

## **CLOAKING A PERFECTLY CONDUCTING SPHERE WITH ROTATIONALLY UNIAXIAL NIHILITY MEDIA IN MONOSTATIC RADAR SYSTEM**

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**Abstract**—In this paper, the backscattering properties of a perfect electric conducting sphere coated with layered anisotropic media whose constitutive parameters are close to nihility are investigated. We show that the backscattering is more sensitive to the radial constitutive parameters than to the tangential ones. Compared with isotropic case, the anisotropic media with small axial parameters have the potential to yield more reduction of backscattering magnitude on coated perfectly conducting spheres.

### **1. INTRODUCTION**

Cloaking an object with suitable electromagnetic materials has attracted attention for many years [1, 2, 3, 4, 5, 6]. One of the cloaking techniques utilizes low-positive, near-zero permittivity covers to induce “invisibility”, which has been suggested in the quasistatic (Rayleigh) limit for spherical and cylindrical objects, i.e., the radius of a scatterer being small compared with the operating wavelength [1]. To cloak larger objects comparable to or larger than the operating wavelength, coordinate transformation method has been proposed, which requires

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the cloak to be an anisotropic inhomogeneous media with every permittivity and permeability element independently controlled [2]. Unlike bistatic observation (transmitter and receiver in different locations) from  $0^\circ$  to  $360^\circ$  angle, in a monostatic radar detection (transmitter and receiver in the same location), only the backscattering is the key in hiding objects. Therefore, we can achieve much simpler electromagnetic parameters for the coating media, compared with the cloaking media.

In [7] the scattering properties of a nihility [8, 9, 10] sphere have been studied, and it is discovered that the backscattering efficiency of a nihility sphere is identically zero with a cost that its extinction and forward-scattering are higher than those of a perfectly conducting sphere. Soon the similar phenomena have been reported [11] through a different visual angle and draw our attention to uniaxial anisotropic medium whose axial permittivity and permeability parameter values tend to zero [12].

Since the medium with strict zero permittivity and zero permeability does not exist [10, 13], instead of ideal nihility, in this paper, we investigate the backscattering effect of a PEC sphere coated by media with small permittivity and permeability parameters, which can be possibly realized within the recent advancement of metamaterials [14, 15]. In view of the present metamaterial designing and fabricating technique [16, 17], this anisotropic metamaterial with small parameter values can be more easily realized than the materials for cloak.

In the following Section 2, we firstly present the theoretical formulation for a general scattering problem of a sphere wrapped by multi-layer radially uniaxial media. Based on the formulation, the backscattering properties of a PEC sphere coated with one-layer anisotropic medium are analyzed and discussed in Section 3. It is demonstrated that the coating has the potential to behave better as a backscattering eliminator when its radial constitutive parameters' ( $\epsilon_r$  and  $\mu_r$ ) values are carefully chosen. The numerical results are provided for one-layer system. However, any finite number of layers can be solved in like manner. Such multi-layered nihility scheme is useful to gain a maximal drop of the backscattering amplitude.

