Progress In Electromagnetics Research, PIER 102, 227–248, 2010

AN APPROXIMATE UTD RAY SOLUTION FOR THE RADIATION AND SCATTERING BY ANTENNAS NEAR A JUNCTION BETWEEN TWO DIFFERENT THIN PLANAR MATERIAL SLAB ON GROUND PLANE

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Abstract—A new, approximate, uniform geometrical theory of diffraction (UTD) based ray solutions are developed for describing the high frequency electromagnetic (EM) wave radiation/coupling mechanisms for antennas on or near a junction between two different thin planar slabs on ground plane. The present solution is obtained by extending the normal incidence solution in order to treat the more general case of skew (or oblique) incidence (three-dimensional 3-D). Plane wave (for oblique or skew incidence) and spherical wave illumination are all considered in this work. Unlike most previous works, which analyze the plane wave scattering by such structures via the Wiener-Hopf (W-H) or Maliuzhinets (MZ) methods, the present development can also treat problems of the radiation by and coupling between antennas near or on finite material coatings on large metallic platforms. In addition, the present solution does not contain the complicated split functions of the W-H solutions nor the complex MZ functions. Unlike the latter methods based on approximate boundary conditions, the present solutions, which are developed via a heuristic

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spectral synthesis approach, recover the proper local plane wave Fresnel reflection and transmission coefficients and surface wave constants of the material slabs. There is a very good agreement, with less than $\pm 1\,\mathrm{dB}$ differences when the numerical results based on the presented UTD solution for a material junction are compared with that of the MZ solution.

1. INTRODUCTION

A new, approximate, UTD [1, 2] based ray solutions are developed for describing the high frequency EM wave radiation/coupling mechanisms for antennas on or near a junction between two different thin planar slabs on ground plane. This work is useful for analyzing the radiation and scattering from edges on electrically large complex platforms. Platforms involving modern naval ships often contain material treatments over their otherwise metallic surfaces to control their scattering. Also modern antenna platforms may be built from composite materials. Furthermore, in many of these naval applications, there could be several antennas mounted on the same platform for multi-functional communication systems; thus it is important not only to be able to predict the effects of the platform on the performance of antennas placed thereon, but it is also important to predict the effects of mutual coupling between such antennas on a common platform. It is possible to control antenna mutual coupling by inserting material treatments around antennas in order to decrease the coupling. It is therefore clear that efficient and reliable computational tools for analyzing and accurately predicting the performance of such antennas which operate in the presence of material coated complex metallic platforms are crucial to the design and the development of modern antenna systems for naval and other applications.

It is noted that material coatings can be classified as double positive/double negative (DPS/DNG). DPS materials are those which exhibit positive values of electrical permittivity and permeability while DNG materials are supposed to exhibit negative values for these quantities [3–6]. One can also have materials with one of their electrical parameters positive with the other being negative; all of these types of materials are included in the UTD solutions developed in this work.

In this paper, it is of interest to extend the normal incidence solution as discussed in [7] in order to treat the more general case of skew (or oblique) incidence (three-dimensional 3-D). Plane wave (for oblique or skew incidence) and spherical wave illumination are considered here. The geometry of the problem is shown in Fig. 1(a).

Previous works dealing with the analytical solutions via the W-H solution to diffraction by a junction between two different thin planar material slabs on a perfect electric conductor (PEC) ground plane [8, 9] generally replace the original coated metallic surfaces or material slabs by approximate impedance boundary condition. The latter approximation allows one to arrive at a rigorous analytical solution to the resulting approximate problem configuration. These previous works primarily address the scattering problem in which the illumination is a uniform plane wave that is incident on the thin material discontinuity. In contrast, the present work is expected to be very useful not only to the analysis of scattering situations but also to antenna problems which are equally importance from a practical standpoint. Alternatively, the MZ is another option for solving the configuration with the thin material discontinuities. All of these solutions [10–15] are based on the MZ method. Unlike W-H and MZ solutions, the solution developed in this work recover the proper local plane wave Fresnel reflection and transmission coefficients (FRTCs), and surface wave constants, respectively, for the actual material, and they also allow the material to be both double positive (DPS) or double negative (DNG). Furthermore, the present works provides solutions for finite sources on or near such structures. In addition, it is important to note that the expressions present in this paper are appropriately approximated via physical reasoning so that they can be made free of the complicated integral forms of the W-H split (or factorization) and MZ functions.

This paper is organized as follows. Section 2 describes how one can arrive with an ansatz for the problem of a junction between two different thin planar material slabs on ground plane with a skew incident plane wave. The ansatz is very useful for arriving the present approximate UTD ray solution of the particular problem with a skew incident plane wave excitation. The extension of the approximate UTD solution to treat the case of spherical wave excitation is discussed in Section 3. The solutions from Sections 2 and 3 are in the form of a plane wave spectral (PWS) integral. One can asymptotically evaluate, in closed form, the PWS integral by using the steepest descent method discussed in Section 4. The total and scattered fields from canonical problems of interest are calculated in Section 5 using the present UTD solution and are shown to compare very well with the MZ solution obtained from [10]. It is noted that all of the fields in this work are assumed to have an $e^{j\omega t}$ time dependence which is suppressed throughout the paper.

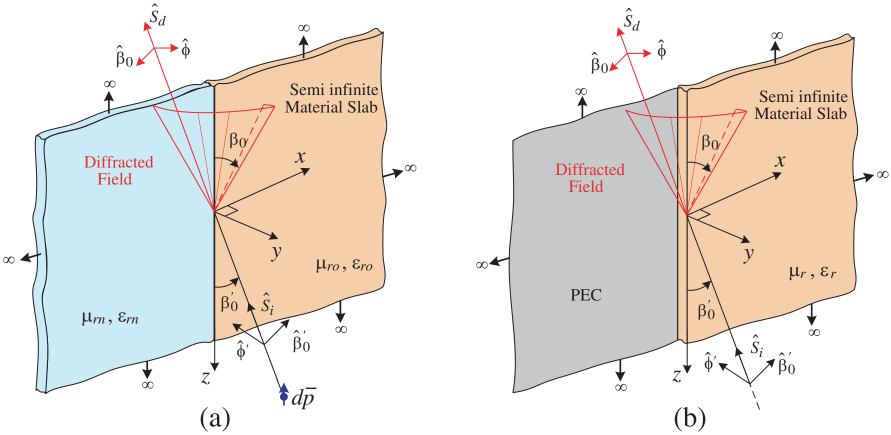

Figure 1. Canonical of interest. (a) 3-D junction between two different, thin, planar DPS/DNG material slabs on a PEC ground plane illuminated by a $\hat{z}$-directed current moment. (b) Thin, planar DPS/DNG material half plane on an entire PEC ground plane illuminated by a skew incident plane wave excitation.

