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ELECTROMAGNETIC FIELDS IN A CAVITY FILLED WITH SOME NONSTATIONARY MEDIA

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Abstract—The paper presents an analytical approach to treat the problem of transient oscillations in a cavity uniformly filled with nonstationary medium, which is characterized by time-varying permittivity and conductivity. Closed-form solutions are found for some transient excitations and medium parameters.

1. INTRODUCTION

Interaction of electromagnetic fields with media that have time varying properties is of significant interest in different applications. Among such phenomena we should mention propagation of electromagnetic pulses in modulated dielectric waveguides, which finds applications in a range of areas including pulse generation, compression, reshaping and filtering, wavelength conversion and terahertz wave generation. Analysis of such phenomena also provides useful insight onto behavior of high speed switches, ultra-short pulse lasers etc. More generally, these problems are of significant importance for understanding of optical communications technology and quantum electronics [6–11].

The problem of interaction of transient electromagnetic fields in a cavity with time-varying filling is the first step in studying non-stationary media. Change in medium parameters significantly

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influences spectral, energetic, and temporal characteristics of electromagnetic fields existing (free oscillations) and being excited (forced oscillations) in such a cavity. Among examples of non-stationary filling in a microwave cavity that requires for studying transient processes, we can mention media with ultrafast chemical reactions, light pumping (e.g., in masers). Another important example is explosive destroying ferromagnetic and piezoelectric properties of medium that leads to ultrafast change of electric and magnetic properties and creates transient electromagnetic fields. Gas discharge in high intensity fields can also be considered as a process with non-stationary and dispersive medium. In general, the changes in medium properties are described by some complicated dependences that are typically close to some combination of exponentials. In our study, we will take some specific time dependences that allow to obtain closed-form solution and are at the same time rather representative.

Among the first publications on electromagnetic fields in a cavity with nonstationary medium we should mention paper [2] where the permittivity of the cavity filling is taken in a form of Epshtein transition. The solution to this problem without external currents was presented as combination of hypergeometric functions. Cavity eigenfrequency is defined in the initial and stationary moments, after changing its permittivity. The next relevant work is [3]. By means of Integral Equation Method [1] the problem of electromagnetic field in the rectangular resonator with time-varying permittivity by harmonic law was solved. For full filling of the cavity and small modulation amplitude the analytical expression for electric field strength is obtained, but only numerical investigation of electromagnetic field characteristics in the resonator with wide range changing of nonstationary medium parameters is possible. In paper [3] resonance frequencies and instability oscillation zones, depending on permittivity parameters and fullness parameter, are found, but transient processes in the cavity are not considered. Later work [12] presents the evaluation of electromagnetic field in rectangular cavity with cylindrical nonstationary obstacle. The cylindrical insert has got harmonically changing permittivity and conductivity with time. For small modulation amplitudes the characteristic equation for TE wave, permitting to define electromagnetic field closely and in the area of main parametric resonance, is obtained. By numeric analysis of the obtained solution in [12] instability zones and characteristic index for wide range of modulation frequencies are defined. Like [3], this work allows to investigate parameters of the system only numerically and does not give an overview of transient processes in the cavity.

The first attempts of applying evolutionary approach to studying transient processes in microwave cavities with non-stationary media were made in [15], though consideration there was mainly aimed at spatial field distribution in a cavity with some specific geometry.

In this paper, we present some temporal dependences of parameters of media that fill the cavity. Resonance frequencies of the cavity filled with time-varying medium are analytically obtained, and transient processes are investigated.

The second part of the paper contains the general statement of the problem of electromagnetic fields in the cavity filled with time-varying medium in the frame of Evolutionary Approach to Electromagnetics in Time Domain. Then, the periodic and pulse changes of the resonator filling permittivity are presented in part three. Analytical solutions for mode amplitudes and properties of electromagnetic fields behavior for nonstationary conductivity of the cavity filling are obtained in part four.

2. PROBLEM STATEMENT AND GOVERNING EQUATIONS FOR TRANSIENT ELECTROMAGNETIC FIELDS

The cavity under study is bounded with a singly-connected closed PEC surface (Figure 1). Considered resonator is filled with linear homogeneous medium. This problem is reduced to solving Maxwell equations

$$\begin{aligned}\nabla\times\boldsymbol{\mathcal{H}}(\mathbf{r},t)&=\varepsilon_0\frac{\partial}{\partial t}\boldsymbol{\mathcal{E}}(\mathbf{r},t)+\frac{\partial}{\partial t}\boldsymbol{\mathcal{P}}(\mathbf{r},t)+\boldsymbol{\mathcal{J}}_\sigma\left(\boldsymbol{\mathcal{E}},\boldsymbol{\mathcal{H}}\right)+\boldsymbol{\mathcal{J}}_e(\mathbf{r},t),\\ \nabla\times\boldsymbol{\mathcal{E}}(\mathbf{r},t)&=\mu_0\frac{\partial}{\partial t}\boldsymbol{\mathcal{H}}(\mathbf{r},t)+\mu_0\frac{\partial}{\partial t}\boldsymbol{\mathcal{M}}(\mathbf{r},t)+\boldsymbol{\mathcal{J}}_m(\mathbf{r},t),\end{aligned}\tag{1}$$

with the constitutive relations for vectors of polarization and magnetization, $\boldsymbol{\mathcal{J}}_e(\mathbf{r},t)$, $\boldsymbol{\mathcal{J}}_m(\mathbf{r},t)$ are given impressed electric and magnetic currents. In (1) the expression for electric flux density is $\boldsymbol{\mathcal{D}}(\mathbf{r},t)=\varepsilon_0\boldsymbol{\mathcal{E}}(\mathbf{r},t)+\boldsymbol{\mathcal{P}}(\mathbf{r},t)$, and for magnetic flux density the expression is $\boldsymbol{\mathcal{B}}(\mathbf{r},t)=\mu_0\left(\boldsymbol{\mathcal{H}}(\mathbf{r},t)+\boldsymbol{\mathcal{M}}(\mathbf{r},t)\right)$.

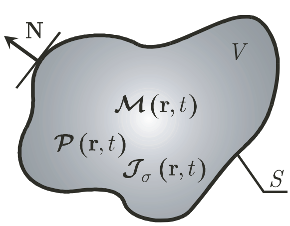

Figure 1. Geometry of the cavity.

Within the frame of Evolutionary Approach to Electromagnetics in Time Domain (TD) [4, 5, 13] (Mode Expansion in TD) the sought electromagnetic fields $\boldsymbol{\mathcal{E}}(\mathbf{r}, t)$, $\boldsymbol{\mathcal{H}}(\mathbf{r}, t)$ are expanded into series in terms of cavity modes:

$$\boldsymbol{\mathcal{E}}(\mathbf{r}, t) = \sum_{n=1}^{\infty} e_n(t)\, \mathbf{E}_n(\mathbf{r}) - \sum_{\alpha=1}^{\infty} a_\alpha(t)\, \nabla\Phi_\alpha(\mathbf{r}), \tag{2}$$

$$\boldsymbol{\mathcal{H}}(\mathbf{r}, t) = \sum_{n=1}^{\infty} h_n(t)\, \mathbf{H}_n(\mathbf{r}) - \sum_{\beta=1}^{\infty} b_\beta(t)\, \nabla\Psi_\beta(\mathbf{r}), \tag{3}$$

The solenoidal cavity modes can be found as solutions to the following boundary eigenvalue problems

$$\begin{cases} \nabla \times \mathbf{H}_n(\mathbf{r}) = -i\omega_n\varepsilon_0\mathbf{E}_n(\mathbf{r}); \\ \nabla \times \mathbf{E}_n(\mathbf{r}) = i\omega_n\mu_0\mathbf{H}_n(\mathbf{r}); \\ \mathbf{n} \times \mathbf{E}_n(\mathbf{r})|_S = 0, \text{ or } \mathbf{n} \cdot \mathbf{H}_n(\mathbf{r})|_S = 0. \end{cases} \tag{4}$$

Irrotational modes occurring in the expansions correspond to transient Coulomb and Ampere fields in the bounded cavity that are closely coupled with charges and currents. They are defined by the following eigenvalue problems

$$\left(\nabla^2+\eta_\alpha^2\right)\Phi_\alpha=0,\ \ \Phi_\alpha|_S=0 \text{ and } \left(\nabla^2+\nu_\alpha^2\right)\Psi_\beta=0,\ \ \left.\frac{\partial}{\partial\mathbf{N}}\Psi_\beta\right|_S = 0. \tag{5}$$