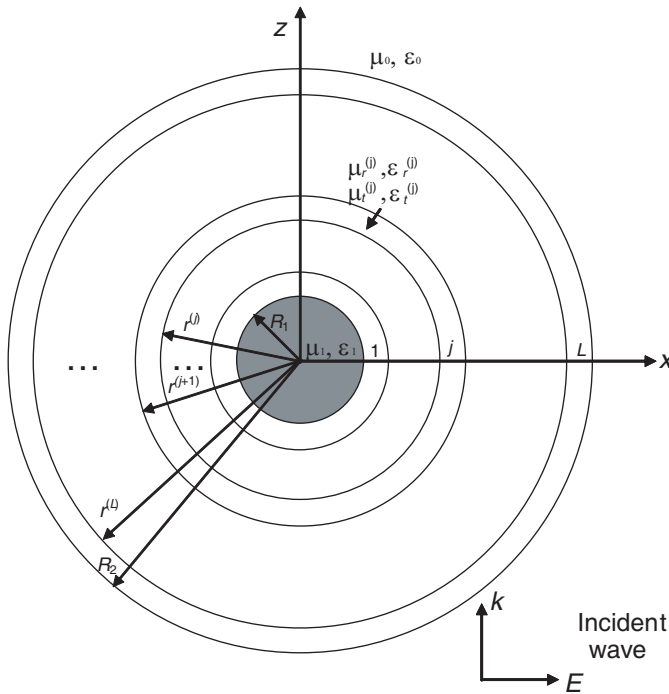
## 2. FORMULATION

In this section, vector potential formulation for electromagnetic scattering problem of an isotropic sphere (not confined to PEC) coated with multilayered rotationally uniaxial media is investigated. As shown in Fig. 1, an isotropic sphere ( $\epsilon_1, \mu_1$ ) of radius  $R_1$  is located at the origin

of the coordinate system, and the spherical region  $R_1 < r < R_2$  is occupied by the multilayered rotationally symmetric anisotropic media, which in the  $j$ -th layer is characterized by

$$\begin{aligned} \bar{\bar{\epsilon}} &= (\epsilon_r^{(j)} - \epsilon_t^{(j)})\hat{r}\hat{r} + \epsilon_t^{(j)}\bar{\bar{I}}, \\ \bar{\bar{\mu}} &= (\mu_r^{(j)} - \mu_t^{(j)})\hat{r}\hat{r} + \mu_t^{(j)}\bar{\bar{I}}, \end{aligned} \tag{1}$$

where  $\bar{\bar{I}} = \hat{r}\hat{r} + \hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}$ ,  $\epsilon_t^{(j)}$  and  $\mu_t^{(j)}$  are the permittivity and permeability along the  $\hat{\theta}$  and  $\hat{\phi}$  directions (parallel to the sphere surface),  $\epsilon_r^{(j)}$  and  $\mu_r^{(j)}$  along the  $\hat{r}$  direction (perpendicular to the sphere surface) for the  $j$ -th layer. The incident plane wave is taken to be linearly polarized in  $x$  direction, traveling along the  $z$  axis. The time dependence of  $e^{-i\omega t}$  is suppressed.



**Figure 1.** Configuration of scattering of plane wave by a sphere coated with a  $L$ -layer shell. Each layer within  $r^{(j)} < r < r^{(j+1)}$  is a radially uniaxial homogeneous medium with permittivity tensor  $\bar{\bar{\epsilon}} = \epsilon_r^{(j)}\hat{r}\hat{r} + \epsilon_t^{(j)}\hat{\theta}\hat{\theta} + \epsilon_t^{(j)}\hat{\phi}\hat{\phi}$  and permeability tensor  $\bar{\bar{\mu}} = \mu_r^{(j)}\hat{r}\hat{r} + \mu_t^{(j)}\hat{\theta}\hat{\theta} + \mu_t^{(j)}\hat{\phi}\hat{\phi}$ .  $r^{(1)} = R_1$ ,  $r^{(L+1)} = R_2$ ,  $j$  ( $1 \leq j \leq L$ ) represents the layer number.

Since the electromagnetic fields can be deduced from the corresponding scalar potentials, which are composed of a superposition of Bessel functions, associated Legendre polynomials, and harmonic functions [18, 19, 20], the scalar potentials for the incident fields ( $r > R_2$ ), scattered fields ( $r > R_2$ ), and the fields inside the isotropic sphere ( $r < R_1$ ) can be expressed by the same notations as those in [19], while the fields for each coating layer of the multilayered anisotropic shell (within  $r^{(j)} < r < r^{(j+1)}$ ) are given by the fractional-order Riccati-Bessel functions of the first and third kinds as shown in Eq. (5):

$$\Phi_{\text{TM}}^i = \frac{\cos \phi}{\omega} \sum_n a_n \psi_n(k_0 r) P_n^1(\cos \theta), \tag{2}$$

$$\Phi_{\text{TE}}^i = \frac{\sin \phi}{\omega \eta_0} \sum_n a_n \psi_n(k_0 r) P_n^1(\cos \theta),$$

$$\Phi_{\text{TM}}^s = \frac{\cos \phi}{\omega} \sum_n a_n T_n^{(M)} \zeta_n(k_0 r) P_n^1(\cos \theta), \tag{3}$$

$$\Phi_{\text{TE}}^s = \frac{\sin \phi}{\omega \eta_0} \sum_n a_n T_n^{(N)} \zeta_n(k_0 r) P_n^1(\cos \theta),$$