2. ANSATZ FOR THE OBLIQUELY INCIDENT PLANE WAVE ILLUMINATION CASE WITH ONE FACE BEING PEC

The solutions to corresponding 3-D problems (skew incidence) in Fig. 1(a) can be obtained by extending the two dimensional (2-D) solution [7] via an approach similar to that in [8]. It is known that the normal field components E_y and H_y satisfy the Helmholtz scalar equation and impedance boundary conditions independently. This leads to a decoupled solution separately for E_y and H_y. Thus it is convenient to start an ansatz, based on the simplification of a related effective 2-D W-H solution [9] for the normal field components in the case of a unit amplitude, plane wave at skew incidence when it is applied to the special case in Fig. 1(b) where the n-face ($x < 0$, $y = 0$, z) is assumed to be a PEC. In particular, the PWS integral for the diffraction of an obliquely incident plane wave by a two part grounded material slab is first constructed from the ansatz provided by the W-H solution [8]. Thus, the normal components of total field for $y > 0$ (free space) for the problem of interest may be expressed as

$$\bar{U}_y = \bar{U}_y^i + \bar{U}_y^s \tag{1}$$

where $\bar{U}_y$ denotes $\hat{y}\begin{bmatrix} E_y \\ \eta_o H_y \end{bmatrix}$. Here E_y represents the total electric field for the TE case and H_y represents the total magnetic field for the TM case. Also η_o is the intrinsic impedance of free space. The $\bar{U}_y^s$ is $\hat{y}\begin{bmatrix} E_y^s \\ \eta_o H_y^s \end{bmatrix}$. Note that E_y^s (or H_y^s) is the $\hat{y}$-directed electric (or magnetic) scattered field. The incident uniform plane wave $\bar{U}_y^i$ is $\hat{y}\begin{bmatrix} E_y^i \\ \eta_o H_y^i \end{bmatrix}$, where E_y^i (or H_y^i) is the $\hat{y}$-directed electric (or magnetic) incident field, which is given by

$$\bar{U}_y^i = \bar{U}_{oy} e^{(jk'_x x + jk'_y y + jk'_z z)} \tag{2}$$

where $\bar{U}_{oy}$ denotes $\hat{y}\begin{bmatrix} E_{oy} \\ \eta_o H_{oy} \end{bmatrix}$. The E_{oy} and H_{oy} are assumed to be unity for convenience. The k'_x, k'_y, and k'_z are given by

$$k'_x = k \sin\beta'_o \cos\phi'; \quad k'_y = k \sin\beta'_o \sin\phi'; \quad k'_z = k \cos\beta'_o \tag{3}$$

with $0 < \beta'_o < \pi$ and $0 \leq \phi' \leq \pi$.

Following the form of the W-H solution for the canonical two part problem in [8], the scattered field $\bar{U}_y^s$ can also be expressed as

$$\bar{U}_y^s = \bar{\bar{\mathcal{R}}}^o(\phi') \bar{U}_{oy} e^{(jk'_x x - jk'_y y + jk'_z z)} + \bar{U}_y^p \tag{4}$$

where $\bar{\bar{\mathcal{R}}}^o(\phi')$ is the o-face FRC, namely

$$\bar{\bar{\mathcal{R}}}^o(\phi') = \begin{bmatrix} \mathcal{R}_e^o(\phi') & 0 \\ 0 & \mathcal{R}_h^o(\phi') \end{bmatrix} \tag{5}$$

where

$$\mathcal{R}_{e,h}^o(\phi') = \frac{P_{e,h}^o(\phi')}{Q_{e,h}^o(\phi')} \tag{6}$$

with

$$P_{e,h}^o(\phi') = \sin\phi' - \delta_{e,h}^o / \sin\beta'_o \tag{7}$$

and

$$Q_{e,h}^o(\phi') = \sin\phi' + \delta_{e,h}^o / \sin\beta'_o \tag{8}$$

where

$$\delta_e^o = -j\mathcal{Y}_d \mathcal{N} \cot(\mathcal{N}\tau k_d), \quad \delta_h^o = j\mathcal{Z}_d \mathcal{N} \tan(\mathcal{N}\tau k_d) \tag{9}$$

with $k_d = k\sqrt{\epsilon_r \mu_r}$, $\mathcal{Z}_d = \sqrt{\mu_r/\epsilon_r}$, $\mathcal{Y}_d = 1/\mathcal{Z}_d$, $\mathcal{N} = \sqrt{1 - \eta \sin^2\beta'_o \sin^2\phi'}$ and $\eta = 1/\mu_r \epsilon_r$. The first term on the RHS of (4) is chosen here to correspond to the field reflected from an "unperturbed" surface of a thin material slab of infinite extent on a PEC plane with the same material and thickness as that on the o-face ($x > 0$, $y = 0$, z). The second term on the RHS of (4) constitutes "perturbation" to the first term which results from the fact that the actual problem in Fig. 1(b) contains a PEC for the n-face ($x < 0$, $y = 0$, z). The "perturbed" field $\bar{U}_y^p$ can be expressed in a manner similar to that done earlier in [7] for the 2-D case. One can start to rewrite (5) in [7] as

$$u_z^p(\rho,\phi) \cong -\frac{1}{2\pi j}\int_{C_\alpha} d\alpha \mathcal{R}_{e,h}(\alpha)\left[\frac{u_{oz}}{\cos\alpha+\cos\phi'}\right]e^{-jk\rho\cos(\alpha-\phi)}. \quad (10)$$

where the following identity has been employed, namely:

$$\sec\left(\frac{\alpha+\phi'}{2}\right)\mp\sec\left(\frac{\alpha-\phi'}{2}\right)=\frac{4}{\cos\alpha+\cos\phi'}\left\{\begin{matrix}\sin\alpha/2\sin\phi'/2\\ \cos\alpha/2\cos\phi'/2\end{matrix}\right\} \quad (11)$$

with

$$\mathcal{R}_e(\alpha)=[\mathcal{R}_e^o(\alpha)-\mathcal{R}_e^n(\alpha)]\,2\cos\alpha/2\cos\phi'/2$$
$$\mathcal{R}_h(\alpha)=[\mathcal{R}_h^o(\alpha)-\mathcal{R}_h^n(\alpha)]\,2\sin\alpha/2\sin\phi'/2$$

and u_{oz} is E_{oz} (or H_{oz}) for the TE (or TM) case. Then the integral in (10) can be expressed in the $\tilde{k}_x$ plane (rectangular coordinate system) as

$$u_z^p(x,y) \cong -\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{d\tilde{k}_x}{\tilde{k}_y}\mathcal{R}_{e,h}(\alpha)\left[\frac{u_{oz}}{\tilde{k}_x+\tilde{k}'_x}\right]e^{-j\tilde{k}_x x-j\tilde{k}_y y}. \quad (12)$$

in which $\tilde{k}_x = k\cos\alpha$, $\tilde{k}_y = k\sin\alpha$ and $\tilde{k}'_x = k\cos\phi'$. The u_z^p denotes the perturbed field corresponding to E_z^p (or H_z^p) for TE (or TM) and the u_{oz} denotes E_{oz} (or H_{oz}) for TE (or TM) which is assumed to be unity here. Next one can obtain the 2-D normal component of the perturbed field u_y^p from u_z^p in (12) by using Maxwell's equations. One can then employ the inverse Fourier transform and with k replaced by $k_t = k\sin\beta_o$. The 3-D PWS integral can be conjectured from 2-D PWS integral as

$$\bar{U}_y^p(x,y,z) \sim -\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{dk_x}{k_y}\bar{\bar{\mathcal{R}}}(\alpha)\cdot\left[\frac{\bar{U}_{oy}}{k_x+k'_x}+\overline{W}\right]e^{(-jk_x x-jk_y y+jk'_z z)} \quad (13)$$

where $\bar{U}_y^p$ denotes $\hat{y}\begin{bmatrix}E_y^p\\ \eta_o H_y^p\end{bmatrix}$. The k_x and k_y are given by

