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ACCURATE SYNTHESIS FORMULAS OBTAINED BY USING A DIFFERENTIAL EVOLUTION ALGORITHM FOR CONDUCTOR-BACKED COPLANAR WAVEGUIDES

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Abstract—In this paper, accurate synthesis formulas obtained by using a differential evolution (DE) algorithm for conductor-backed coplanar waveguides (CBCPWs) are presented. The synthesis formulas are useful to microwave engineers for accurately calculating the physical dimensions of CBCPWs. The results of the synthesis formulas are compared with the theoretical and experimental results available in the literature. A full-wave electromagnetic simulator IE3D and experimental results are obtained in this work. The average percentage error of the synthesis formulas obtained by using DE algorithm is computed as 0.67% for 1086 CBCPW samples having different electrical parameters and physical dimensions, as compared with the results of quasi-static analysis.

1. INTRODUCTION

The conductor-backed coplanar waveguide (CBCPW) has the advantage of mechanical strength, heat sinking ability, and lower characteristic impedance compared with conventional coplanar waveguide in designing microwave integrated circuits (MICs). Among these advantages, CBCPWs allow easy implementation of mixed coplanar/microstrip circuits, reduce radiation effects, and raise effective permittivity. These and several other advantages make CBCPWs ideally suit for MIC as well as monolithic MIC (MMIC) applications [1–12].

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Various types of CBCPW structures have been presented and analyzed in the literature [4–11]. The CBCPWs were analyzed by using the spectral domain approach (SDA) [4], conformal mapping method (CMM) [5, 6], and quasi-static SDA [7]. The dispersion characteristics of CBCPWs have been obtained by using the rigorous mode matching method [8], alternative formulation of the transverse resonance technique [9], full-wave SDA and Galerkin's method [10], and a hybrid two-dimensional finite-difference time-domain/Marquardt curve-fitting technique [11]. The synthesis formulas were also proposed in [12].

In this paper, accurate synthesis formulas obtained by using the differential evolution (DE) [13, 14] algorithm are presented for CBCPWs. DE algorithm, one of the evolutionary algorithms, has been proven to be a highly efficient technique for solving numerical optimization problems. It is recognized that DE algorithm is practical and powerful optimization tools for a variety of microwave engineering problems [15–23]. The synthesis formulas proposed here are used to successfully compute the physical dimensions of CBCPWs. The validity and accuracy of the proposed synthesis formulas are verified by comparing their results with those of experimental works [3], CMM [5], synthesis formulas [12], a full-wave electromagnetic simulator IE3D [24], and experimental works realized in this study. It was shown that the synthesis formulas proposed in this paper provide more accurate results than the synthesis formulas proposed in [12].

2. DIFFERENTIAL EVOLUTION (DE) ALGORITHM

DE algorithm is an exceptionally simple, fast and robust computation method for solving optimization problems [13, 14]. DE algorithm uses only a few control parameters, and these remain fixed throughout the entire optimization procedure.

DE algorithm operates on a population with N_{POP} chromosomes, and each chromosome is a symbolic representation of the vector consisting of the N_{PAR} optimization parameters. To establish a starting point for optimum seeking, the population must be initialized. The initial population is created with random values selected from within the given boundaries:

$$v_{i,j}^{P} = v_j^{\min} + M_j \cdot \left(v_j^{\max} - v_j^{\min} \right), \quad j = 1,\ 2,\ \ldots,\ N_{PAR} \tag{1}$$

where M_j is a random number, uniformly distributed between 0 and 1, and $v_j^{\min}$ and $v_j^{\max}$ represent the minimum and maximum permissible values of the jth parameter, respectively.

After the initialization, the algorithm goes into genetic evolution, and three genetic operations, namely, mutation, crossover and selection are executed in sequence. In this work, the mutation operation, which generates a mating partner for each individual by producing a difference vector called the mutant vector, is used. Mutant vector $v_{i,G+1}$ is produced by

$$v_{i,G+1} = t_{r1,G} + F \cdot (t_{r2,G} - t_{r3,G}) \tag{2}$$

where r_1, r_2, and r_3 belong to the set $\{1, 2, \ldots, N_{POP}\}$, and $t_{r1,G}$, $t_{r2,G}$, and $t_{r3,G}$ represent three random individuals chosen in the current generation, G, to produce the mutant vector for the next generation, $v_{i,G+1}$. The random numbers r_1, r_2, and r_3 should be different from the running index i. F is the real scaling factor which controls the amplification of the differential variation between two random vectors.

