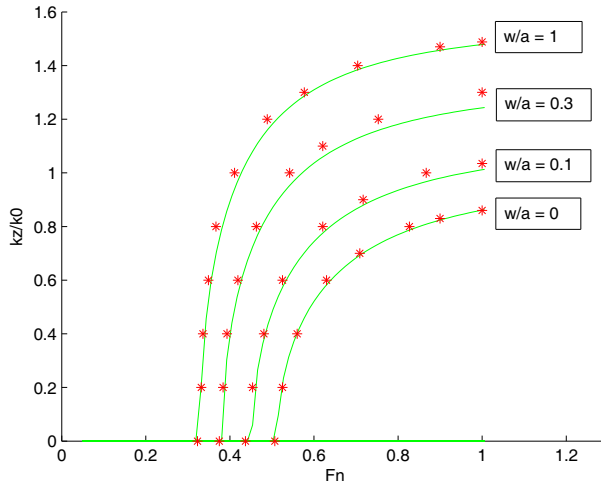
Figure 2. The variation of the normalized propagation constant according to the normalized frequency $F_n = \frac{a}{\lambda}$ for a slab of various widths w of Fig. 1. ———— : Our results with the TOM. ∗∗∗∗∗∗∗: Results of Ref. [13] with the variation method.

The obtained curves with the TOM of the normalized propagation constant λ/λ_g (it is the ratio of wavelength of open space to the wavelength of guide) according to the normalized frequency $F_n = \frac{a}{\lambda}$, shown with full features, are compared in Fig. 2 with the curves shown with asterisks which are obtained through the variation calculations given in Ref. [13]. We obtain a good agreement.

Now let us consider the guide of Fig. 1, studied by Ref. [7], of the parameters given by Table 1. It is a guide with laminated ferrite (TT1-414 from trans-tech).

Table 1. Parameters of the guide of Fig. 1, (ferrite TT1-414 from trans-tech).

a mm	b mm	h	w mm	ε_{r2}
72.136	33.06	b	19.05	11.3

Figure 3(a) shows our results of simulation with the TOM of the normalized propagation constant of the first four modes in the band [2.2–3] GHz. The ferrite is magnetized longitudinally with $\frac{M}{M_S} = 0.6$ and $4\pi M_S = 750$ G. Throughout the 15 modes, we obtain the stability of convergence of the constant of propagation (see Fig. 3(b)). The variation of our results with that of Ref. [7] is from 0.5% to 2.84 GHz. A good agreement is then obtained.