$$k_x = k\sin\beta_o\cos\alpha; \quad k_y = k\sin\beta_o\sin\alpha.$$

The $\overline{W}$ is an unknown constant column vector which was absent in the 2-D situation and $\overline{W}$ is $\hat{y}\begin{bmatrix} A \\ B \end{bmatrix}$. It is necessary to introduce this unknown constant in this 3-D situation to suppress the nonphysical poles, which may occur in the tangential field components E_z and H_z for the skew incidence case. In addition, this unknown constant $\overline{W}$ will make the 3-D PWS integral in (13) to recover the 2-D PWS integral when $\beta'_o \to \pi/2$ or at normal incidence. This unknown constant $\overline{W}$ will be determined later. It is noted that one can obtain the same PWS integral as shown in (13) if the same ansatz as explained in [7] is used to heuristically synthesize the PWS integral from the available 3-D W-H in [8]. The $\bar{\bar{\mathcal{R}}}$ in (13) is given by

$$\bar{\bar{\mathcal{R}}}(\alpha) = \begin{bmatrix} \mathcal{R}_h(\alpha) & 0 \\ 0 & \mathcal{R}_e(\alpha) \end{bmatrix} \tag{14}$$

In the above, the reflection coefficient $\mathcal{R}_e^n(\alpha) = -1$ and $\mathcal{R}_h^n(\alpha) = 1$ for the n-face because it is PEC in this special canonical problem. It is important to note that the $\bar{U}_y^p(x, y, z)$ in (13) can be recovered from the $u_y^p(x, y)$ when the plane wave is normal incident ($\beta'_o = \pi/2$) where $\cos\beta'_o = \hat{s}^i \cdot \hat{z}$. From (13), one can easily obtain the vector potentials $\bar{A} = \hat{y}A_y$ and $\bar{F} = \hat{y}F_y$ directly in terms of E_y^p and H_y^p from the usual relations between fields and potentials [16]. Thus, from (13), one can obtain

$$A_y = -\frac{\omega\mu_o\epsilon_o}{2\pi}\int_{-\infty}^{\infty}\frac{dk_x}{k_y}\mathcal{R}_h(\alpha)\left[\frac{E_{oy}}{k_x + k'_x} + A\right]\frac{1}{k^2 - k_y^2}e^{(-jk_x x - jk_y y + jk'_z z)} \tag{15}$$

and

$$F_y = -\frac{\omega\mu_o\epsilon_o}{2\pi\eta_o}\int_{-\infty}^{\infty}\frac{dk_x}{k_y}\mathcal{R}_e(\alpha)\left[\frac{\eta_o H_{oy}}{k_x + k'_x} + B\right]\frac{1}{k^2 - k_y^2}e^{(-jk_x x - jk_y y + jk'_z z)} \tag{16}$$

where ϵ_o and μ_o are the permittivity and permeability of free space as usual. Next all the remaining components of the external electric and magnetic fields can be found from these vector potentials. It follows that the "perturbed" tangential field components E_z^p and H_z^p can thus be expressed as

$$\begin{aligned} E_z^p(x, y, z) \sim \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{dk_x}{k_y(k^2 - k_y^2)}&\left\{kk_x\mathcal{R}_e(\alpha)\left[\frac{\eta_o H_{oy}}{k_x + k'_x} + B\right]\right. \\ &\left. + k_z k_y\mathcal{R}_h(\alpha)\left[\frac{E_{oy}}{k_x + k'_x} + A\right]\right\}e^{(-jk_x x - jk_y y + jk'_z z)} \end{aligned} \tag{17}$$

$$\eta_o H_z^p(x,y,z) \sim \frac{1}{2\pi j}\int\limits_{-\infty}^{\infty} \frac{dk_x}{k_y(k^2-k_y^2)}\left\{k_y k_z \mathcal{R}_e(\alpha)\left[\frac{\eta_o H_{oy}}{k_x+k'_x}+B\right]\right.$$
$$\left. -kk_x\mathcal{R}_h(\alpha)\left[\frac{E_{oy}}{k_x+k'_x}+A\right]\right\} e^{(-jk_x x-jk_y y+jk'_z z)} \quad (18)$$

where $k_y^2 = k_t^2 - k_x^2$ and $k_z^2 = k^2 - k_t^2$. One notes that from the two preceding equations there are two poles at $k_x = \pm jk'_z$ in (17) and (18), whose residues introduce spurious field contributions which do not have a physical meaning. Therefore, one needs to remove those spurious residues. The latter can be suppressed by using the unknown constant $\overline{W}$, which was introduced earlier in (13), only for this purpose, namely to suppress those spurious two poles. It follows from (17) that

$$\pm j\mathcal{R}_e^{\pm}(\alpha)\left[\frac{\eta_o H_{oy}}{k_x^{\pm}+k'_x}+B\right] = -\mathcal{R}_h^{\pm}(\alpha)\left[\frac{E_{oy}}{k_x^{\pm}+k'_x}+A\right] \quad (19)$$

where the superscript $\pm$ corresponds to the residue at the pole location $k_x = \pm jk'_z$. One can solve for the unknown constants A and B from (19).

It is more convenient to evaluate the integration in the angular spectral domain; hence, one introduces a transformation, $k_x = k\sin\beta_o\cos\alpha$, $k_y = k\sin\beta_o\sin\alpha$, and $k_z = k\cos\beta_o$, in which the α is a complex angular spectral variable. Also one replaces the x and y by the cylindrical polar coordinate quantities $\rho\cos\phi$ and $\rho\sin\phi$, respectively with $\rho = \sqrt{x^2+y^2}$. The expressions in (17) and (18) can be expressed in the new α-domain as follows:

$$E_z^p(\bar{r}) \sim -\frac{1}{2\pi j}\int_{C_\alpha} d\alpha \frac{\sin\beta_o}{\Delta(\alpha)}\left\{\cos\alpha\mathcal{R}_e(\alpha)\left[\frac{\eta_o H_{oy}}{\cos\alpha+\cos\phi'}+B\right]\right.$$
$$\left.+\cos\beta_o\sin\alpha\mathcal{R}_h(\alpha)\left[\frac{E_{oy}}{\cos\alpha+\cos\phi'}+A\right]\right\}e^{-jk\rho\sin\beta_o\cos(\alpha-\phi)}e^{jkz\cos\beta'_o} \quad (20)$$

$$\eta_o H_z^p(\bar{r}) \sim -\frac{1}{2\pi j}\int_{C_\alpha} d\alpha \frac{\sin\beta_o}{\Delta(\alpha)}\left\{-\cos\alpha\mathcal{R}_h(\alpha)\left[\frac{E_{oy}}{\cos\alpha+\cos\phi'}+A\right]\right.$$
$$\left.+\cos\beta_o\sin\alpha\mathcal{R}_e(\alpha)\left[\frac{\eta_o H_{oy}}{\cos\alpha+\cos\phi'}+B\right]\right\}e^{-jk\rho\sin\beta_o\cos(\alpha-\phi)}e^{jkz\cos\beta'_o} \quad (21)$$

where $\Delta(\alpha) = 1-\sin^2\beta_o\sin^2\alpha$ and

$$A = \frac{\sin\beta_o\cos\beta_o}{\Delta(\phi')}\left[\xi\mathcal{R}^+\mathcal{R}^- E_{oy} - \{\cos\phi'\tan\beta_o + j\zeta\}\eta_o H_{oy}\right]$$