Solution of boundary problems (4) and (5) is a completely separate task that has been solved for a number of canonic geometries and can be solved using numerous known computational techniques (like MoM, FEM, etc.) for any arbitrary geometry. In this study, we assume that this part of the whole problem is already solved by some method, and the results (eigenvalues and eigenfunctions) are known. The main focus is on the time evolution of the fields that is described by the mode amplitudes. Time dependences of the fields are described by the mode amplitudes $e_n(t)$, $h_n(t)$, $a_\alpha(t)$, $b_\beta(t)$. In the same way, one can expand the initial fields as well as the impressed electric and magnetic currents $\boldsymbol{\mathcal{J}}_e(\mathbf{r}, t)$ and $\boldsymbol{\mathcal{J}}_h(\mathbf{r}, t)$

$$\boldsymbol{\mathcal{E}}_0(\mathbf{r}) = \sum_{n=1}^{\infty} e_n^0\mathbf{E}_n(\mathbf{r}) - \sum_{\alpha=1}^{\infty} a_\alpha^0\nabla\Phi_\alpha(\mathbf{r}), \tag{6}$$

$$\boldsymbol{\mathcal{H}}_0(\mathbf{r}) = \sum_{n=1}^{\infty} h_n^0\mathbf{H}_n(\mathbf{r}) - \sum_{\beta=1}^{\infty} b_\beta^0\nabla\Psi_\beta(\mathbf{r}), \tag{7}$$

$$\varepsilon_0^{-1}\boldsymbol{\mathcal{J}}_e(\mathbf{r}, t) = \sum_{n=1}^{\infty} j_n^e(t)\mathbf{E}_n(\mathbf{r}) - \sum_{\alpha=1}^{\infty} j_\alpha^e(t)\nabla\Phi_\alpha(\mathbf{r}), \tag{8}$$

$$\mu_0^{-1}\boldsymbol{\mathcal{J}}_h(\mathbf{r},t) = \sum_{n=1}^{\infty} j_n^h(t)\mathbf{H}_n(\mathbf{r}) - \sum_{\beta=1}^{\infty} j_\beta^h(t)\nabla\Psi_\beta(\mathbf{r}). \quad (9)$$

By substituting expansions (2), (3) and (6)–(9) into Maxwell equations and further applying the orthogonality conditions

$$\frac{\varepsilon_0}{V}\int_V \mathbf{E}_n(\mathbf{r})\cdot\mathbf{E}_m^*(\mathbf{r})dV = \frac{\mu_0}{V}\int_V \mathbf{H}_n(\mathbf{r})\cdot\mathbf{H}_m^*(\mathbf{r})dV = \delta_{nm}, \quad (10)$$

$$-\frac{\varepsilon_0}{V}\int_V \mathbf{E}_n(\mathbf{r})\cdot\nabla\Phi_\alpha^*(\mathbf{r})dV = -\frac{\mu_0}{V}\int_V \mathbf{H}_n(\mathbf{r})\cdot\nabla\Psi_\beta^*(\mathbf{r})dV = 0, \quad (11)$$

one can obtain a second order system of integro-differential equations, namely

$$\frac{d}{dt}e_n(t)+ik_nh_n(t)=-j_n^e(t)-\frac{1}{V}\int_V\left\{\frac{\partial}{\partial t}\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{E}})+\boldsymbol{\mathcal{J}}_\sigma(\boldsymbol{\mathcal{E}},\boldsymbol{\mathcal{H}})\right\}\mathbf{E}_n^*(\mathbf{r})dV, \quad (12)$$

$$\frac{d}{dt}h_n(t)+ik_ne_n(t)=-j_n^h(t)-\frac{\mu_0}{V}\int_V\left\{\frac{\partial}{\partial t}\boldsymbol{\mathcal{M}}(\boldsymbol{\mathcal{H}})\right\}\mathbf{H}_n^*(\mathbf{r})dV, \quad (13)$$

$$\frac{d}{dt}a_\alpha(t)=-j_\alpha^e(t)+\frac{1}{V}\int_V\left\{\frac{\partial}{\partial t}\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{E}})+\boldsymbol{\mathcal{J}}_\sigma(\boldsymbol{\mathcal{E}},\boldsymbol{\mathcal{H}})\right\}\nabla\Phi_\alpha^*(\mathbf{r})dV, \quad (14)$$

$$\frac{d}{dt}b_\beta(t)=-j_\beta^h(t)+\frac{\mu_0}{V}\int_V\left\{\frac{\partial}{\partial t}\boldsymbol{\mathcal{M}}(\boldsymbol{\mathcal{H}})\right\}\nabla\Psi_\alpha^*(\mathbf{r})dV, \quad (15)$$

with initial conditions

$$e_n(0)=\frac{1}{V}\int_V\boldsymbol{\mathcal{E}}_0(\mathbf{r})\mathbf{E}_n^*(\mathbf{r})dv,\ h_n(t)=\frac{1}{V}\int_V\boldsymbol{\mathcal{H}}_0(\mathbf{r})\mathbf{H}_n^*(\mathbf{r})dv, \quad (16)$$

$$a_\alpha(0)=-\frac{1}{V}\int_V\boldsymbol{\mathcal{E}}_0(\mathbf{r})\nabla\Phi_\alpha^*(\mathbf{r})dv,\ b_\beta(0)=-\frac{1}{V}\int_V\boldsymbol{\mathcal{H}}_0(\mathbf{r})\nabla\Psi_\beta^*(\mathbf{r})dv. \quad (17)$$

Mode amplitudes of impressed currents are defined as

$$j_n^e(t)=\frac{1}{V}\int_V\boldsymbol{\mathcal{J}}_e(\mathbf{r},t)\mathbf{E}_n^*(\mathbf{r})dv,\ j_n^h(t)=\frac{1}{V}\int_V\boldsymbol{\mathcal{J}}_m(\mathbf{r},t)\mathbf{H}_n^*(\mathbf{r})dv, \quad (18)$$

$$j_\alpha^e(t)=-\frac{1}{V}\int_V\boldsymbol{\mathcal{J}}_e(\mathbf{r},t)\nabla\Phi_\alpha^*(\mathbf{r})dv,\ j_\beta^h(t)=-\frac{1}{V}\int_V\boldsymbol{\mathcal{J}}_m(\mathbf{r},t)\nabla\Psi_\beta^*(\mathbf{r})dv. \quad (19)$$

Thus, to describe evolution of electromagnetic fields in the cavity with linear, homogeneously filling it is necessary to find solution of equation system (12)–(15) with initial conditions (16), (17) for mode amplitudes of electric and magnetic field strength. This approach is very convenient for nonstationary question, because the separation of temporal and spatial parts of the problem, taking into account an arbitrary time dependence of the fields and dielectric medium properties, is realized. This gives simple analytical solutions in many cases.

Let us define the constitutive relation as follows

$$\boldsymbol{\mathcal{P}}(\mathbf{r},t) = \varepsilon_0\chi_e(t)\boldsymbol{\mathcal{E}}(\mathbf{r},t), \quad \varepsilon(t) = 1+\chi_e(t), \tag{20}$$

$$\boldsymbol{\mathcal{M}}(\mathbf{r},t) = \chi_m(t)\boldsymbol{\mathcal{H}}(\mathbf{r},t), \quad \mu(t) = 1+\chi_m(t), \tag{21}$$

$$\boldsymbol{\mathcal{J}}_\sigma(\mathbf{r},t) = \sigma(t)\boldsymbol{\mathcal{E}}(\mathbf{r},t), \tag{22}$$

By substituting expansions for the electric and magnetic field strength and taking into account orthogonality conditions of the basis vectors, we write evolutionary equations for electromagnetic field mode amplitudes in the resonator filled with arbitrary time-varying medium

$$\frac{d}{dt}\left(\varepsilon(t)e_n(t)\right) + \sigma(t)e_n(t)/\varepsilon_0 + ik_nh_n(t) = -j_n^e(t), \quad e_n(0) = e_n^0, \tag{23}$$

$$\frac{d}{dt}\left(\mu(t)h_n(t)\right) + ik_ne_n(t) = -j_n^h(t), \qquad h_n(0) = h_n^0, \tag{24}$$

$$\frac{d}{dt}\left(\varepsilon(t)a_\alpha(t)\right) + \sigma(t)a_\alpha(t)/\varepsilon_0 = -j_\alpha^e(t), \qquad a_\alpha(0) = a_\alpha^0, \tag{25}$$

$$\frac{d}{dt}\left(\mu(t)b_\beta(t)\right) = -j_\beta^h(t), \qquad b_\beta(0) = b_\beta^0. \tag{26}$$

After specifying time dependence of permittivity, permeability and conductivity of the filling, the temporal electromagnetic process, caused by the presence of time-varying medium, should be analyzed.

3. CAVITY WITH TIME-VARYING PERMITTIVITY

For presenting investigation of the electromagnetic fields in the resonator, filled with the time-varying permittivity medium (but permeability and conductivity of the medium are constant), let us write the permittivity in the following general form

$$\varepsilon(t) = \frac{\varepsilon_s}{1-\gamma\eta(t)}, \quad |\gamma| < 1, \tag{27}$$

where γ describes the deviation of the permittivity from the stationary value ε_s, and function $\eta(t)$ defines the time dependence of the permittivity.