$$\Phi_{\text{TM}}^{\text{int}} = \frac{\cos \phi}{\omega} \sum_n c_n^{(M)} \psi_n(k_1 r) P_n^1(\cos \theta), \tag{4}$$

$$\Phi_{\text{TE}}^{\text{int}} = \frac{\sin \phi}{\omega \eta_0} \sum_n c_n^{(N)} \psi_n(k_1 r) P_n^1(\cos \theta),$$

$$\Phi_{\text{TM}}^{(j)} = \frac{\cos \phi}{\omega} \sum_n [a_{jn}^{(M)} \psi_{\nu_1}(k_t^{(j)} r) + a_{jn}^{(M)} R_{jn}^{(M)} \zeta_{\nu_1}(k_t^{(j)} r)] P_n^1(\cos \theta), \tag{5}$$

$$\Phi_{\text{TE}}^{(j)} = \frac{\sin \phi}{\omega \eta_0} \sum_n [a_{jn}^{(N)} \psi_{\nu_2}(k_t^{(j)} r) + a_{jn}^{(N)} R_{jn}^{(N)} \zeta_{\nu_2}(k_t^{(j)} r)] P_n^1(\cos \theta),$$

where  $a_n = \frac{(-i)^{-n}(2n+1)}{n(n+1)}$ ,  $n$  refers to the mode numbers which are integers,  $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ ,  $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ ,  $k_1 = \omega\sqrt{\mu_1\epsilon_1}$ , and  $k_t^{(j)} = \omega\sqrt{\mu_t^{(j)}\epsilon_t^{(j)}}$ .  $\psi_\nu$  and  $\zeta_\nu$  are Riccati-Bessel functions of the first and third kinds, respectively. For the anisotropic shells, the functions' orders are fractional and different for both TM and TE modes:

$$\begin{aligned} \nu_1 &= \sqrt{\frac{\epsilon_t}{\epsilon_r} n(n+1) + \frac{1}{4} - \frac{1}{2}}, \\ \nu_2 &= \sqrt{\frac{\mu_t}{\mu_r} n(n+1) + \frac{1}{4} - \frac{1}{2}}, \end{aligned} \tag{6}$$

$\nu_1$  and  $\nu_2$  represent the anisotropy effect in the coating layer.  $T_n^{(M)}$ ,  $T_n^{(N)}$ ,  $c_n^{(M)}$ ,  $c_n^{(N)}$ ,  $a_{jn}^{(M)}$ ,  $a_{jn}^{(N)}$ ,  $R_{jn}^{(M)}$ , and  $R_{jn}^{(N)}$  are unknown expansion coefficients, thereinto,  $R_{jn}^{(M)}$  and  $R_{jn}^{(N)}$  are defined as the general reflection coefficients of  $n$ -th order in the  $j$ -th layer for both modes. In order to solve these coefficients, we match the boundary conditions on each spherical surface between adjacent layers. The coefficients for TM waves form a group of equations, and the ones for TE waves form the other. If the shell is composed of  $L$  layers, there will be  $(L + 1)$  boundaries which will give rise to  $(2L + 2)$  equations for each mode. Solving the unknowns requires inverting a  $(2L + 2) \times (2L + 2)$  square matrix, which is straightforward but tedious.

However, using an agile method introduced in [21] for obtaining the reflection and transmission by layered media, we are able to deal with the problem in a much simpler way, which avoids inverting the big  $(2L + 2) \times (2L + 2)$  square matrix. The relationship of neighboring general reflection coefficients ( $R_{jn}^{(M)}$  and  $R_{j-1n}^{(M)}$ ,  $R_{j-1n}^{(N)}$  and  $R_{jn}^{(N)}$ ) can be found. For example, for TE waves ( $2 \leq j \leq L$ ),  $R_{jn}^{(N)}$  is expressed in terms of  $R_{j-1n}^{(N)}$ :

$$R_{jn}^{(N)} = -\frac{\sqrt{\epsilon_t^{(j)} \mu_t^{(j)}} [\text{LT}] - \sqrt{\epsilon_t^{(j-1)} \mu_t^{(j-1)}} [\text{RT}]}{\sqrt{\epsilon_t^{(j)} \mu_t^{(j)}} [\text{LB}] - \sqrt{\epsilon_t^{(j-1)} \mu_t^{(j-1)}} [\text{RB}]}, \quad (7)$$