Following mutation operation, in order to control the amount of diversity of the mutant vectors, the crossover is used. The crossover may partially suppress the effect of mutation by forming a trial vector $p_{ji,G+1}$

$$p_{ji,G+1} = \begin{cases} v_{ji,G+1}, & R_j \leq P_{CR} \\ t_{ji,G}, & \text{otherwise} \end{cases} \tag{3}$$

where R_j is a real random number in the range of $[0, 1]$, and P_{CR} is the probability of a real-valued crossover factor.

Finally, the selection operation is used to produce better offspring. Each child competes with its parent, and survives only if its fitness value is better. Following this, the next round of genetic evolution then begins.

The above procedure continues until a termination criterion is attained or a predetermined generation number is reached.

3. SYNTHESIS FORMULAS OBTAINED FROM DE ALGORITHM

The cross-section of a CBCPW is illustrated in Fig. 1. In this figure, s is the central strip width, w is the slot width, and h is thickness of the dielectric substrate with relative permittivity ε_r.

In this paper, accurate synthesis formulas for computing the physical dimensions of CBCPWs are obtained with the use of DE algorithm. These synthesis formulas are essentially derived from the data set. The data set used in this work has been obtained from the respective quasi-static analysis results [5] and contains 1086 samples. The design parameter ranges of CBCPWs used in these samples are $2 \leq \varepsilon_r \leq 50$, $0.1 \leq s/h \leq 5.5$, $0.1 \leq w/h \leq 1.90$,

$20\,\mu\text{m} \leq h \leq 3000\,\mu\text{m}$, and the respective characteristic impedance is $10\ \Omega \leq Z_0 \leq 220\,\Omega$.

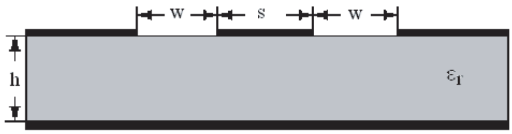

Figure 1. Cross-section of a CBCPW.

In this paper, first, the CBCPW parameters related to the physical dimensions are determined by using the results available in the literature. After this determination, models for the physical dimensions are chosen, then the unknown coefficient values of the models are determined by the DE algorithm. It is clear from the literature [5] that four parameters ε_r, h, s, and w are needed to determine the characteristic impedance of a CBCPW. The first design step is the selection of a suitable dielectric substrate (ε_r, h) for a CBCPW having a required characteristic impedance Z_0. Then, the physical dimensions w and s are determined. In this paper, two synthesis formulas for w and s are presented. The first synthesis formula calculates the slot width $w(Z_0,\ s,\ \varepsilon_r, h)$ for a given dielectric substrate (ε_r, h) and a required characteristic impedance Z_0 by choosing an appropriate strip width s. The second synthesis formula computes the strip width $s(Z_0,\ w,\ \varepsilon_r, h)$ for a given dielectric substrate (ε_r, h) and a required characteristic impedance Z_0 by choosing an appropriate slot width w.

To find proper computer-aided design (CAD) models for the physical dimensions of CBCPWs, many experiments were carried out in this study. After many trials, the following synthesis models, which produce very good results, were chosen to determine the slot and strip widths, respectively,

$$w = \frac{h \cdot \left(\alpha_1 \cdot x^{\alpha_2} + \alpha_3 \cdot y^{\alpha_4} + \alpha_5 \cdot \left(\frac{s}{h}\right)^{\alpha_6}\right)}{\left(\alpha_7 \cdot \left(x^{\alpha_8} \cdot \left(\frac{s}{h}\right)^{\alpha_9} + \alpha_{10} \cdot \left(\frac{s}{h}\right)^{\alpha_{11}} \cdot (y + \alpha_{12})^{\alpha_{13}}\right)^{\alpha_{14}} \cdot \left(x^{\alpha_{15}} \cdot (y + \alpha_{16})^{\alpha_{17}}\right) + \alpha_{18} \cdot \left(\frac{s}{h}\right)^{\alpha_{19}}\right)} \tag{4}$$