After substituting these medium parameters, the system of evolutionary Equations (23)–(26) is rewritten as

$$\frac{d}{dt}x_n(t) + 2\rho x_n(t) + i\frac{k_n}{\mu}y_n(t) = -j_n^e(t) + 2\rho\gamma\eta(t)x_n(t),$$
$$x_n(0) = \frac{\varepsilon_s}{1-\gamma\eta(0)}e_n^0 \tag{28}$$

$$\frac{d}{dt}y_n(t) + i\frac{k_n}{\varepsilon_s}x_n(t) = -j_n^h(t) + i\frac{k_n\gamma}{\varepsilon_s}\eta(t)x_n(t), \quad y_n(0) = \mu h_n^0 \tag{29}$$

$$\frac{d}{dt}z_\alpha(t) + 2\rho\left(1-\gamma\eta(t)\right)z_\alpha(t) = -j_\alpha^e(t), \quad z_\alpha(0) = \frac{\varepsilon_s}{1-\gamma\eta(0)}a_\alpha^0 \tag{30}$$

$$\frac{d}{dt}\mu b_\beta(t) = -j_\beta^h(t), \quad b_\beta(0) = b_\beta^0 \tag{31}$$

where $\rho = \sigma/2\varepsilon_s\varepsilon_0$, and $x_n(t) = \varepsilon(t)e_n(t)$, $z_\alpha(t) = \varepsilon(t)a_\alpha(t)$, $y_n(t) = \mu h_n(t)$; e_n^0, h_n^0, a_α^0, b_β^0 are initial conditions of the mode amplitudes.

The equations for irrotational mode amplitudes (30), (31) are the first order linear ordinary differential equations with variable coefficients. Their solution can be found by variable separation and direct integration; the result can be presented in the following form:

$$z_\alpha(t) = e^{-2\rho\int_0^t(1-\gamma\eta(t'))dt'}\left[\frac{\varepsilon_s}{1-\gamma\eta(0)}a_\alpha^0 - \int_0^t j_\alpha^e(t')e^{2\rho\int_0^{t'}(1-\gamma\eta(t''))dt''}dt'\right] \tag{32}$$

$$b_\beta(t) = b_\beta^0 - \frac{1}{\mu}\int_0^t j_\beta^h(t')dt'. \tag{33}$$

The system of evolutionary equations for solenoidal electromagnetic mode amplitudes can be written in a matrix form as follows:

$$\frac{d}{dt}\mathbf{X}(t) + \mathbf{Q}_0\cdot\mathbf{X}(t) = \mathbf{RHS}\left(t,\mathbf{X}\right), \qquad \mathbf{X}(0) = \mathbf{X}_0 \tag{34}$$

where

$$\mathbf{RHS}\left(t,\mathbf{X}\right) = \mathbf{F}(t) + \gamma\eta(t)\mathbf{Q}_1\cdot\mathbf{X}(t), \quad \mathbf{X}(t) = \begin{pmatrix} x_n(t) \\ y_n(t) \end{pmatrix},$$
$$\mathbf{F}(t) = \begin{pmatrix} -j_n^e(t) \\ -j_n^h(t) \end{pmatrix}, \qquad \mathbf{X}_0 = \begin{pmatrix} \varepsilon_s e_n^0 \\ \mu h_n^0 \end{pmatrix},$$
$$\mathbf{Q}_0 = \begin{pmatrix} 2\rho & i\frac{k_n}{\mu} \\ i\frac{k_n}{\varepsilon_s} & 0 \end{pmatrix}, \qquad \mathbf{Q}_1 = \begin{pmatrix} 2\rho & 0 \\ i\frac{k_n}{\varepsilon_s} & 0 \end{pmatrix}.$$

This system of ordinary differential equations with constant coefficients can be solved in a closed form [14]. Solenoidal mode amplitudes in new designations $x_n(t)$, $y_n(t)$ are obtained as

$$x_n(t) = r_e(t) + \frac{k_n\gamma}{\sqrt{\varepsilon_s\mu}} \int\limits_0^t e^{-\rho(t-t')}\eta(t')\sin\omega_n(t-t')x_n(t')dt', \qquad (35)$$

$$y_n(t) = r_h(t) + i\frac{k_n\gamma}{\varepsilon_s} \int\limits_0^t e^{-\rho(t-t')}\eta(t')\cos\omega_n(t-t')x_n(t')dt', \qquad (36)$$

where $\omega_n = \sqrt{\frac{k_n^2}{\varepsilon_s\mu} - \rho^2}$ is the eigenfrequency of the cavity filled with medium that permittivity is ε_s, and the slowness of changing permittivity with time is

$$\frac{\rho}{k_n},\ \frac{\rho}{\omega_n} \ll 1. \qquad (37)$$

Electric and magnetic auxiliary function, $r_e(t)$ and $r_h(t)$, are free functions of integral equation system (35), (36) and equal to

$$r_e(t) = e^{-\rho t}\left(x_n^0\cos\omega_n t - i\sqrt{\frac{\varepsilon_e}{\mu}}y_n^0\sin\omega_n t\right)$$
$$-\int\limits_0^t e^{-\rho(t-t')}\left(j_n^e(t')\cos\omega_n(t-t') - i\sqrt{\frac{\varepsilon_s}{\mu}}j_n^h(t')\sin\omega_n(t-t')\right)dt', \qquad (38)$$

$$r_h(t) = e^{-\rho t}\left(y_n^0\cos\omega_n t - i\sqrt{\frac{\mu}{\varepsilon_e}}x_n^0\sin\omega_n t\right)$$
$$-\int\limits_0^t e^{-\rho(t-t')}\left(j_n^h(t')\cos\omega_n(t-t') - i\sqrt{\frac{\mu}{\varepsilon_s}}j_n^e(t')\sin\omega_n(t-t')\right)dt'. \qquad (39)$$

These functions define behavior of the mode amplitudes (in new designations) of electric and magnetic fields in the cavity, filled with the medium that permittivity is ε_s.

3.1. Double Exponential Pulse Change of Permittivity

Let us consider the evolution of electromagnetic fields in the cavity with double exponential pulse medium permittivity, written as follows

$$\eta(t) = \left(e^{-\alpha t} - e^{-\beta t}\right),\ \Rightarrow\ \varepsilon(t) = \varepsilon_{\exp 2}(t) = \frac{\varepsilon_s}{1-\gamma(e^{-\alpha t} - e^{-\beta t})},\ \alpha < \beta. \qquad (40)$$

Before investigating of solenoidal mode amplitudes, we write the irrotational mode amplitude of electric field (32), taking into account the given time dependence of permittivity:

$$\begin{aligned} z_\alpha(t) &= e^{-2\rho t} \exp\left[-2\rho\gamma\left(\frac{e^{-\alpha t}}{\alpha} - \frac{e^{-\beta t}}{\beta}\right)\right] \\ &\quad \times \left\{\varepsilon_e a_\alpha^0 - \int_0^t j_\alpha^e(t') \exp\left(2\rho\left[t' + \gamma\left(\frac{e^{-\alpha t'}}{\alpha} - \frac{e^{-\beta t'}}{\beta}\right)\right]\right) dt'\right\}. \end{aligned} \tag{41}$$

The magnetic irrotational mode amplitude is defined in (33). In the absence of field sources, the irrotational part of magnetic field exists as static field, but the irrotational part of electric field decreases by exponent function from its initial value to zero.

Taking into account the permittivity (40) for solenoidal part of electromagnetic field the resolvent of integral Equation (35) is obtained as

$$\begin{aligned} R_{\exp 2}(t, t') &= \frac{k_n \gamma}{\sqrt{\mu\varepsilon_s}} e^{-\rho(t-t')} \left(e^{-\alpha t'} - e^{-\beta t'}\right) \\ &\quad \times \sin\left(\omega_n(t-t') + \frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}}\left(\frac{e^{-\alpha t}}{\alpha} - \frac{e^{-\beta t}}{\beta} - \frac{e^{-\alpha t'}}{\alpha} + \frac{e^{-\beta t'}}{\beta}\right)\right), \end{aligned} \tag{42}$$

with the following convergence condition

$$\frac{k_n\gamma(\beta-\alpha)}{2\alpha\beta\sqrt{\mu\varepsilon_e}} \leq 1. \tag{43}$$

Thus, sought amplitudes should be written in quadratures

$$x_n(t) = r_e(t) + \int_0^t R_{\exp 2}(t, t') r_e(t') dt', \tag{44}$$

$$\begin{aligned} y_n(t) &= r_h(t) + i\frac{k_n}{\varepsilon_s} \int_0^t e^{-\rho(t-t')} \left(e^{-\alpha t'} - e^{-\beta t'}\right) \cos\omega_n(t-t') \\ &\quad \times \left\{r_e(t) + \int_0^t R_{\exp 2}(t, t') r_e(t') dt'\right\} dt', \end{aligned} \tag{45}$$

functions $r_e(t)$ and $r_h(t)$ are described in (38), (39).