where

$$\begin{aligned} \text{LT} &= \psi'_\nu(\xi_t^{(j-1)})\psi_\nu(\xi_t^{(j)}) + R_{j-1n}^{(N)}\zeta'_\nu(\xi_t^{(j-1)})\psi_\nu(\xi_t^{(j)}), \\ \text{RT} &= \psi_\nu(\xi_t^{(j-1)})\psi'_\nu(\xi_t^{(j)}) + R_{j-1n}^{(N)}\zeta_\nu(\xi_t^{(j-1)})\psi'_\nu(\xi_t^{(j)}), \\ \text{LB} &= \psi'_\nu(\xi_t^{(j-1)})\zeta_\nu(\xi_t^{(j)}) + R_{j-1n}^{(N)}\zeta'_\nu(\xi_t^{(j-1)})\zeta_\nu(\xi_t^{(j)}), \\ \text{RB} &= \psi_\nu(\xi_t^{(j-1)})\zeta'_\nu(\xi_t^{(j)}) + R_{j-1n}^{(N)}\zeta_\nu(\xi_t^{(j-1)})\zeta'_\nu(\xi_t^{(j)}), \end{aligned}$$

thereinto,  $\xi_t^{(j-1)} = k_t^{(j-1)}r^{(j)}$ ,  $\xi_t^{(j)} = k_t^{(j)}r^{(j)}$ ,  $r^{(j)}$  is the radius of the interface between the  $(j - 1)$ -th and the  $j$ -th layer ( $r^{(1)} = R_1$ ). At  $r = R_1$ , we can get the exact value of the general reflection coefficient for the first layer:

$$R_{1n}^{(N)} = -\frac{\sqrt{\frac{\epsilon_t^{(1)}}{\epsilon_1}}\psi_n(\xi_1)\psi'_\nu(\xi_t^{(1)}) - \sqrt{\frac{\mu_t^{(1)}}{\mu_1}}\psi'_n(\xi_1)\psi_\nu(\xi_t^{(1)})}{\sqrt{\frac{\epsilon_t^{(1)}}{\epsilon_1}}\psi_n(\xi_1)\zeta'_\nu(\xi_t^{(1)}) - \sqrt{\frac{\mu_t^{(1)}}{\mu_1}}\psi'_n(\xi_1)\zeta_\nu(\xi_t^{(1)})}, \quad (8)$$

where  $\xi_1 = k_1 R_1$  and  $\xi_t^{(1)} = k_t^{(1)} R_1$ . By substitution all the general reflection coefficients in other layers all the reflection coefficients can be obtained from inside to outside. Once we have  $R_{Ln}^{(N)}$  for the outmost layer of the shell, by applying the boundary conditions at  $r = R_2$ , the scattering coefficients  $T_n^{(N)}$  can be solved as:

$$T_n^{(N)} = -\frac{\sqrt{\frac{\epsilon_0}{\epsilon_t^{(L)}}} [\text{TN1}] \psi'_n(\xi_0) - \sqrt{\frac{\mu_0}{\mu_t^{(L)}}} [\text{TN2}] \psi_n(\xi_0)}{\sqrt{\frac{\epsilon_0}{\epsilon_t^{(L)}}} [\text{TN1}] \zeta'_n(\xi_0) - \sqrt{\frac{\mu_0}{\mu_t^{(L)}}} [\text{TN2}] \zeta_n(\xi_0)}, \quad (9)$$

where

$$\begin{aligned} \text{TN1} &= \psi_\nu(\xi_t^{(L)}) + R_{Ln}^{(N)} \zeta_\nu(\xi_t^{(L)}), \\ \text{TN2} &= \psi'_\nu(\xi_t^{(L)}) + R_{Ln}^{(N)} \zeta'_\nu(\xi_t^{(L)}), \end{aligned}$$

$\xi_0^{(L)} = k_0^{(L)} R_2$ , and  $\xi_t^{(L)} = k_t^{(L)} R_2$ . The scattering coefficients  $T_n^{(M)}$  for TM waves can be obtained similarly, which guarantees that the fields in free space (outside the shell) are all attainable. The fields in the shells and the inner sphere can also be solved straightforwardly if we keep on seeking the remaining unknown coefficients  $a_{jn}^{(M)}$ ,  $a_{jn}^{(N)}$ ,  $c_n^{(M)}$ , and  $c_n^{(N)}$  from the outer layer to inner layer.