and

$$s = \frac{h \cdot \left(\beta_1 \cdot x^{\beta_2} \cdot y^{\beta_3} \cdot \left(\frac{w}{h}\right)^{\beta_4} + \beta_5 \cdot x + \beta_6 \cdot y + \beta_7 \cdot \left(\frac{w}{h}\right) + \beta_8 \cdot x^2 + \beta_9\right)^{\beta_{10}}}{\beta_{11} \cdot x^{\beta_{12}} \cdot y^{\beta_{13}} \cdot \left(\frac{w}{h}\right)^{\beta_{14}} + \beta_{15} \cdot x + \beta_{16} \cdot y + \beta_{17} \cdot \left(\frac{w}{h}\right)} \tag{5}$$

with

$$x = \frac{Z_0}{\eta_0} \tag{6}$$

$$y = \varepsilon_r \tag{7}$$

where $\eta_0 = 120\pi\,\Omega$ is the intrinsic impedance of free space, $(\alpha_1, \alpha_2, \ldots, \alpha_{19})$ and $(\beta_1, \beta_2, \ldots, \beta_{17})$ are the unknown coefficients. In this paper, the coefficient values are optimally found by the DE algorithm. The following slot and strip width formulas are then obtained by substituting these optimum coefficient values into Eqs. (4) and (5):

$$w = \frac{h \cdot \left(2.15 \cdot x^{0.894} - 1.61 \cdot y^{-0.339} + 8.03 \cdot \left(\frac{s}{h}\right)^{0.96}\right)}{\left(7.804 \cdot \left(x^{2.434} \cdot \left(\frac{s}{h}\right)^{2.438} + 1.253 \cdot \left(\frac{s}{h}\right)^{-0.019} \cdot (y+0.124)^{-1.208}\right)^{-6.972} \cdot \left(x^{-3.109} \cdot (y + 0.232)^{-9.96}\right) - 7.881 \cdot \left(\frac{s}{h}\right)^{0.954}\right)} \tag{8}$$

and

$$s = \frac{h \cdot \left(19.212 \cdot x^{1.204} \cdot y^{0.634} \cdot \left(\frac{w}{h}\right)^{-0.24} - 13.31 \cdot x + 0.58 \cdot y + 0.235 \cdot \left(\frac{w}{h}\right) + 17.4 \cdot x^2 + 1.145\right)^{-0.813}}{2.827 \cdot x^{3.76} \cdot y^{1.325} \cdot \left(\frac{w}{h}\right)^{-1.076} + 0.828 \cdot x - 0.00011 \cdot y - 0.0021 \cdot \left(\frac{w}{h}\right)} \tag{9}$$

The proposed synthesis formulas are valid for the ranges of $s/h \leq 10/(1 + \ln\varepsilon_r)$, $w/h \leq 10/[3(1 + \ln\varepsilon_r)]$, $2 \leq \varepsilon_r \leq 50$, and $10\,\Omega \leq Z_0 \leq 220\,\Omega$. In the DE algorithm optimization process, the values of population size, mutation rate, and crossover rate are taken as 30, 0.8, and 0.8, respectively.

4. NUMERICAL RESULTS AND DISCUSSION

In order to show the validity and accuracy of the proposed synthesis formulas given in (8) and (9), the results of the synthesis formulas obtained by using DE algorithm are compared with the results of respective quasi-static analysis [5] in Figs. 2 and 3. The results of synthesis formulas proposed by Yildiz and Turkmen [12] are also given in these figures for comparison. Figs. 2 and 3, respectively, illustrate the quasi-static analysis [5] contours, the slot width $w(Z_0, s, \varepsilon_r, h)$ results obtained by first synthesis formula and the strip width $s(Z_0, w, \varepsilon_r, h)$ results obtained by second synthesis formula for CBCPWs with $\varepsilon_r = 12.9$ and $h = 200\,\mu\text{m}$ and a required characteristic impedance. It