We consider two cases of excitation currents. One of them has mode amplitudes in form of prompt pulse; the other has mode amplitudes which are a harmonic signal. We write prompt pulse mode amplitudes as

$$j_n^e(t) = A_n \delta(t - t_0), \;\; j_\alpha^e(t) = A_\alpha \delta(t - t_0), \;\; j_n^h(t) = 0, \;\; j_\beta^h(t) = 0. \quad (46)$$

With these mode amplitudes of excitation currents electric and magnetic auxiliary functions (35) become

$$\begin{aligned} r_e(t) &= -A_n e^{-\rho(t-t_0)} \cos\omega_n(t - t_0) \\ r_h(t) &= iA_n \sqrt{\frac{\mu}{\varepsilon_s}} e^{-\rho(t-t_0)} \sin\omega_n(t - t_0), \end{aligned}$$

and the system (44), (45) is transformed to

$$\begin{aligned} x_n(t) &= -A_n e^{-\rho(t-t_0)} \cos\omega_n(t - t_0) \\ &\quad -A_n \int_0^t R_{\exp 2}(t, t') e^{-\rho(t'-t_0)} \cos\omega_n(t' - t_0) dt', \end{aligned} \quad (47)$$

$$\begin{aligned} y_n(t) &= iA_n \sqrt{\frac{\mu}{\varepsilon_s}} e^{-\rho(t-t_0)} \sin\omega_n(t - t_0) \\ &\quad + i\frac{k_n}{\varepsilon_s} \int_0^t e^{-\rho(t-t')} \left(e^{-\alpha t'} - e^{-\beta t'}\right) \cos\omega_n(t - t') x_n(t') dt'. \end{aligned} \quad (48)$$

So, mode amplitudes of electromagnetic field in the cavity, excited by

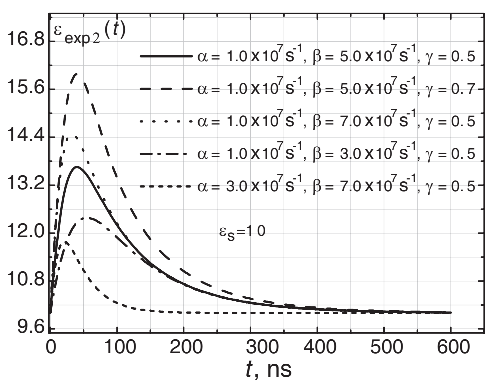

Figure 2. Permittivity time dependence.

currents with mode amplitudes (46), are

$$
\begin{aligned}
e_n(t) &= -\frac{A_n}{\varepsilon_s}e^{-\rho(t-t_0)}\left(1-\gamma\left(e^{-\alpha t}-e^{-\beta t}\right)\right)\\
&\quad\times\cos\left(\omega_n(t-t_0)-\frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}}\left(\frac{1-e^{-\alpha t}}{\alpha}-\frac{1-e^{-\beta t}}{\beta}\right)\right) \qquad (49)\\
h_n(t) &= i\frac{A_n}{\sqrt{\mu\varepsilon_s}}e^{-\rho(t-t_0)}\left(1-\gamma\left(e^{-\alpha t}-e^{-\beta t}\right)\right)\\
&\quad\times\sin\left(\omega_n\left(t-t_0\right)-\frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}}\left(\frac{1-e^{-\alpha t}}{\alpha}-\frac{1-e^{-\beta t}}{\beta}\right)\right), \qquad (50)\\
z_\alpha(t) &= -\frac{A_\alpha}{\varepsilon_{\exp 2}\left(t\right)}\exp\left[-2\rho\left(t+\gamma\left(\frac{e^{-\alpha t}}{\alpha}-\frac{e^{-\beta t}}{\beta}\right)\right)\right]\\
&\quad\times\exp\left(2\rho\left(t_0+\gamma\left(\frac{e^{-\alpha t_0}}{\alpha}-\frac{e^{-\beta t_0}}{\beta}\right)\right)\right), \qquad (51)\\
b_\beta\left(t\right) &= 0. \qquad (52)
\end{aligned}
$$

In addition, these analytical expressions correspond to free oscillations in the cavity with initial conditions $x_n^0 = -A$, $y_n^0 = 0$, at the moment $t = t_0$. Thus, inherent solenoidal field in the resonator filled with double exponent pulse medium permittivity (40) decreases by exponent function with the frequency, which change flips time dependence of permittivity (Figure 3).

If mode amplitudes of excitation currents have harmonic time

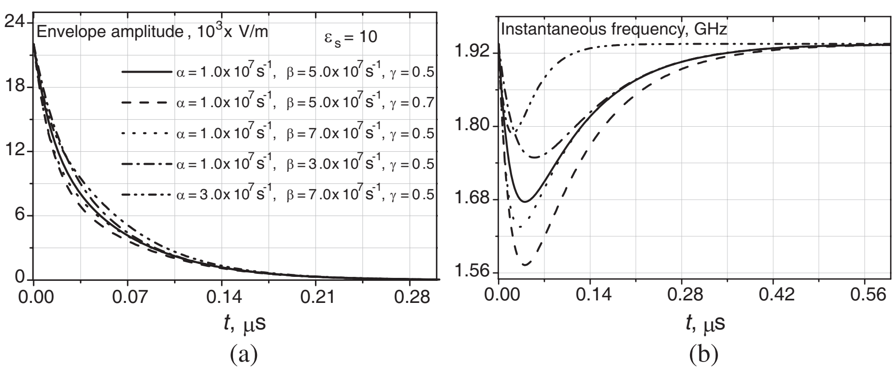

Figure 3. Time dependence of envelope amplitude and instantaneous frequency of $e_n(t)$. $A = 10$, $k_n/2\pi = 10.599\,\text{GHz}$, $t_0 = 0$, $\sigma/2\varepsilon_0\varepsilon_s = 2\times10^7 s^{-1}$.

dependence, such as

$$j_n^e(t) = A_n \cos \Omega_n t, \;\; j_\alpha^e(t) = A_\alpha \cos \Omega_\alpha t, \;\; j_n^h(t) = 0, \;\; j_\beta^h(t) = 0 \quad (53)$$

the integral Equation (35) is modified to

$$x_n(t) = \frac{A_n \left(\sin\left(\Omega_n t - \xi\right) - e^{-\rho t} \sin\left(\omega_n t - \xi\right)\right)}{2\sqrt{\rho^2 + \Delta\omega^2}} + \int_0^t R_{\exp 2}(t,t') \frac{A_n \left(\sin\left(\Omega_n t' - \xi\right) - e^{-\rho t'} \sin(\omega_n t' - \xi)\right)}{2\sqrt{\rho^2 + \Delta\omega^2}} dt', \quad (54)$$

where $\Delta\omega = \omega_n - \Omega_n$ and $\xi = \arctan\left(\rho/\Delta\omega\right)$. Therefore, electromagnetic field mode amplitudes in quadratures are obtained as

$$e_n(t) = \frac{1}{\varepsilon_{\exp 2}(t)} \left\{ r_e(t) + \int_0^t R_{\exp 2}(t,t') r_e(t') dt' \right\}, \quad (55)$$

$$h_n(t) = r_h(t) + i \frac{k_n}{\mu \varepsilon_s} \int_0^t e^{-\rho(t-t')} \left(e^{-\alpha t'} - e^{-\beta t'}\right) \cos \omega_n (t - t') \times \left\{ r_e(t') + \int_0^{t'} R_{\exp 2}(t', t'') r_e(t'') dt'' \right\} dt', \quad (56)$$

$$a_\alpha(t) = \frac{1}{\varepsilon_{\exp 1}(t)} e^{-2\rho t} \exp\left(-2\rho\gamma T e^{-t/T}\right) \times \left\{ \varepsilon_s a_\alpha^0 - \int_0^t A_\alpha^e \exp\left[2\rho\left(t' + \gamma T e^{-t'/T}\right)\right] \cos \Omega_\alpha t' dt' \right\}, \quad (57)$$

with auxiliary functions

$$r_e(t) = \frac{A_n}{2\sqrt{\rho^2 + \Delta\omega^2}} \left(\sin(\Omega_n t - \xi) - e^{-\rho t} \sin(\omega_n t - \xi)\right) \quad (58)$$

$$r_h(t) = i \frac{A_n^e}{2\sqrt{\Delta\omega^2 + \rho^2}} \sqrt{\frac{\mu}{\varepsilon_s}} \left(\cos(\Omega_n t - \xi) - e^{-\rho t} \cos(\omega_n t - \xi)\right) \quad (59)$$