$$B = -\frac{\sin\beta_o\cos\beta_o}{\Delta(\phi')}\left[\{\cos\phi'\tan\beta_o + j\zeta\}E_{oy} + \xi\eta_o H_{oy}\right]$$

with $\xi = \frac{2}{\mathcal{R}^+ + \mathcal{R}^-}$, $\zeta = \frac{\mathcal{R}^+ - \mathcal{R}^-}{\mathcal{R}^+ + \mathcal{R}^-}$, and $\mathcal{R}^\pm = \frac{\mathcal{R}_h^\pm}{\mathcal{R}_e^\pm}$, in which

$$\mathcal{R}^\pm = \frac{\left[\mathcal{R}_h\left(\alpha_o^\pm\right) - 1\right] \sin \alpha_o^\pm/2 \sin \phi'/2}{\left[\mathcal{R}_e\left(\alpha_o^\pm\right) + 1\right] \cos \alpha_o^\pm/2 \cos \phi'/2}$$

with $\alpha_o^\pm = \pi/2 \pm j \ln((1 + \cos \beta_o)/\sin \beta_o)$ by using $\cos^{-1} z = \pi/2 + j \ln(jz + \sqrt{1 - z^2})$. On using the identity,

$$\frac{4}{\cos \alpha + \cos \phi'} \begin{Bmatrix} \sin \alpha/2 \sin \phi'/2 \\ \cos \alpha/2 \cos \phi'/2 \end{Bmatrix} = \sec\left(\frac{\alpha + \phi'}{2}\right) \mp \sec\left(\frac{\alpha - \phi'}{2}\right) \quad (22)$$

and after some manipulations, one can have the "perturbed" field in a compact form, namely,

$$\begin{aligned} \bar{U}_{pw}^p(\bar{r}) \sim -\frac{1}{2\pi j} \int_{C_\alpha} d\alpha \frac{\sin \beta_o}{\Delta(\alpha)\Delta(\phi')} \Big\{ & C(\alpha) \overline{\overline{T}}(\alpha) \cdot \bar{\bar{\mathcal{D}}}^c(\alpha, \phi') \\ & + \overline{\overline{T}}_u(\alpha) \cdot \overline{\overline{U}}(\alpha, \phi') + \overline{\overline{T}}_v(\alpha) \cdot \overline{\overline{V}}(\alpha, \phi') \Big\} \\ & \cdot \overline{\overline{T}}(\phi') \cdot \bar{U}_{oz} e^{-jk\rho \sin \beta_o \cos(\alpha - \phi)} e^{jkz \cos \beta_o'} \end{aligned} \quad (23)$$

where $C(\alpha) = \cos^2 \beta_o - \sin^2 \beta_o \cos \alpha \cos \phi'$, $\bar{U}_{pw}^p = \hat{z} \begin{bmatrix} E_z^p \\ \eta_o H_z^p \end{bmatrix}$, and

$$\overline{\overline{T}}(\alpha) = \begin{bmatrix} \cos \alpha & \cos \beta_o \sin \alpha \\ \cos \beta_o \sin \alpha & -\cos \alpha \end{bmatrix}; \quad \overline{\overline{T}}_u(\alpha) = \begin{bmatrix} -\cos \alpha & \cos \beta_o \sin \alpha \\ \cos \beta_o \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\overline{\overline{T}}_v(\alpha) = \begin{bmatrix} -\cos \beta_o \sin \alpha & \cos \alpha \\ \cos \alpha & \cos \beta_o \sin \alpha \end{bmatrix}$$

$$\overline{\overline{U}}(\alpha, \phi') = \begin{bmatrix} U_e & 0 \\ 0 & U_h \end{bmatrix}; \qquad \overline{\overline{V}}(\alpha, \phi') = \begin{bmatrix} V_h & 0 \\ 0 & V_e \end{bmatrix}$$

$$\bar{\bar{\mathcal{D}}}^c(\alpha, \phi') = \begin{bmatrix} \mathcal{D}_e^c & 0 \\ 0 & \mathcal{D}_h^c \end{bmatrix}; \quad \bar{U}_{oz} = \hat{z} \begin{bmatrix} E_{oz} \\ \eta_o H_{oz} \end{bmatrix},$$

with

$$\begin{aligned} U_e &= \sin \beta_o \cos \beta_o 2 \cos \alpha/2 \cos \phi'/2 (\mathcal{R}_e + 1) j\zeta \\ U_h &= \sin \beta_o \cos \beta_o 2 \sin \alpha/2 \sin \phi'/2 (\mathcal{R}_h - 1) j\zeta \\ V_e &= \sin \beta_o \cos \beta_o 2 \cos \alpha/2 \cos \phi'/2 (\mathcal{R}_e + 1) \xi \mathcal{R}^+ \mathcal{R}^- \\ V_h &= \sin \beta_o \cos \beta_o 2 \sin \alpha/2 \sin \phi'/2 (\mathcal{R}_h - 1) j\xi. \end{aligned}$$

The dyads $\overline{\overline{T}}$, $\overline{\overline{T}}_u$, $\overline{\overline{T}}_v$, $\overline{\overline{U}}$ and $\overline{\overline{V}}$ in the above are expressed in matrix form for convenience. The $\mathcal{D}_{e,h}^c$ are defined as

$$\mathcal{D}_{e,h}^c(\alpha, \phi') = \pm \frac{1}{2} \left[\mathcal{R}_{e,h}^o(\alpha) - \mathcal{R}_{e,h}^n(\alpha)\right] \left[\sec\left(\frac{\alpha - \phi'}{2}\right) \pm \sec\left(\frac{\alpha + \phi'}{2}\right)\right], \quad (24)$$

with $\mathcal{R}^o_{e,h}(\alpha)$ from (6) and $\mathcal{R}^n_{e,h}(\alpha) = \mp 1$. It is noted that one can write $\bar{U}_{oy}$ in terms of $\bar{U}_{oz}$ by employing incident vector potentials, A^i_z and F^i_z, which can be obtained via inspection, namely:

$$A^i_z = \frac{j\omega\mu_o\epsilon_o}{k^2 - k'^2_z}E^i_z; \quad F^i_z = \frac{j\omega\mu_o\epsilon_o}{k^2 - k'^2_z}H^i_z$$

where $E^i_z = E_{oz}e^{(jk'_x x + jk'_y y + jk'_z z)}$ and $H^i_z = H_{oz}e^{(jk'_x x + jk'_y y + jk'_z z)}$. The normal field components of incident field denoted by $\bar{U}_{oy}$ can then be obtained from the incident vector potentials, A^i_z and F^i_z. The result provides the transformation matrix $\bar{\bar{T}}(\phi')$ given above.