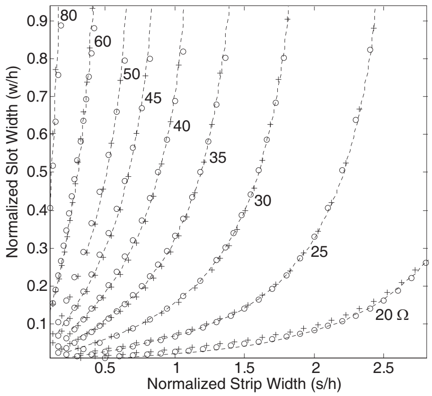

Figure 2. Comparison of the $w(Z_0, s, \varepsilon_r, h)$ results obtained by using the first synthesis formula proposed in this paper, the synthesis formula proposed in [12] and the quasi-static analysis [5] contours for CBCPWs ($\varepsilon_r = 12.9$ and $h = 200\,\mu\text{m}$). $- - -$ Analysis [5], $\circ\ \circ\ \circ$ Yildiz and Turkmen [12], $+ + +$ First Synthesis Formula.

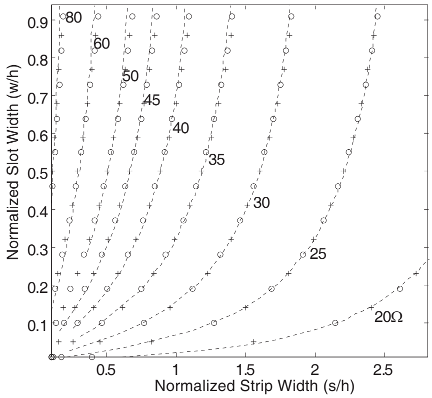

Figure 3. Comparison of the $s(Z_0, w, \varepsilon_r, h)$ results obtained by using the second synthesis formula proposed in this paper, the synthesis formula proposed in [12] and the quasi-static analysis [5] contours for CBCPWs ($\varepsilon_r = 12.9$ and $h = 200\,\mu\text{m}$). $- - -$ Analysis [5], $\circ\ \circ\ \circ$ Yildiz and Turkmen [12], $+ + +$ Second Synthesis Formula.

is clear from Figs. 2 and 3 that there is a very good agreement between the results of quasi-static analysis [5] and the synthesis formulas proposed in this paper. This good agreement supports the validity of the synthesis formulas proposed here. The average percentage errors of the synthesis formulas obtained by using DE algorithm and the synthesis formulas proposed Yildiz and Turkmen [12] are computed as 0.67% and 0.98%, respectively, for 1086 CBCPW samples, as compared with the results of quasi-static analysis [5]. Therefore, better accuracy with respect to the previous synthesis formulas [12] is obtained.

The characteristic impedances calculated by using the results of synthesis Formulas (8) and (9) for a given s and w, respectively, are compared with those of quasi-static analysis [5] for CBCPWs with $\varepsilon_r = 10.2$ and $h = 635\,\mu\text{m}$ in Fig. 4. In this figure, the characteristic impedance results are plotted with respect to the shape ratio $(s+w)/h$ for four different s/h values. It can be seen from Fig. 4 that the results of the synthesis formulas are in very good agreement with the results of quasi-static analysis. It is also apparent from this figure that there is a very good self-consistent agreement between the first and second synthesis formulas.

In this paper, three different CBCPWs are fabricated on RT/duroid laminates ($\varepsilon_r = 6.15$ and $h = 1270\,\mu$m) by using the printed circuit board (PCB) excavation technique. The characteristic impedances of these CBCPWs are calculated from the measured S-parameters for 1.75 GHz. We also calculated the characteristic impedances by using a full-wave electromagnetic simulator IE3D [24]. In Table 1, the results of the synthesis formulas obtained by using DE algorithm are compared with the results of measured [3], CMM [5], the formulas presented by Yildiz and Turkmen [12], IE3D [24], and experimental works realized in this study. In this table, Z_0' is the measured characteristic impedance values, w' and s' represent the measured geometrical dimensions of slot and strip widths of CBCPWs, respectively. Also, Z_0 represents the characteristic impedance values obtained from CMM and IE3D by using w' and s'. Z_{0w} and Z_{0s} are the final-check quasi-static analysis results calculated by using the w and s values obtained from the first and second synthesis formulas, respectively. As it can be seen from Table 1, a close agreement is obtained between the theoretical and experimental results. It is also clear that the best results are obtained from Eq. (8) with respect to the experimental results.