Figures 3(c) and 4 show the phenomenon of bifurcation of the
modes (see 5th mode in Fig. 3(c)) due to the anisotropy of ferrite. In
Fig. 5, we show the existence of the complex modes in these types of
structures. The total complex modes in positive z and negative z are
cancelled. These modes depend on the dimensions of the guide.
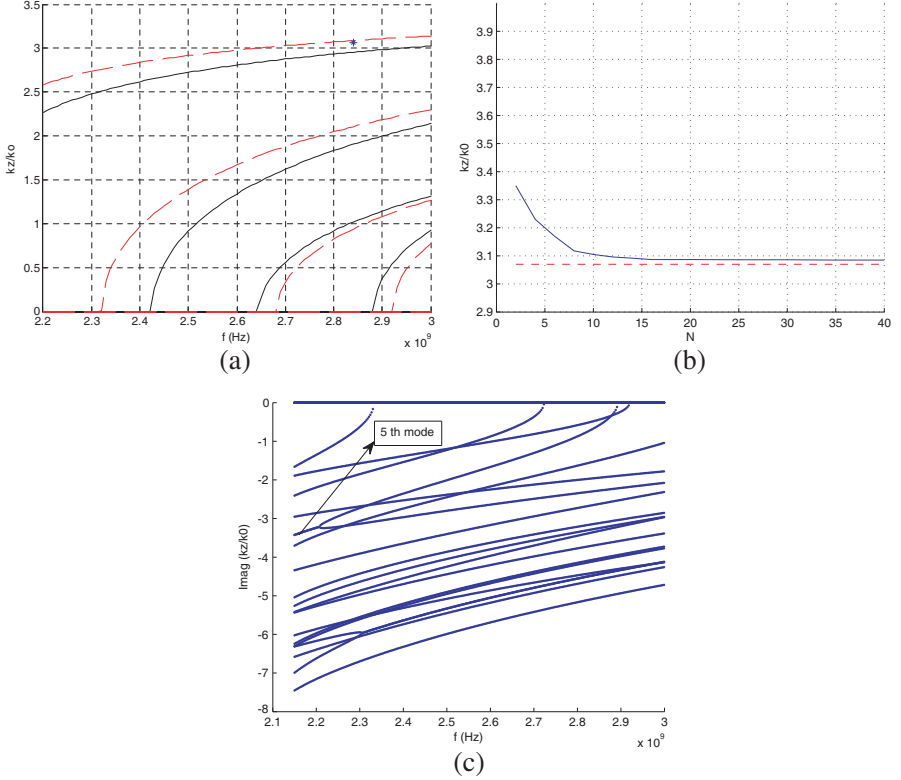
Figure 3. (a) The variation of the normalized propagation constant
according to the frequency. ———: Our results with the TOM
for a demagnetized ferrite. ------: Our results with the TOM for
$\frac{M}{M_S} = 0.6$, $(k_z/k_0 = 3.087$ with 2.84 GHz.) *: The value of Ref. [7],
$(k_z/k_0 = 3.070$ with 2.84 GHz). (b) The curve of convergence of the
normalized propagation constant (k_z/k_0) for $\frac{M}{M_S} = 0.6$ with 2.84 GHz:
———: Our results with the TOM for $\frac{M}{M_S} = 0.6$. ------: The value
of Ref. [7], $(k_z/k_0 = 3.070$ with 2.84 GHz). (c) The variation of the
imaginary part of (k_z/k_0) according to the frequency. Our results with
the TOM for $\frac{M}{M_S} = 0.6$.
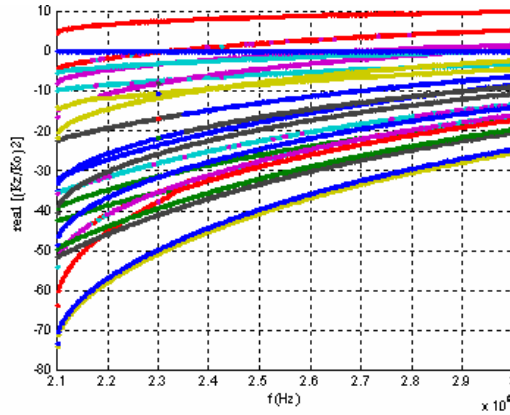
Figure 4. The variation of the real part of $\left[(k_z/k_0)^2\right]$ according to the frequency (TOM).
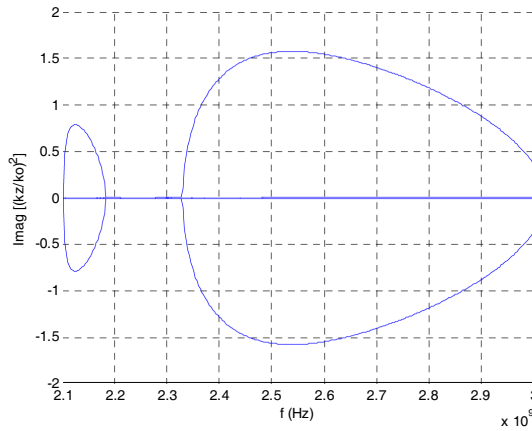
Figure 5. The variation of the Imaginary part of $\left[(k_z/k_0)^2\right]$ according to the frequency (TOM).

In general, the metal guide completely filled with ferrite will be thus a particular case of our study, by extrapolating the dimensions of the guide as follows

$$w = a \tag{19}$$

The terms of discontinuities $\bar{\bar{D}}_\varepsilon$ and $\bar{\bar{D}}_\mu$ are cancelled. We obtain the same results of simulations as those given by Ref. [10].

4. CONCLUSION

In conclusion, the extension of the TOM in the case of the guides of metal rectangular waves partially filled with longitudinally magnetized ferrite is presented. The modes of a higher order and complex modes have been obtained. This structure presents applications in circuits at magnetic order such as the phase-converters.

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