### 3. RESULTS

Examples of numerical results are provided in this section. We first analyze the plane-wave response of a PEC sphere coated with isotropic media whose permittivity and permeability are smaller than vacuum. Then we focus on the anisotropic coating case. When the coating has a parameters of  $\mu_0$  and  $\epsilon_0$ , we can get the scattering of a raw PEC sphere, which will be used for comparison. In all of our calculations in this section, the frequency is fixed at 10 GHz, the radius of the PEC sphere is  $R_1 = 3\lambda_0$ , and the truncation of the summations is chosen to be 50, at which the convergence has been verified to be acceptable.

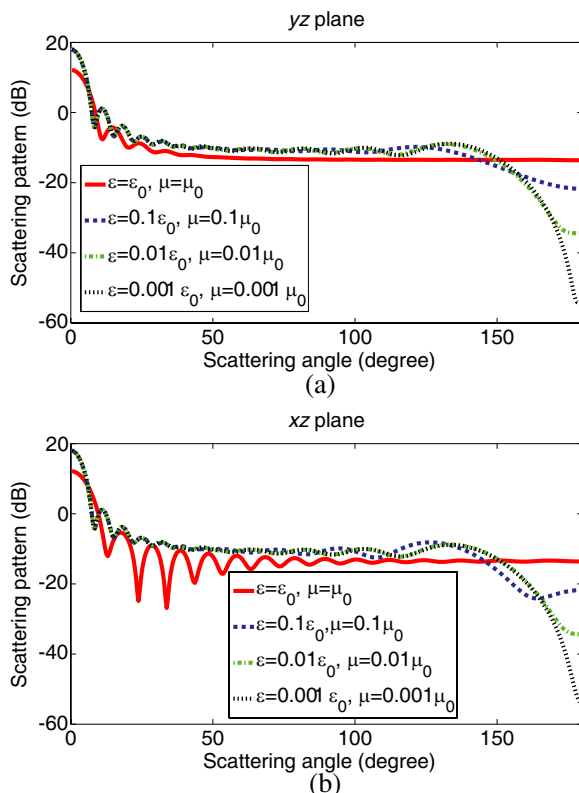
#### 3.1. Isotropic Coating

Three examples are presented in this subsection with logarithmic degressive relative constitutive parameters of the isotropic coatings in comparison with no coating of a PEC sphere. The background material is air. Fig. 2 plots the bistatic scattering as a function of the scattering angle  $\theta$  for a PEC sphere coated by different isotropic shells

with decrescent values of permittivity and permeability in two different planes. The vertical axis represents the normalized differential cross sections,  $\frac{|S_1(\theta)|^2}{k_0^2 \pi R_2^2}$ ,  $\frac{|S_2(\theta)|^2}{k_0^2 \pi R_2^2}$ , where  $S_1(\theta)$  and  $S_2(\theta)$  are defined by:

$$\begin{aligned} S_1(\theta) &= - \sum_n \frac{(2n + 1)}{n(n + 1)} \left[ T_n^{(M)} \pi_n(\theta) + T_n^{(N)} \tau_n(\theta) \right], \\ S_2(\theta) &= - \sum_n \frac{(2n + 1)}{n(n + 1)} \left[ T_n^{(M)} \tau_n(\theta) + T_n^{(N)} \pi_n(\theta) \right]. \end{aligned} \tag{10}$$

In the above two equations  $\pi_n(\theta)$  and  $\tau_n(\theta)$  are related to the associated Legendre functions [19]. For the configuration shown in Fig. 2,  $S_1(\theta)$  and  $S_2(\theta)$  illustrate the scattering patterns in the  $yz$  and  $xz$  planes, corresponding to Figs. 2(a) and 2(b) respectively.