In this paper, the synthesis models, which are simpler and more complicated than the models given by Eqs. (4) and (5), were also tired. It was observed that the results of simpler models are not in good

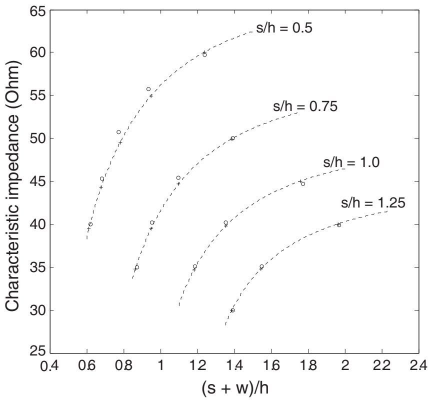

Figure 4. Comparisons of the characteristic impedances calculated by using the results of the first synthesis Formula (8) for a given s; the results of the second synthesis Formula (9) for a given w; and the quasi-static analysis [5] for CBCPWs ($\varepsilon_r = 10.2$ and $h = 635\,\mu$m). $- - -$ Analysis [5], $+ + +$ First Synthesis Formula, $\circ \circ \circ$ Second Synthesis Formula.

Table 1. Comparisons of the characteristic impedance results of the present synthesis formulas, experimental works [3], CMM [5], synthesis formulas proposed by Yildiz and Turkmen [12], IE3D [24], and experimental works realized in this study.

Measured			CMM [5]	IE3D [24]	Yildiz and Turkmen [12]		Present Results	
w' (μm)	s' (μm)	Z'_0 (Ω)	Z_0 (Ω)	Z_0 (Ω)	Z_{0w} (Ω)	Z_{0s} (Ω)	Z_{0w} (Ω)	Z_{0s} (Ω)
350	1150	47.17*	49.49	51.89	49.72	49.64	48.83	50.10
400	1250	47.48*	49.15	52.16	49.35	49.32	48.60	49.77
450	1350	47.25*	48.66	52.30	48.82	48.83	48.22	49.27
50	51	48	50	51.75	50.48	50.23	50.01	50.28
20	27	46	50	51.35	50.67	51.02	49.40	50.81
10	14	44	50	41.67	48.58	50.64	48.01	51.15

*Measured in this paper for CBCPWs with $\varepsilon_r = 6.15$ and $h = 1270$ μm and the remainder measured in [3] for CBCPWs with $\varepsilon_r = 12.9$ and $h = 100$ μm.

agreement with the theoretical and experimental results available in the literature, and that the more complicated models provide only a little improvement in the results, at the expense of the simplicity of the synthesis formulas. The advantages of the proposed synthesis formulas are simplicity and accuracy.

Similar good results are obtained for all CBCPW samples to be designed with different electrical parameters and physical dimensions. The results obtained from the proposed synthesis formulas clearly illustrate the performance of DE algorithm in obtaining high quality solutions. DE algorithm can be applied to obtain synthesis formulas for other transmission lines by choosing proper models.

5. CONCLUSION

In this paper, synthesis formulas obtained by using DE algorithm are presented for computing accurately the physical dimensions of CBCPWs. The characteristic impedance values calculated by using the results of these synthesis formulas are in good agreement with the theoretical and experimental results available in the literature and experimental results obtained in this study. This good agreement supports the validity and accuracy of the synthesis formulas proposed here. The synthesis formulas allow the designers to determine the physical dimensions of CBCPWs for the required design specifications in a very simple and convenient way, rather than by the iteration approach of applying the analysis technique.

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