Irrotational mode amplitude of magnetic field strength is zero, as in previous case. Figure 4 presents solenoidal mode amplitude of electric field strength with numerically calculated quadratures of (55). As expected, we obtain complicated amplitude-phase modulation agreed by permittivity time dependence, difference of

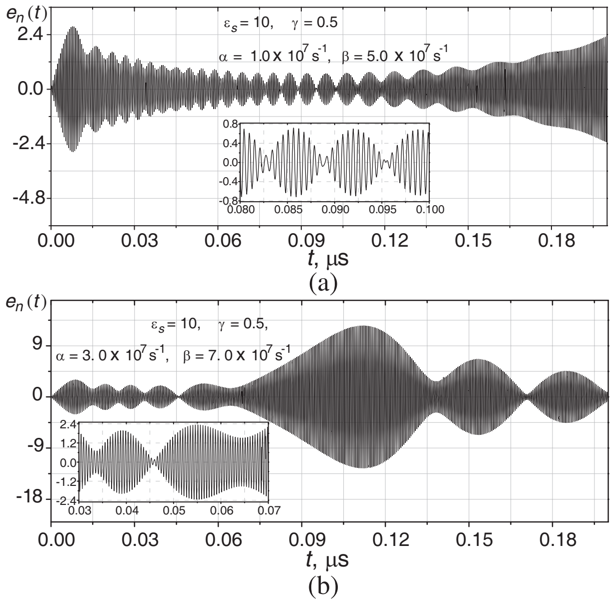

Figure 4. Solenoidal mode amplitude of electric field strength, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7\, s^{-1}$.

steady eigenfrequency and external signal frequency $\Delta\omega$ and difference of unsteady eigenfrequency and external signal frequency. At the time interval presented on Figure 4, the change of the oscillation amplitude is made under influence of function $\varepsilon_{\exp 2}(t)$; the change of the oscillation frequency is made under influence of difference of time-dependent eigenfrequency, changing with $\varepsilon_{\exp 2}(t)$, and frequency of mode amplitudes of excitation currents.

3.2. Periodic Permittivity Alternating

Let us investigate the behavior electromagnetic fields in the cavity that filling has the following periodic time dependence

$$\eta(t) = \sin(\omega_\varepsilon t), \ \Rightarrow \ \varepsilon(t) = \varepsilon_{\sin}(t) = \varepsilon_s / \left(1 - \gamma \sin(\omega_\varepsilon t)\right). \qquad (60)$$

The irrotational mode amplitude of electric field strength (32) yields

$$\begin{aligned} z_\alpha(t) \ = \ & \exp\left[-2\rho\left(t + \frac{\gamma}{\omega_\varepsilon}\cos\omega_\varepsilon t\right)\right] \\ & \times \left\{\varepsilon_e a_\alpha^0 - \int_0^t j_\alpha^e(t') \exp\left(2\rho\left(t' + \frac{\gamma}{\omega_\varepsilon}\cos\omega_\varepsilon t\right)\right) dt'\right\}, \qquad (61) \end{aligned}$$

The magnetic irrotational mode amplitude is defined in (33). In the absence of field sources, the irrotational part of magnetic field exists as static field, but the irrotational part of electric field has the appearance of oscillations with frequency ω_ε relative to the damped exponential curve, conditioned by losses in the dielectric.

Taking into account the permittivity time dependence (60), the resolvent of integral Equation (35), determining solenoidal mode amplitudes, is obtained as

$$\begin{aligned} R_{\sin}(t,t') &= \frac{k_n\gamma}{\sqrt{\mu\varepsilon_s}} e^{-\rho(t-t')} \sin\omega_\varepsilon t' \\ &\times \sin\left(\omega_n(t-t') + \frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}} \frac{\cos\omega_\varepsilon t - \cos\omega_\varepsilon t'}{\omega_\varepsilon}\right), \end{aligned} \quad (62)$$

with the following convergence condition

$$k_n\gamma/2\omega_\varepsilon\sqrt{\mu\varepsilon_s} \leq 1. \quad (63)$$

The expressions for finding the required functions should be written in quadratures

$$x_n(t) = r_e(t) + \int_0^t R_{\sin}(t,t') r_e(t') dt' \quad (64)$$

$$\begin{aligned} y_n(t) &= r_h(t) + i\frac{k_n}{\varepsilon_s} \int_0^t e^{-\rho(t-t')} \sin(\omega_\varepsilon t') \cos\omega_n(t-t') \\ &\times \left\{ r_e(t') + \int_0^{t'} R_{\sin}(t',t'') r_e(t'') dt'' \right\} dt' \end{aligned} \quad (65)$$

where electric $r_e(t)$ and magnetic $r_h(t)$ auxiliary functions are defined in (38) and (39) respectively.

Let us consider free oscillations in the resonator, filled with such medium. Free oscillations in the cavity give an idea of oscillation process change introduced by exactly nonstationary dielectric. For this, the mode amplitudes are easily found analytically

$$e_n(t) = e_n^0 e^{-\rho t} \left(1 - \gamma\sin(\omega_\varepsilon t)\right) \cos\left(\omega_n t + \frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}} \frac{\cos\omega_\varepsilon t}{\omega_\varepsilon}\right), \quad (66)$$

$$\begin{aligned} h_n(t) &= e_n^0 e^{-\rho t} \Big\{ -i\sqrt{\frac{\varepsilon_s}{\mu}} \sin\omega_n t \\ &+ i\frac{k_n\gamma}{\mu} \int_0^t \sin(\omega_\varepsilon t') \cos\omega_n(t-t') \cos\left(\omega_n t' + \frac{k_n\gamma}{2\sqrt{\mu\varepsilon_s}} \frac{\cos\omega_\varepsilon t'}{\omega_\varepsilon}\right) dt' \Big\}, \end{aligned} \quad (67)$$

$$a_\alpha(t) = a_\alpha^0 \left(1 - \gamma \sin \omega_\varepsilon t\right) \exp\left[-2\rho\left(t + \frac{\gamma}{\omega_\varepsilon}\cos\omega_\varepsilon t\right)\right]. \tag{68}$$

$$b_\beta(t) = b_\beta^0. \tag{69}$$

Thus, free electromagnetic field in the cavity filled with periodic medium permittivity (60) has amplitude-frequency modulation of oscillations; the modulation frequency is equal to ω_ε; the deviation from steady-state level is determined by parameter γ.

For investigation of the forced oscillations, the initial conditions of the electromagnetic field are given zero, and the external currents have the following harmonic mode amplitudes

$$j_n^e(t) = A_n \cos\Omega_n t,\ \ j_\alpha^e(t) = A_\alpha \cos\Omega_\alpha t,\ \ j_n^h(t) = 0,\ \ j_\beta^h(t) = 0. \tag{70}$$

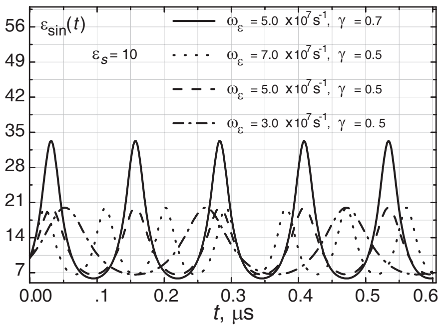

Figure 5. Permittivity time dependence.

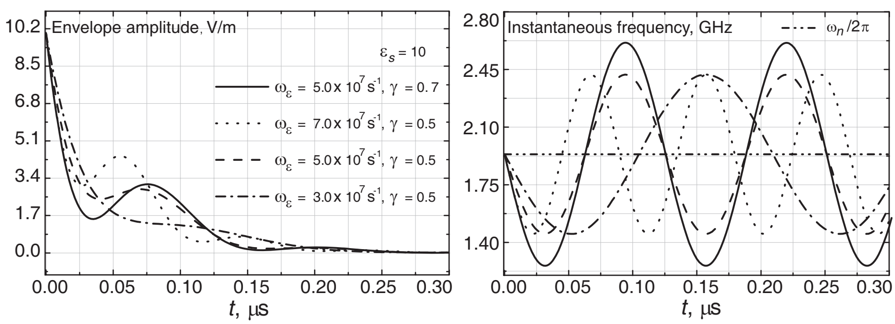

Figure 6. Time dependence of envelope amplitude and instantaneous frequency of $e_n(t)$. $e_n^0 = 10$, $k_n/2\pi = 10.599\,\text{GHz}$, $\omega_n/2\pi = 1.935\,\text{GHz}$, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7 s^{-1}$.