**Figure 2.** Normalized differential cross sections for a perfectly conducting sphere coated with different isotropic media in (a)  $yz$  planes and (b)  $xz$  planes. The size of the coated layer are ( $R_1 = 3\lambda_0$ ,  $R_2 = 4\lambda_0$ ).

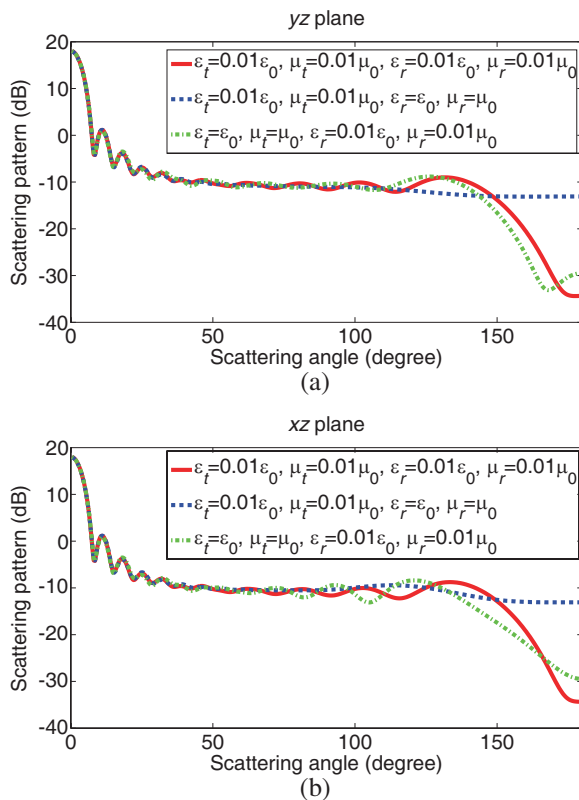
From these two sub-figures, we see that the scattering cross section is more and more suppressed at 180 degrees as the relative permittivity and permeability of the coating get smaller and smaller. So the fourth curve (black dotted line) has the lowest backscattering due to its smallest permittivity and permeability values, which is closest to the nihility sphere case [7]. The reason is that when a PEC sphere is wrapped by nihility, no wave can penetrate and exist in it, thus the sphere coated by nihility acts like it is a nihility sphere. On the other hand, the forward-scattering of the coated PEC spheres are all higher than the PEC sphere without coating. The detailed relationship between the backscattering magnitude  $Q_{back}$  and isotropic constitutive parameters will be discussed in Section 3.3 later.

### 3.2. Anisotropic Coating

Considering the fact that achieving the small permittivity and permeability values might be easier in one or two directions than all the three, we will focus on the rotationally anisotropic coating with small constitutive parameter values. In order to study the backscattering property in this situation, we present the impact of each constitutive parameter including  $\epsilon_t, \mu_t, \epsilon_r,$  and  $\mu_r$  on the scattering of a wrapped PEC sphere. First, the pairs of tangential ( $\epsilon_t, \mu_t$ ) and radial ( $\epsilon_r, \mu_r$ ) parameters of anisotropic media are decreased separately to compare with isotropic case, and the results are shown in Fig. 3, where the relative permittivity and permeability values are chosen to be 0.01 in this example. It is indicated that the small values of  $\epsilon_r$  and  $\mu_r$  play more effective roles than  $\epsilon_t$  and  $\mu_t$  on reducing the backscattering of the coated PEC sphere, however, the case where all the four parameters are small still performs the best shielding backscattering, which is illustrated as the red solid line in Fig. 3.

In more detail, Fig. 4 shows the impact of every individual constitutive parameter of the anisotropic coating for a PEC sphere. In the figure legend, if not shown, the default constitutive parameters for each curve denote that their values are equal to the vacuum's permittivity  $\epsilon_0$  or permeability  $\mu_0$ . For example,  $\epsilon_t = 0.01\epsilon_0$  shown in the legend of Fig. 4 represents the coated layer has the following parameters:  $\epsilon_t = 0.01\epsilon_0, \epsilon_r = \epsilon_0,$  and  $\mu_t = \mu_r = \mu_0$ . From Fig. 4 we see that the first and the second curves (only one tangential constitutive parameter is small) are both higher than the case where either  $\mu_r$  or  $\epsilon_r$  is small, in the range near 180 degrees. So they are both worse than isotropic case when the coating is the air, which indicates that either of tangential parameters ( $\epsilon_t, \mu_t$ ) cannot help to reduce the backscattering. The rest four curves all have lower backscattering than the air case as the red lines shown in Fig. 2. Besides, solely small  $\epsilon_r$  value of the