Next, the (23) can be evaluated by using the SDP asymptotic integration technique when κ is large, where $\kappa = k\rho\sin\beta_o$. One can rewrite (23) symbolically as

$$\bar{U}^p_{pw} \sim \int_{C_\alpha} d\alpha \overline{\mathcal{F}}(\alpha)e^{\kappa f(\alpha)}, \quad 0 \le \phi \le \pi \tag{25}$$

where

$$\begin{aligned}\overline{\mathcal{F}} \;=\; & -\frac{1}{2\pi j}\frac{\sin\beta_o}{\Delta(\alpha)\Delta(\phi')}\Big\{C(\alpha)\bar{\bar{T}}(\alpha)\cdot\bar{\bar{\mathcal{D}}}^c(\alpha,\phi') + \bar{\bar{T}}_u(\alpha)\cdot\bar{\bar{U}}(\alpha,\phi') \\ & +\bar{\bar{T}}_v(\alpha)\cdot\bar{\bar{V}}(\alpha,\phi')\Big\}\cdot\bar{\bar{T}}(\phi')\cdot\bar{U}_{oz}e^{jk\cos\beta'_o z}\end{aligned}$$

and $f(\alpha) = -j\cos(\alpha - \phi)$. It is noted that the $\overline{\mathcal{F}}$ has poles at $\alpha_r = \pi - \phi'$, and α_{swo}. Deforming the contour C_α to the SDP contour allows one to express (23) as

$$\begin{aligned}&\bar{U}^p_{pw} \sim -2\pi j[Res\{\overline{\mathcal{F}}(\alpha_r)e^{\kappa f(\alpha_r)}\}U(\alpha_r - \phi) \\ &+Res\{\overline{\mathcal{F}}(\alpha_{swo})e^{\kappa f(\alpha_{swo})}\}U(\alpha_{swo} - \phi)] + \int_{SDP} d\alpha\overline{\mathcal{F}}(\alpha)e^{\kappa f(\alpha)}\end{aligned} \tag{26}$$

where α_r is the GO reflected wave pole which provides the GO reflected field contributions, $\bar{U}^{ro}_{pw}$ and $\bar{U}^{rn}_{pw}$. The α_{swo} is the SW pole, which yields either a forward surface wave (FSW) or a backward surface wave (BSW) field contribution, $\bar{U}^{sw}_{pw}$. The $U(\cdot)$ is the usual Heaviside unit step function. Applying Cauchy's residue theorem, one can obtain $\bar{U}^{ro}_{pw}$, $\bar{U}^{rn}_{pw}$, and $\bar{U}^{sw}_{pw}$ as follows:

$$\begin{aligned}\bar{U}^{ro}_{pw} \;=\; & -\frac{1}{\Delta(\phi')}\bar{\bar{T}}(\pi - \phi')\cdot\bar{\bar{\mathcal{R}}}^o(\phi')\cdot\bar{\bar{T}}(\phi') \\ & \cdot\bar{U}_{oz}e^{jk\rho\sin\beta_o\cos(\phi+\phi')}e^{jkz\cos\beta'_o}U(\pi - \phi' - \phi)\end{aligned} \tag{27}$$

$$\bar{U}^{rn}_{pw} \;=\; \bar{\bar{\mathcal{R}}}^n\cdot\bar{U}_{oz}e^{jk\rho\sin\beta_o\cos(\phi+\phi')}e^{jkz\cos\beta'_o}U(\pi - \phi' - \phi) \tag{28}$$

$$\begin{aligned}\bar{U}_{pw}^{swo} &= \frac{C(\alpha_{sw}^o)}{\Delta(\alpha_{sw}^o)\Delta(\phi')}\overline{\overline{T}}(\alpha_{sw}^o)\cdot\ \bar{\bar{R}}^{swo}(\alpha_{sw}^o)\cdot\overline{\overline{T}}(\phi') \\ &\quad\cdot\bar{U}_{oz}e^{jk\rho\sin\beta_o\cos(\alpha_{sw}^o-\phi)}e^{jkz\cos\beta_o'}U(\alpha_{sw}^o-\phi) \end{aligned} \tag{29}$$

where $\bar{\bar{\mathcal{R}}}^o$ is defined in (5), $\bar{\bar{\mathcal{R}}}^n = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$, and $\bar{\bar{R}}^{swo} = \begin{bmatrix}R_e^{swo} & 0\\ 0 & R_h^{swo}\end{bmatrix}$. The $R_{e,h}^{swo}$ is defined as

$$R_{e,h}^{swo}(\alpha_{sw}^o,\phi') = \pm\frac{P_{e,h}^o(\alpha_{sw}^o)}{2Q_{e,h}^{o'}(\alpha_{sw}^o)}\left[\sec\left(\frac{\alpha_{sw}^o-\phi'}{2}\right)\pm\sec\left(\frac{\alpha_{sw}^o+\phi'}{2}\right)\right]. \tag{30}$$

The $Q_{e,h}^{o'}(\alpha_{sw}^o)$ is the derivative of $Q_{e,h}^o(\alpha)$ in (8) with respect to α and evaluated at $\alpha=\alpha_{sw}^o$. The $\bar{U}_{pw}^{ro}$ and $\bar{U}_{pw}^{rn}$ represents the GO fields reflected from o and n-face, respectively. A closed form evaluation of the SDP integral in (26) via the non-uniform steepest descent method, yields the non uniform diffracted field $\bar{U}_{pw}^d$ as

$$\bar{U}_{pw}^d \sim \frac{1}{\Delta(\phi)\Delta(\phi')\sin\beta_o}\left[C(\phi,\phi')\overline{\overline{T}}(\phi)\cdot\bar{\bar{\mathcal{D}}}^c(\phi,\phi')\cdot\overline{\overline{T}}(\phi')+\overline{\overline{W}}\right]\cdot\bar{U}_{oz}\frac{e^{-jks}}{\sqrt{s}} \tag{31}$$

where $\bar{\bar{\mathcal{D}}}^c = \begin{bmatrix}\mathcal{D}_e^c & 0\\ 0 & \mathcal{D}_h^c\end{bmatrix}$ with $\mathcal{D}_{e,h}^c$ is defined as

$$\mathcal{D}_{e,h}^c(\phi,\phi') = \pm\frac{1}{2}\left[\mathcal{R}_{e,h}^o(\phi)-\mathcal{R}_{e,h}^n(\phi)\right]\left[\sec\left(\frac{\phi-\phi'}{2}\right)\pm\sec\left(\frac{\phi+\phi'}{2}\right)\right], \tag{32}$$

and where $\mathcal{R}_{e,h}^{o,n}$ are defined above. The $\overline{\overline{W}}$ is given by

$$\overline{\overline{W}} = -\frac{e^{-j\pi/4}}{\sqrt{2\pi k}}\left[\overline{\overline{T}}_u(\phi)\cdot\overline{\overline{U}}+\overline{\overline{T}}_v(\phi)\cdot\overline{\overline{V}}\right]\cdot\overline{\overline{T}}(\phi'). \tag{33}$$

The total field $\bar{U}_{pw}$ can be obtained by

$$\bar{U}_{pw} \sim \bar{U}_{pw}^i + \bar{U}_{pw}^s. \tag{34}$$

The $\hat{z}$-directed tangential component of a uniform incident plane wave $\bar{U}_{pw}^i$ is $\hat{z}\begin{bmatrix}E_{pw}^i\\ \eta_o H_{pw}^i\end{bmatrix}$, where E_{pw}^i (or H_{pw}^i) is the electric (or magnetic) incident field, which is given by

$$\bar{U}_{pw}^i = \bar{U}_{oz}e^{(jk_x'x+jk_y'y+jk_z'z)} \tag{35}$$

where as before $\bar{U}_{oz}$ denotes $\hat{z}\begin{bmatrix}E_{oz}\\ \eta_o H_{oz}\end{bmatrix}$. The E_{oz} and H_{oz} are assumed to be unity here. The $\bar{U}_{pw}^s$ can be found by using the same approach for finding the $\bar{U}^p_{pw}$ from $\bar{U}^p_y$. One can then obtain the $\bar{U}^s_{pw}$ as

$$\bar{U}^s_{pw} \sim \frac{1}{\Delta(\phi')} \bar{\bar{T}}(\pi-\phi')\cdot\bar{\bar{R}}^o(\phi')\cdot\bar{\bar{T}}(\phi')\cdot\bar{U}_{oz} e^{jk\rho\sin\beta_o\cos(\phi+\phi')} e^{jkz\cos\beta'_o} + \bar{U}^p_{pw}. \tag{36}$$

By substituting (36) and (26) (together with (27)–(31)) into (34), one can obtain the total $\hat{z}$-directed tangential field components $\bar{U}_{pw}$.