The integral Equation (35) is transformed to

$$x_n(t) = \frac{A_n \left(\sin\left(\Omega t - \xi\right) - e^{-\rho t} \sin\left(\omega_n t - \xi\right)\right)}{2\sqrt{\rho^2 + \Delta\omega^2}} + \int_0^t R_{\sin}(t, t') \frac{A_n \left(\sin\left(\Omega t' - \xi\right) - e^{-\rho t'} \sin\left(\omega_n t' - \xi\right)\right)}{2\sqrt{\rho^2 + \Delta\omega^2}} dt'. \quad (71)$$

where $\Delta\omega = \omega_n - \Omega_n$ and $\xi = \arctan \frac{\rho}{\Delta\omega}$.

Irrotational mode amplitudes become

$$a_\alpha(t) = \frac{-A_\alpha}{\varepsilon_s} (1 - \gamma \sin \omega_\varepsilon t) \exp\left[-2\rho \left(t + \frac{\gamma}{\omega_\varepsilon} \cos \omega_\varepsilon t\right)\right] \times \int_0^t \cos(\Omega_\alpha t') \exp\left(2\rho \left(t' + \frac{\gamma}{\omega_\varepsilon} \cos \omega_\varepsilon t\right)\right) dt', \quad (72)$$

$$b_\beta(t) = 0. \quad (73)$$

Expressions (71) and (65) are easy-to-use as direct formulas for numeric calculation of time dependence of solenoidal mode amplitudes of electromagnetic field.

Temporal process of solenoidal electric field strength mode amplitude is presented in Figures 7–9. On this graphs one can see the amplitude-frequency modulation of oscillations, which form depends on the behavior of permittivity with time. The periodic amplitude modulation is conditioned by periodic time dependence of dielectric permittivity. The excitation frequency is chosen equal to eigenfrequency of the resonator, filled with the medium that permittivity has steady value ε_s, but eigenfrequency of the cavity with time-varying medium fluctuates with varying of $\varepsilon(t)$. Accordingly, the frequency modulation is determined by the difference of these frequencies.

During the previous consideration of time-varying permittivity the velocity of establishment of oscillation process was defined by losses in the medium and exponential varying of dielectric parameters. In this case, the velocity of establishment of field frequency is determined only by medium losses. The periodic variation of the oscillation amplitude has constant character, because it is supported by undamped periodic change of ε.

Figure 10 presents time dependence of envelope amplitude and instantaneous frequency of solenoidal mode amplitude of electric field strength, corresponding to oscillating process in Figure 8.

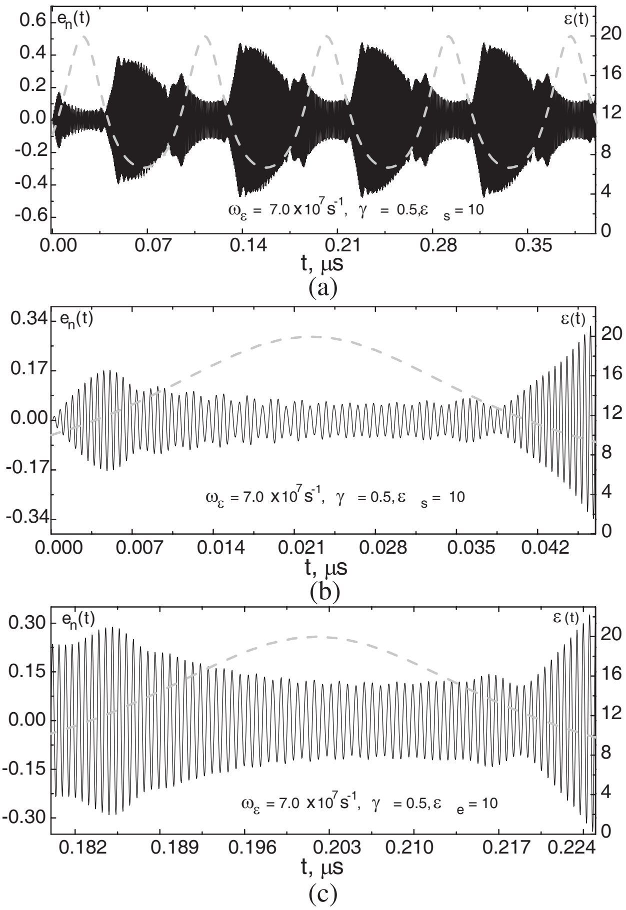

Figure 7. Solenoidal mode amplitude of electric field strength (solid line), permittivity (dotted line). $A = 10^9$, $k_n/2\pi = 10.599\,\mathrm{GHz}$, $\omega_n/2\pi = 1.935\,\mathrm{GHz}$, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7 s^{-1}$.

4. CAVITY WITH TIME-VARYING CONDUCTIVITY

In this part of the paper we investigate the properties of electromagnetic fields in the cavity, filled with medium that conductivity depends upon time as

$$\sigma(t) = \sigma_0 \left(1 - e^{-t/T}\right). \tag{74}$$

Permittivity and permeability of the medium are supposed to be invariable, that is $\varepsilon(t) \equiv \varepsilon$, $\mu(t) \equiv \mu$.

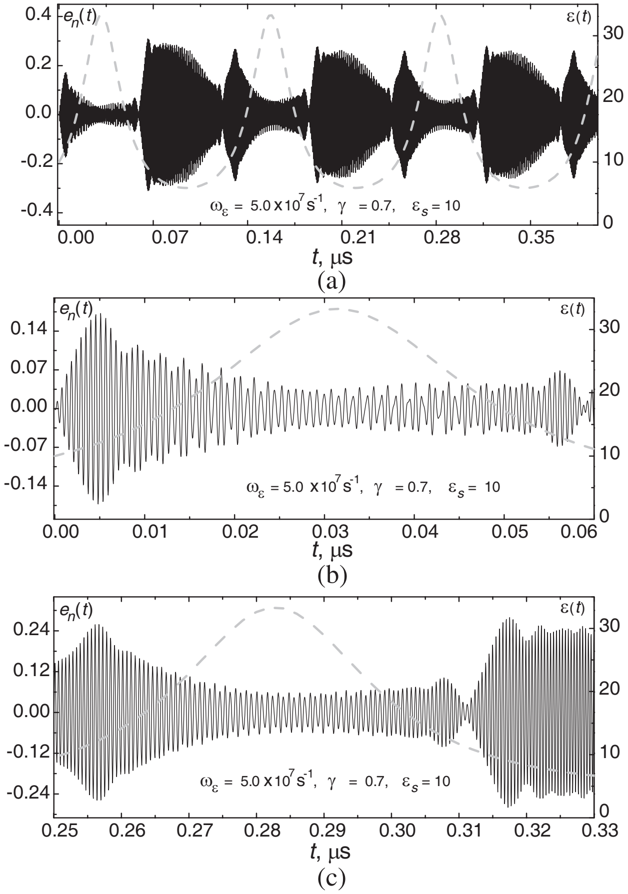

Figure 8. Solenoidal mode amplitude of electric field strength (solid line), permittivity (dotted line). $A = 10^9$, $k_n/2\pi = 10.599\,\mathrm{GHz}$, $\omega_n/2\pi = 1.935\,\mathrm{GHz}$, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7 s^{-1}$.

The system of evolutionary Equations (23)–(26) is rewritten to

$$\frac{d}{dt}x_n(t) + 2\rho x_n(t) + i\frac{k_n}{\mu}y_n(t) = -j_n^e(t) + 2\rho\eta(t)x_n(t); \tag{75}$$