**Figure 3.** Comparison of the normalized differential cross sections for a perfectly conducting sphere coated with isotropic medium and anisotropic media ( $R_1 = 3\lambda_0, R_2 = 4\lambda_0$ ) in (a)  $yz$  planes and (b)  $xz$  planes.

coating works better than  $\mu_r$  due to the PEC core. Otherwise, if there is a perfect magnetic conducting (PMC) sphere inside the coating, the fact will be opposite according to the principle of duality [21], meaning that  $\mu_r$  will act more significantly than  $\epsilon_r$ , which has been validated by our calculation.

### 3.3. Discussions

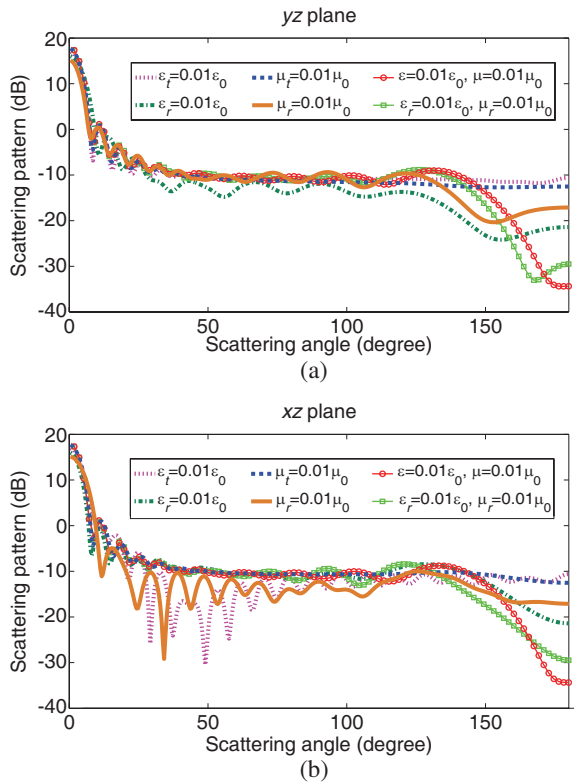
The performance of coating's invisibility for a PEC sphere is determined by its constitutive parameters. As we have already shown, the small values of constitutive parameters result in a lowered backscattering and concomitant higher forward-scattering regardless

of isotropic or anisotropic coating, but with different extents. In this subsection, we discuss the effect of the constitutive parameters on the backscattering properties from a different viewpoint.

Make use of the special values when  $\theta = \pi$  [22],

$$\begin{aligned} \pi_n(\pi) &= (-1)^{n+1} \frac{n(n+1)}{2}, \\ \tau_n(\pi) &= (-1)^n \frac{n(n+1)}{2}, \end{aligned} \tag{11}$$

the two equations from Eq. (10) will get the same expression because of the square, and then the unique normalized differential backscattering



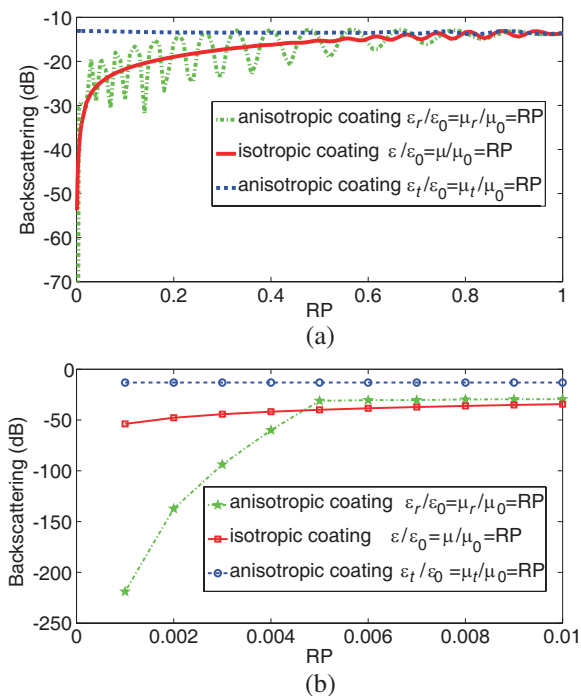
**Figure 4.** Normalized differential cross sections for a perfectly conducting sphere coated by anisotropic media ( $R_1 = 3\lambda_0$ ,  $R_2 = 4\lambda_0$ ) in (a)  $yz$  planes and (b)  $xz$  planes.