It is important to note that the solution in (31) still satisfies all the crucial physical properties, such as the PEC boundary condition on the n-face, the Karp-Karal lemma on the o-face despite the approximations used to arrive at (23). Furthermore, the approximated solution in (31) still recovers the PEC solution when the material slab is removed. Thus, the solution in (26) (and (31)), which is based on the approximate expression of (23), clearly retains many of the important physical properties, thereby lending more confidence to the heuristic approximation of (23). However, the analytical expression for the approximate diffracted $\bar{U}^d_{pw}$ in (31) does not satisfy reciprocity, but is expected to provide numerical results which nearly satisfy the reciprocity principle. On the other hand, the technique shown in [7] can be easily applied to restore the reciprocity condition into the $\bar{U}^d_{pw}$ in (31). The desired ansatz is now established by the set of Equations (34)–(36) and (23), respectively which allows one to obtain a corresponding solution for the case of spherical wave incidence.

3. EXTENSION TO TREAT THE CASE OF SPHERICAL WAVE EXCITATION WITH ONE FACE BEING PEC

A UTD solution for a thin planar material half plane on an entire PEC ground plane illuminated by spherical wave or an elemental current moment is developed in this section. A spherical wave with an arbitrary field polarization transverse to the incident ray direction, $\hat{s}^i$, can be created by superimposing the fields of $\hat{z}$-directed electric and magnetic current moments at the origin of the spherical wave. The incident, $\hat{z}$-directed, electric field, E^i_z, (or the magnetic field, H^i_z) at an observer location $\bar{r}(\rho,\phi,z)$, which is produced by an electric (or magnetic) current moment of strength $d\bar{p}_e = \hat{z}dp_{ez}$ (or $d\bar{p}_m = \hat{z}dp_{mz}$) at $\bar{r}'(\rho',\phi',z)$, respectively, can be expressed as

$$\bar{U}^i_z \equiv \hat{z}\begin{Bmatrix} E^i_z \\ H^i_z \end{Bmatrix} = -jk\hat{z}\begin{Bmatrix} Z_o dp_{ez} \\ Y_o dp_{mz} \end{Bmatrix} \sin^2\beta_o \tilde{G}_o\left(k\left|\bar{r}-\bar{r}'\right|\right) \tag{37}$$

and

$$\tilde{G}_o\left(k\left|\bar{r}-\bar{r}'\right|\right) = \frac{e^{-jkS^i}}{4\pi S^i} \tag{38}$$

$$S^i = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi') + (z - z')^2}. \tag{39}$$

If one assumes that the current moment is sufficiently far from the o face so that the incident spherical wave generates "locally" plane at the line of discontinuity (or edge), then one may employ (34) and (36) obtained for the skew incident plane wave case to provide the $\hat{z}$ components of the scattered field for the spherical wave incidence case as:

$$\begin{aligned}\bar{U}_z^s \sim -jk \begin{Bmatrix} Z_o dp_{ez} \\ Y_o dp_{mz} \end{Bmatrix} \sin^2 \beta_o \frac{1}{\Delta(\phi')} \\ \cdot \bar{\bar{T}}(\pi - \phi') \cdot \bar{\bar{\mathcal{R}}}^o(\phi') \cdot \bar{\bar{T}}(\phi') \cdot \bar{U}_{oz} \frac{e^{-jkS^r}}{\sqrt{S^r}} + \bar{U}_z^p\end{aligned} \tag{40}$$

where the first term on the RHS of (40) represents as before the field scattered from the "unperturbed" structure, which is assumed to be a thin planar material slabs of infinite extent with PEC backing that has the same thickness and electrical parameters as the PEC backed material pertaining to the o-face in the actual or original problem geometry of Fig. 1(b). Also the $\bar{U}_{oz}$ in (40) is defined in Section 2. Under the present assumption of source far from the surface at $y = 0$, one can show that the unperturbed scattered field is asymptotically given by the first term on the RHS of (40) which is the GO reflected field, where $\bar{\bar{\mathcal{R}}}^o$ is the FRC for this unperturbed material surface with PEC ground plane, and S^r is the GO ray path corresponding to the GO field reflected from that unperturbed surface. The $\bar{\bar{\mathcal{R}}}^o(\phi')$ is defined in (5). Also,

$$S^r = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi + \phi') + (z - z')^2}. \tag{41}$$

The $\bar{U}_z^p$ in (40) represents the "perturbation" to the first term on the RHS of (40). By employing an inverse Fourier integral transformation, one can find the $\bar{U}_z^p$ for the 3-D spherical wave incidence case from the u_z^p for the 2-D cylindrical wave incidence case as explained in [17], namely

$$\bar{U}_z^p = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{C_\alpha} \left(-\frac{1}{2\pi j} \right) \bar{\mathcal{A}}(\alpha, \phi') G_o[k_t s(\alpha)] e^{-jk_z(z'-z)} \, d\alpha \right] dk_z. \tag{42}$$

The $\bar{\mathcal{A}}(\alpha, \phi')$ is an appropriate spectral amplitude or weight function, and $G_o[k_t s(\alpha)]$ is defined as

$$G_o[ks(\alpha)] = \frac{-j}{4} H_o^{(2)}[ks(\alpha)] \tag{43}$$

with k replaced by k_t, in which $k_t^2 = k^2 - k_z^2$. By following the steps in [17], one can further rewrite (42) in terms of the modified cylindrical

Bessel function of the second kind and of order zero, $K_o[jk_t s(\alpha)]$, to yield

$$\bar{U}_z^p = -\frac{1}{8\pi^3 j}\int_{C_\alpha} d\alpha \bar{\mathcal{A}}(\alpha,\phi')\int\limits_{-\infty}^{\infty} dk_z K_o[jk_t s(\alpha)]e^{-jk_z(z'-z)} \tag{44}$$

where for large $k_t s(\alpha)$

$$K_o[j\vartheta(\varsigma)] = \sqrt{\frac{\pi}{2j\vartheta(\varsigma)}}e^{-j\vartheta(\varsigma)}.$$

It is noted that $K_o(j\vartheta) = -j\frac{\pi}{2}H_o^{(2)}(\vartheta)$. One can next apply the following identity

$$\int_{-\infty}^{\infty} dk_z H_o^{(2)}[k_t s(\alpha)]e^{-jk_z(z'-z)} \equiv 2j\frac{e^{-jkS(\alpha)}}{S(\alpha)} \tag{45}$$

in (44). This leads to

$$\bar{u}_z^p = -\frac{1}{2\pi j}\int_{C_\alpha} d\alpha \bar{\mathcal{A}}(\alpha,\phi')\frac{e^{-jkS(\alpha)}}{4\pi S(\alpha)} \tag{46}$$

with

$$S(\alpha) = \sqrt{\rho^2+\rho'^2+2\rho\rho'\cos(\alpha-\phi)+(z-z')^2} \tag{47}$$