$$\frac{d}{dt}y_n(t) + i\frac{k_n}{\varepsilon}x_n(t) = -j_n^h(t); \tag{76}$$

$$\frac{d}{dt}z(t) + 2\rho(1-\eta(t))z(t) = -j_\alpha^e(t); \tag{77}$$

$$\frac{d}{dt}\left(\mu b_\beta(t)\right) = -j_\beta^h(t). \tag{78}$$

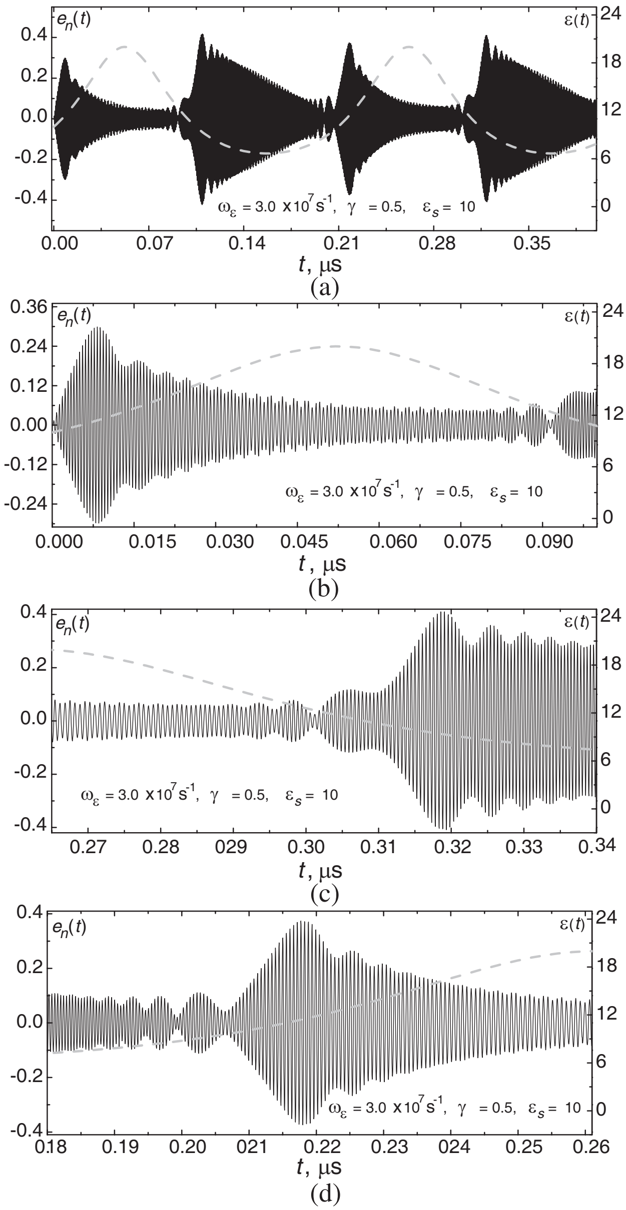

Figure 9. Solenoidal mode amplitude of electric field strength (solid line), permittivity (dotted line). $A = 10^9$, $k_n/2\pi = 10.599\,\text{GHz}$, $\omega_n/2\pi = 1.935\,\text{GHz}$, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7 s^{-1}$.

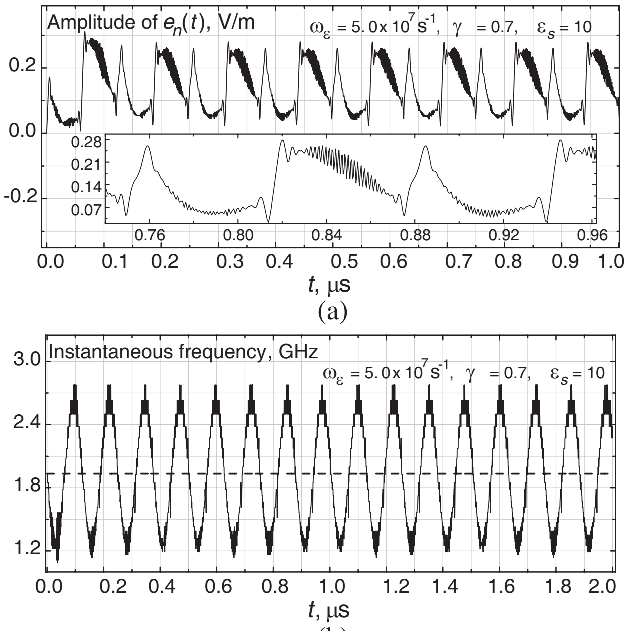

Figure 10. (a) Envelope amplitude and (b) instantaneous frequency of solenoidal mode amplitude of electric field strength. $A = 10^9$, $k_n/2\pi = 10.599\,\text{GHz}$, $\omega_n/2\pi = 1.935\,\text{GHz}$, $\sigma/2\varepsilon_0\varepsilon_s = 2 \times 10^7 s^{-1}$.

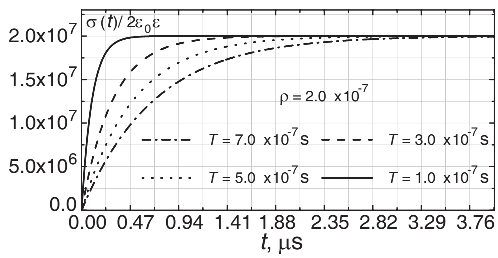

Figure 11. Time dependence of medium conductivity.

for new convenient sought and given function and values $x_n(t) = \varepsilon e_n(t)$, $y_n(t) = \mu h_n(t)$, $z(t) = \varepsilon a_\alpha(t)$, $\eta(t) = e^{-t/T}$, $\rho = \sigma_0/2\varepsilon_0\varepsilon_e$. The initial conditions of these new designations are

$$x_n(0) = \varepsilon e_n^0 = x_n^0, \quad y_n(0) = \mu h_n^0 = y_n^0, \tag{79}$$

$$z_\alpha(0) = \varepsilon a_\alpha^0 = z_\alpha^0, \quad b_\beta(0) = b_\beta^0. \tag{80}$$

Firstly, we write the irrotational mode amplitudes of electromagnetic field. Similar to (30), (31), Equations (77), (78) are the first order linear ordinary differential equation with variable coefficients, and their solution can be found by separating variables and directly integrating as:

$$z_\alpha(t)=\exp\left[-2\rho\left(t+e^{-t/T}\right)\right]\left(\varepsilon a_\alpha^0-\int_0^t j_\alpha^e(t')\exp\left[2\rho\left(t+e^{-t'/T}\right)\right]dt'\right). \quad (81)$$

$$b_\beta(t)=-\frac{1}{\mu}\int_0^t j_\beta^h(t')\,dt'+b_\beta^0, \quad (82)$$

Thus, in the absence of excitation the irrotational part of magnetic field exists as static field, but the irrotational part of electric field decreases from its initial value to zero by exponent.

The solenoidal problem should be formulated in matrix form

$$\frac{d}{dt}\mathbf{X}(t)+\mathbf{Q}_0\cdot\mathbf{X}(t)=\mathbf{RHS}(t,\mathbf{X}),\quad \mathbf{X}(0)=\mathbf{X}_0 \quad (83)$$

where

$$\begin{aligned}\mathbf{RHS}(t,\mathbf{X}) &= \mathbf{F}(t)+\eta(t)\mathbf{Q}_1\cdot\mathbf{X}(t), & \mathbf{X}(t) &= \begin{pmatrix} x_n(t) \\ y_n(t)\end{pmatrix},\\ \mathbf{F}(t) &= \begin{pmatrix} -j_n^e(t) \\ -j_n^h(t)\end{pmatrix}, & \mathbf{X}_0 &= \begin{pmatrix} \varepsilon e_n^0 \\ \mu h_n^0\end{pmatrix},\\ \mathbf{Q}_0 &= \begin{pmatrix} 2\rho & i\frac{k_n}{\mu} \\ i\frac{k_n}{\varepsilon} & 0\end{pmatrix}, & \mathbf{Q}_1 &= \begin{pmatrix} 2\rho & 0 \\ 0 & 0\end{pmatrix}\end{aligned}$$

The solution of the matrix ordinary differential Equation (83) with constant coefficients is

$$\mathbf{X}(t)=\mathbf{X}_0e^{-t\mathbf{Q}_0}+\int_0^t e^{-(t-t')\mathbf{Q}_0}\mathbf{RHS}(t')dt'. \quad (84)$$

The function $e^{-t\mathbf{Q}_0}$ may be calculated as presented in [14], and integral equations are obtained

$$\begin{aligned}x_n(t)=&\,e^{-\rho t}\left(x_n^0\cos\omega_n t-i\sqrt{\frac{\varepsilon}{\mu}}y_n^0\sin\omega_n t\right)\\ &-\int_0^t e^{-\rho(t-t')}\left(j_n^e(t')\cos\omega_n(t-t')-i\sqrt{\frac{\varepsilon}{\mu}}j_n^h(t')\sin\omega_n(t-t')\right)dt'\end{aligned}$$

$$+2\rho \int_0^t e^{-\rho(t-t')}\eta(t')\cos\omega_n(t-t')x_n(t')dt' \tag{85}$$

$$y_n(t) = e^{-\rho t}\left(y_n^0\cos\omega_n t - i\sqrt{\frac{\mu}{\varepsilon}}x_n^0\sin\omega_n t\right)$$
$$-\int_0^t e^{-\rho(t-t')}\left(j_n^h(t')\cos\omega_n(t-t') - i\sqrt{\frac{\mu}{\varepsilon}}j_n^e(t')\sin\omega_n(t-t')\right)dt'$$
$$-2\rho\, i\sqrt{\frac{\mu}{\varepsilon}}\int_0^t e^{-\rho(t-t')}\eta(t')\sin\omega_n(t-t')x_n(t')dt' \tag{86}$$

where $\omega_n = \sqrt{\frac{k_n^2}{\varepsilon\mu} - \rho^2}$ and the following convergence condition is used

$$\frac{\rho}{k_n},\ \frac{\rho}{\omega_n} \ll 1. \tag{87}$$

The resolvent of integral Equation (85) is found as

$$R_\sigma(t,t') = 2\rho e^{-\rho(t-t')}e^{-t'/T}\cos\omega_n(t-t') \times \exp\left[-\rho T\left(e^{-t/T} - e^{-t'/T}\right)\right], \tag{88}$$