being paid close attention to can be written as:

$$Q_{back} = \frac{1}{k_0^2 \pi R_2^2} \left| \sum_n \frac{(-1)^n (2n + 1)}{2} (T_n^{(M)} - T_n^{(N)}) \right|^2. \quad (12)$$

For different coatings, once their scattering coefficients  $T_n^{(M)}$  and  $T_n^{(N)}$  are known, the normalized differential backscattering can be obtained through the above equation.

Figure 5 compares the calculated backscattering magnitude [see Eq. (12)] for anisotropic and isotropic coatings as functions of their relative permittivity and permeability values. For anisotropic coatings, the green dot-dashed curve represents that only radial relative permittivity (RP =  $\epsilon_r/\epsilon_0$ ) and relative permeability (RP =  $\mu_r/\mu_0$ ) for



**Figure 5.** Comparison of normalized differential backscattering values versus RP values for a perfectly conducting sphere ( $R_1 = 3\lambda_0$ ) coated with anisotropic media and isotropic media. (b) is the enlargement of (a) with RP value from 0.001 to 0.01. for anisotropic media, the RP values for default constitutive parameters are 1 ( $\epsilon_t = \epsilon_0$ ,  $\mu_t = \mu_0$  or  $\epsilon_r = \epsilon_0$ ,  $\mu_r = \mu_0$ ). the thickness of the coating is  $\lambda_0$  ( $R_2 = 4\lambda_0$ ).

anisotropic media are smaller than unity, the tangential ones remain the same as the air ( $\epsilon_t = \epsilon_0, \mu_t = \mu_0$ ); on the contrary, for the blue dashed curve, the tangential RP are changing while the radial ones fixed to vacuum values.

As shown in Fig. 5(a), the green dot-dashed curve oscillates continuously while the blue dashed one is almost a horizontal line, which is influenced little by the changing values of tangential parameters  $\epsilon_t$  and  $\mu_t$  of the anisotropic coating. The normalized differential backscattering for small  $\epsilon_r$  and  $\mu_r$  type coated PEC sphere has a series of local minima possessing lower magnitude than its corresponding isotropic coating, which can be clearly observed in Fig. 5(a). Near these valleys, the PEC sphere can be better shielded from backscattering detection. Fig. 5(b) illustrated the result in the area of  $0.001 \leq \text{RP} \leq 0.01$ . As we can see from Fig. 5(b), when  $\epsilon_r$  and  $\mu_r$  approach nihility, that is  $\text{RP} \rightarrow 0$ , the backscattering of anisotropic media drops much more quickly than the isotropic media, which can be explained as a spatial filter without reflection for normal incidence [12]. The examples we have given in the previous subsection (Figs. 3 and 4) happen at  $\text{RP} = 0.01$  where the anisotropic coating's backscattering (green dot-dashed) is larger than isotropic coating (red solid). Therefore, if we want to cloak a PEC sphere better from a monostatic radar with an anisotropic coating, selective right positions for RP values will be a wise choice.

#### 4. CONCLUSION

In summary, the scattering properties of a coated sphere are studied theoretically. Concentrating on a PEC sphere with anisotropic coating, attention is paid to the backscattering properties which is valuable in monostatic radar detection. Our calculated results show that decreasing the radial constitutive parameters ( $\epsilon_r$  and  $\mu_r$ ) is much more helpful to reduce the backscattering magnitude, while small tangential constitutive parameters ( $\epsilon_t$  and  $\mu_t$ ) have little effect on the elimination of the backscattering. In physical applications, the backscattering of the coated PEC sphere can be controlled by adjusting the constitutive parameters of the anisotropic media, either for enhancement or for reduction.

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