It is important to note that if the current moment is not assumed to be sufficiently far from the o and n faces, then additional contributions (not present in (40)) must be included. Such additional contributions arise because the current moment can in some situations strongly excite SWs directly in the material; these SWs become incident on the discontinuity at “0” to produce a reflected SW and a transmitted SW, as well as a diffracted space wave. The reflected and transmitted SWs can be deduced from the W-H solution to appropriate, simpler, canonical two-part diffraction problems in which the excitation is an incident SW. In the radiation problem, these SW effects are not significant. Only the diffraction of the incident SW by the discontinuity contributes to the radiation field; its effect is discussed in [18] for a line source excitation. The spectral function $\bar{\mathcal{A}}$ is proportional to the strength of the current moment, and it may be expressed as

$$\bar{\mathcal{A}}(\alpha,\phi') \equiv -jk\begin{Bmatrix} Z_o dp_{ez} \\ Y_o dp_{mz}\end{Bmatrix}\sin^2\beta_o \bar{\bar{\mathcal{D}}}(\alpha,\phi')\cdot\bar{U}_{oz} \tag{48}$$

where the unknown spectral weight $\bar{\bar{\mathcal{D}}}$ is to be determined using the ansatz of Section 2. Let $\rho' = s'\sin\beta_o$, $\rho = s\sin\beta_o$, and $z = -s\cos\beta_o$. Thus,

$$S^2(\alpha) = (s+s')^2\left\{1-\frac{2ss'}{(s+s')^2}\sin^2\beta_o[1-cos(\alpha-\phi)]\right\} \tag{49}$$

In order to identify $\bar{\bar{\mathcal{D}}}$, the exponential in (46) with (49) may be approximated by the first two terms of its binomial expansion for large $k\frac{ss'}{s+s'}\sin^2\beta_o$, which is assumed here to be the large parameter (for the asymptotic development). Then, (46) becomes

$$\bar{U}_z^p \sim -\frac{1}{2\pi j}\int_{C_\alpha}\bar{\mathcal{A}}(\alpha,\phi')\frac{e^{-jk(s+s')}}{4\pi S(\alpha)}e^{jk\frac{ss'}{s+s'}\sin^2\beta_o[1-cos(\alpha-\phi)]}\,d\alpha. \quad (50)$$

If the current moment is allowed to receded to infinity, i.e., if $s'\to\infty$, while s is kept finite, then one obtains the scattered field $\bar{U}_{pw}^p$ due to plane wave illumination, namely,

$$\bar{U}_z^p \sim C_o(ks')\bar{U}_{pw}^p \quad (51)$$

where C_o is the current moment factor given by

$$C_o(ks') = -jk\begin{Bmatrix} Z_o dp_{ez} \\ Y_o dp_{mz}\end{Bmatrix}\sin^2\beta_o'\frac{e^{-jks'}}{4\pi s'} \quad (52)$$

and

$$\bar{U}_{pw}^p = -\frac{1}{2\pi j}\int_{C_\alpha}\bar{\bar{\mathcal{D}}}(\alpha,\phi')\cdot\bar{U}_{oz}e^{-jk\rho\sin\beta_o\cos(\alpha-\phi)}e^{jkz\cos\beta_o'}\,d\alpha. \quad (53)$$

By directly comparing (53) with the desired ansatz in (23), one can easily identify $\bar{\bar{\mathcal{D}}}$ by inspection to be

$$\begin{aligned}\bar{\bar{\mathcal{D}}}(\alpha,\phi') \;=\; & \frac{\sin\beta_o}{\Delta(\alpha)\Delta(\phi')}\Big\{C(\alpha)\overline{\overline{T}}(\alpha)\cdot\bar{\bar{\mathcal{D}}}^c(\alpha,\phi') \\ & +\overline{\overline{T}}_u(\alpha)\cdot\overline{\overline{U}}(\alpha,\phi')+\overline{\overline{T}}_v(\alpha)\cdot\overline{\overline{V}}(\alpha,\phi')\Big\}\cdot\overline{\overline{T}}(\phi'). \quad (54)\end{aligned}$$

4. ASYMPTOTIC ANALYSIS

Next one can asymptotically evaluate, in closed form, the integral in (50) by using the steepest descent method. One can start this evaluation by rewriting (50) symbolically as

$$\bar{U}_z^p \sim \int_{C_\alpha} d\alpha\overline{\mathcal{F}}(\alpha)e^{\kappa f(\alpha)}, \quad 0\le\phi\le\pi \quad (55)$$

where the κ denotes $k\frac{ss'}{s+s'}\sin\beta_o^2$, $f(\alpha)=j[1-\cos(\alpha-\phi)]$ and

$$\begin{aligned}\overline{\mathcal{F}} = & -\frac{1}{8\pi^2 j}\begin{Bmatrix} Z_o dp_{ez} \\ Y_o dp_{mz}\end{Bmatrix}\sin^2\beta_o'\frac{\sin\beta_o}{\Delta(\alpha)\Delta(\phi')}\Big\{C(\alpha)\overline{\overline{T}}(\alpha)\cdot\bar{\bar{\mathcal{D}}}^c(\alpha,\phi') \\ & +\overline{\overline{T}}_u(\alpha)\cdot\overline{\overline{U}}(\alpha,\phi')+\overline{\overline{T}}_v(\alpha)\cdot\overline{\overline{V}}(\alpha,\phi')\Big\}\cdot\overline{\overline{T}}(\phi')\cdot\bar{U}_{oz}\frac{e^{-jk(s+s')}}{S(\alpha)}.\end{aligned}$$

After deforming the integral contour of (55) to the steepest descent path (SDP) through the saddle point at $\alpha \equiv \alpha_s = \phi$, it allows one to express (55) for large κ as

$$\begin{aligned}\bar{U}_z^p &\sim -2\pi j Res\left\{\overline{\mathcal{F}}(\alpha_r)e^{\kappa f(\alpha_r)}\right\}U(\alpha_r-\phi)\\ &-2\pi j Res\left\{\overline{\mathcal{F}}(\alpha_{sw}^o)e^{\kappa f(\alpha_{sw}^o)}\right\}U\left(\alpha_{sw}^o-\phi\right)+\int\limits_{SDP} d\alpha\overline{\mathcal{F}}(\alpha)e^{\kappa f(\alpha)}. \quad (56)\end{aligned}$$