and expressions (85), (86) are transformed to

$$x_n(t) = r_e(t) + \int_0^t R_\sigma(t,t')r_e(t')dt',$$
$$r_e(t) = e^{-\rho t}\left(x_n^0\cos\omega_n t - i\sqrt{\frac{\varepsilon}{\mu}}y_n^0\sin\omega_n t\right) \tag{89}$$
$$-\int_0^t e^{-\rho(t-t')}\left(j_n^e(t')\cos\omega_n(t-t') - i\sqrt{\frac{\varepsilon}{\mu}}j_n^h(t')\sin\omega_n(t-t')\right)dt'$$

$$y_n(t) = r_h(t) - 2\rho\cdot i\sqrt{\frac{\mu}{\varepsilon}}\int_0^t e^{-\rho(t-t')}\eta(t')\sin\omega_n(t-t')$$
$$\times\left\{r_e(t') + \int_0^{t'} R_\sigma(t',t'')r_e(t'')dt''\right\}dt'$$

$$r_h(t) = e^{-\rho t}\left(y_n^0 \cos\omega_n t - i\sqrt{\frac{\mu}{\varepsilon}}x_n^0 \sin\omega_n t\right) - \int_0^t e^{-\rho(t-t')}\left(j_n^h(t')\cos\omega_n(t-t') - i\sqrt{\frac{\mu}{\varepsilon}}j_n^e(t')\sin\omega_n(t-t')\right)dt' \tag{90}$$

It is obvious that the electric and magnetic auxiliary functions $r_e(t)$ and $r_h(t)$ define behavior of the mode amplitudes of electric and magnetic fields in the cavity, filled with medium that conductivity has constant value σ_0.

Let us consider free oscillations of electromagnetic fields in a cavity with time-varying conductivity medium. With $h_n^0 = 0$, $b_\beta^0 = 0$ the mode amplitudes of electromagnetic field yield

$$e_n(t) = e_n^0 e^{-\rho\left(t-T\left(e^{-t/T}-1\right)\right)}\cos\omega_n t, \tag{91}$$

$$h_n(t) = -ie_n^0\left\{\sqrt{\frac{\varepsilon}{\mu}}e^{-\rho t}\sin\omega_n t + 2\rho e^{-\rho t}\sqrt{\frac{\varepsilon}{\mu}} \int_0^t e^{\rho T(e^{-t'/T}-1)}e^{-t'/T}\cos(\omega_n t')\sin\omega_n(t-t')dt'\right\}. \tag{92}$$

$$a_\alpha(t) = a_\alpha^0 \exp\left[-2\rho\left(t + e^{-t/T}\right)\right], \quad b_\beta(t) = 0 \tag{93}$$

In general case in a cavity filled with transient conductivity medium the eigenfrequency changes in time with the conductivity, but for this specific example (74) we obtain the case of simple amplitude modulation with constant carrier frequency. In Figure 12, light curves correspond to time dependence of oscillation envelope in the cavity filled with medium, which has constant conductivity σ_0. With time-varying conductivity of the filling, the free oscillations decay rapidly.

For forced oscillations the mode amplitudes of external currents are given as

$$j_n^e(t) = A_n \sin\Omega_n t, \;\; j_\alpha^e(t) = A_\alpha \sin\Omega_\alpha t, \;\; j_n^h(t) = 0, \;\; j_n^h(t) = 0, \tag{94}$$

initial conditions are zero. At this (89) is transformed to

$$e_n(t) = \frac{-A_n\left(\cos(\Omega t - \xi) - e^{-\rho t}\cos(\omega_n t - \xi)\right)}{2\varepsilon\sqrt{\rho^2 + \Delta\omega^2}} - A_n\int_0^t R_\sigma(t,t')\frac{\left(\cos(\Omega t' - \xi) - e^{-\rho t'}\cos(\omega_n t' - \xi)\right)}{2\varepsilon\sqrt{\rho^2 + \Delta\omega^2}}dt'. \tag{95}$$

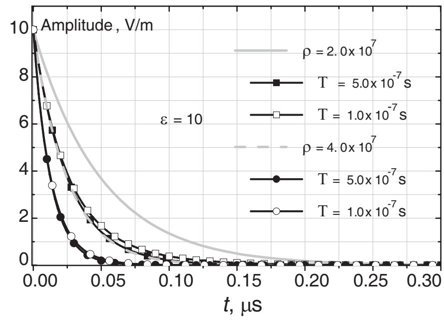

Figure 12. Time-dependence of free oscillations.

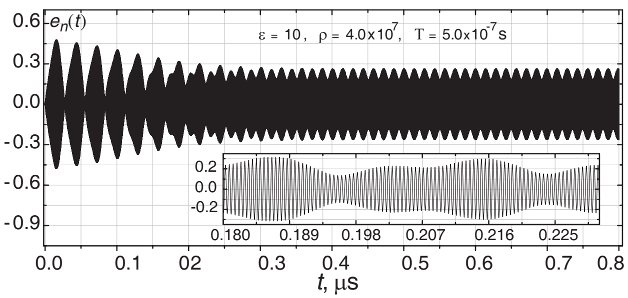

Figure 13. Solenoidal mode amplitude of electric field strength.

where $\Delta\omega = \omega_n - \Omega_n$, $\xi = \arctan \frac{\rho}{\Delta\omega}$.

The irrotational mode amplitudes are as follows

$$a_\alpha(t) = \frac{-A_\alpha}{\varepsilon} \exp\left[-2\rho\left(t + e^{-t/T}\right)\right] \times \int_0^t \sin\left(\Omega_\alpha t'\right) \exp\left[2\rho\left(t + e^{-t'/T}\right)\right] dt', \tag{96}$$

$$b_\beta(t) = 0. \tag{97}$$

The expressions (95) and (90) are easy-to-use as direct formulas for numeric calculation of time dependence of mode amplitudes of electromagnetic field.

Figures 13 and 14 present evolution of solenoidal mode amplitude of electric field strength, excited by harmonic external electric current, for different parameters of conductivity time dependence. As free oscillations in the cavity show, varying of frequency and phase of oscillations does not occur, but alterations to time dependence of oscillations amplitude are introduced. Figure 14 demonstrates

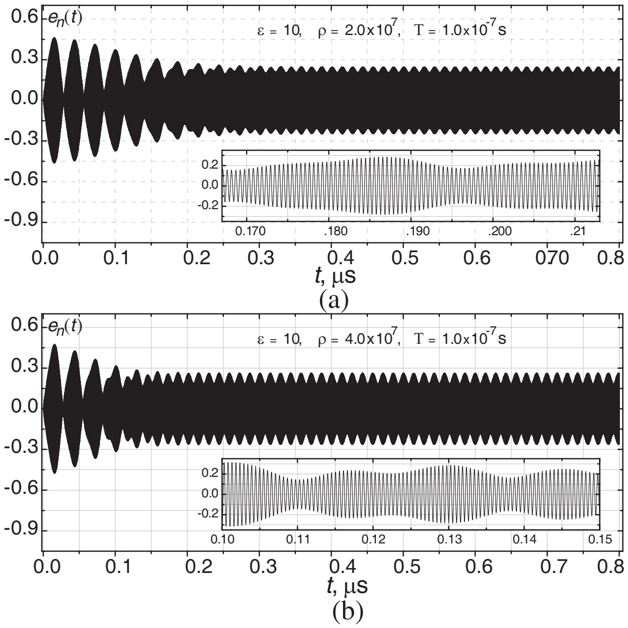

Figure 14. Solenoidal mode amplitude of electric field strength.

oscillations which have the same varying speed of conductivity with time, but different end values ρ.

For big ρ the oscillations steady rapidly. With increase of conductivity varying speed (see Figure 13) the oscillation establishment slows down.

5. CONCLUSION

Transient electromagnetic fields in a cavity filled with time-varying medium have been studied analytically in the time domain. Both time-varying permittivity and conductivity were considered.

For double exponent pulse change of permittivity the amplitude-frequency modulation has temporal character. Rate of forced oscillations steadying in the cavity depends on medium losses and rate of change of ε. Instant frequency change of the transient process is governed by difference of exciting currents frequency and steady and unsteady eigenfrequencies of the filled cavity. For periodic time dependence of permittivity the rate of steadying of wave processes depends on the losses in the medium. Periodic amplitude modulation of electromagnetic oscillations has constant character because it is supported by undamped periodic varying of ε.

A solution has been obtained in a closed-form for mode amplitudes of electromagnetic fields in a cavity filled with medium that has smooth transition of conductivity from some initial to the final value. In this case, the amplitude modulation has temporal character, and frequency change depends on difference between the constant eigenfrequency and the frequency of excitation currents.

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