The reflected field $\bar{U}_z^r$ is given by the sum of the "unperturbed" GO reflected field contained in the first term on the RHS of (40) together with the residue contribution from the pole at $\alpha = \alpha_r = \pi - \phi'$ in (56), as

$$\bar{U}_z^r = -jk\left\{\begin{matrix} Z_o dp_{ez} \\ Y_o dp_{mz}\end{matrix}\right\}\sin^2\beta_o'\frac{1}{\Delta(\phi')}\overline{\overline{T}}(\pi-\phi')\cdot\bar{\bar{\mathcal{R}}}(\phi')\cdot\overline{\overline{T}}(\phi')\cdot\bar{U}_{oz}\frac{e^{-jkS_r}}{4\pi S_r} \quad (57)$$

where S_r is defined in (41), and

$$\bar{\bar{\mathcal{R}}}(\phi') = \begin{cases}\bar{\bar{\mathcal{R}}}^o(\phi') & \text{if } \phi+\phi'<\pi \\ \bar{\bar{\mathcal{R}}}^n(\phi') & \text{if } \phi+\phi'>\pi\end{cases} \quad (58)$$

in which $\bar{\bar{\mathcal{R}}}^{o,n}$ denotes the FRC as defined earlier. Also, $\bar{U}_z^{sw}$ is given by the residue arising from the SW pole $\alpha = \alpha_{sw}^o$ in (56) as

$$\begin{aligned}\bar{U}_z^{sw} &= \frac{C(\alpha_{sw}^o)}{\Delta(\alpha_{sw}^o)\Delta(\phi')}\overline{\overline{T}}(\alpha_{sw}^o)\cdot\bar{\bar{R}}^{swo}(\alpha_{sw}^o)\cdot\overline{\overline{T}}(\phi')\\ &\quad\cdot\bar{U}_{oz}\frac{e^{-jkS(\alpha_{sw}^o,\phi)}}{4\pi S(\alpha_{sw}^o,\phi)}U\left(\alpha_{sw}^o-\phi\right) \quad (59)\end{aligned}$$

It is assumed that the material slab is sufficiently thin so only the lowest TM surface wave can propagate for the material slab with PEC ground plane. Thus, the $\bar{\bar{R}}^{swo} = \begin{bmatrix} 0 & 0 \\ 0 & R_h^{swo}\end{bmatrix}$. The SDP integral, which yields the diffracted field $\bar{U}_z^d$ symbolically is given by

$$\bar{U}_z^d = \int\limits_{SDP}\overline{\mathcal{F}}(\alpha,\phi')e^{\kappa f(\alpha)}\,d\alpha. \quad (60)$$

As for the normal incidence case discussed in [7], one can decompose the spectral function $\overline{\mathcal{F}}(\alpha)$ in the integrand of (60) in to one contains the GO type pole and remaining part containing the SW type pole. The former can be conveniently evaluated by using the Pauli-Clemmow (PC) approach while the latter can be performed by the Van der

Waerden (VDW) approach. The expression for the UTD first order diffracted field is then found to have the general form as

$$\bar{U}_z^d = \bar{U}_z^i(Q_e) \cdot \bar{\bar{D}}(\phi, \phi')A(s, s')e^{-jks} \tag{61}$$

where $\bar{\bar{D}} = \bar{\bar{D}}^{go} + \bar{\bar{D}}^{sw}$. The $\bar{U}_z^i(Q_e)$ represents the incident field at the point of diffraction Q_e, and $A(s, s')$ is a spread factor given by $A(s, s') = \sqrt{\frac{s'}{s(s+s')}}$. Here s is the distance from Q_e to an observation point, and the s' is the distance from Q_e to the source point. The $\bar{\bar{D}}^{go}$ is based on the PC method while $\bar{\bar{D}}^{sw}$ is based on the VDW method; they are given by

$$\bar{\bar{D}}^{go} = \frac{1}{\Delta(\phi)\Delta(\phi')\sin\beta_o}\left[C(\phi, \phi')\overline{\overline{T}}(\phi) \cdot \bar{\bar{D}}^c(\phi, \phi') \cdot \overline{\overline{T}}(\phi') + \overline{\overline{W}}\right], \tag{62}$$

and

$$\bar{\bar{D}}^{sw} = \frac{1}{\Delta(\alpha_{sw}^o)\Delta(\phi')\sin\beta_o}\left[C\left(\alpha_{sw}^o, \phi'\right)\overline{\overline{T}}(\alpha_{sw}^o) \cdot \bar{\bar{D}}^{csw}(\alpha_{sw}^o, \phi') \cdot \overline{\overline{T}}(\phi')\right] \tag{63}$$

where the $\bar{\bar{D}}^c = \begin{bmatrix} D_e^c & 0 \\ 0 & D_h^c \end{bmatrix}$ and $\bar{\bar{D}}^{csw} = \begin{bmatrix} D_e^{sw} & 0 \\ 0 & D_h^{sw} \end{bmatrix}$ with $D_{e,h}^c$ and $D_{e,h}^{sw}$ are given by

$$D_{e,h}^c(\phi, \phi') = \mp\frac{e^{-j\pi/4}}{2\sqrt{2\pi k}}\left[\Gamma_{e,h}^o(\phi, \phi') - \Gamma_{e,h}^n(\phi, \phi')\right]$$
$$\left[\sec\left(\frac{\phi - \phi'}{2}\right)F_{KP}\left(kLa_{go}^-\right) \pm \sec\left(\frac{\phi + \phi'}{2}\right)F_{KP}\left(kLa_{go}^+\right)\right], \tag{64}$$

and

$$D_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^o) = \mp\frac{e^{-j\pi/4}}{2\sqrt{2\pi k}}$$
$$\left[\frac{R_{e,h}^{swo}(\alpha_{sw}^o, \phi')}{\sin\left(\frac{\alpha_{sw}^o - \phi}{2}\right)}\left[1 - F_{KP}\left(kLa_{sw}^o\right)\right] + d_{e,h}^{swo}(\phi, \phi'; \alpha_{sw}^o)\right] \tag{65}$$

$$d_{e,h}^{swo}(\phi, \phi'; \alpha_{sw}^o) = \frac{P_{e,h}^o(\alpha_{sw}^o)}{Q_{e,h}^o(\phi)}\left[\sec\left(\frac{\alpha_{sw}^o - \phi'}{2}\right) \pm \sec\left(\frac{\alpha_{sw}^o + \phi'}{2}\right)\right]. \tag{66}$$

The $\Gamma_{e,h}^o$ is an ad hoc modification so as to preserve reciprocity. It is given by

$$\Gamma_{e,h}^o(\phi') = \frac{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} - \delta_{e,h}^o/\sin\beta_o}{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} + \delta_{e,h}^o/\sin\beta_o} \tag{67}$$

where $\delta_e^o = -j\mathcal{Y}_d N \cot(N\tau k_d)$ and $\delta_h^o = j\mathcal{Z}_d N \tan(N\tau k_d)$ with $N = \sqrt{1 - \eta 4 \sin^2 \beta_o \sin^2 \frac{\phi}{2} \sin^2 \frac{\phi'}{2}}$. The $F_{KP}(kLa_{go}^{\pm})$ is the well-known UTD edge transition functions defined in [1]. The $\Gamma_{e,h}^n = \mp 1$ because the n-face is PEC. It is important to note that the UTD solutions for a junction between two different planar material slabs on a PEC ground plane at skew incidence as shown in Fig. 1(a) can be easily given in the same form as (61)–(64) except the $\Gamma_{e,h}^n(\phi')$ is now

$$\Gamma_{e,h}^n(\phi') = \frac{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} - \delta_{e,h}^n / \sin\beta_o}{2\sin\frac{\phi}{2}\sin\frac{\phi'}{2} + \delta_{e,h}^n / \sin\beta_o} \tag{68}$$

with the proper substitution of n-face electrical permittivity and permeability, ϵ_{rn} and μ_{rn}, respectively.

5. NUMERICAL RESULTS

Numerical results for a DPS material junction shown in Fig. 1(a) based on the work presented in this paper referred to as UTD are compared with the results of the MZ solution [10]. There is a very good agreement, with less than ± 1 dB differences. In Figs. 3 and 4, only the UTD solutions developed in this paper are shown. It is noted

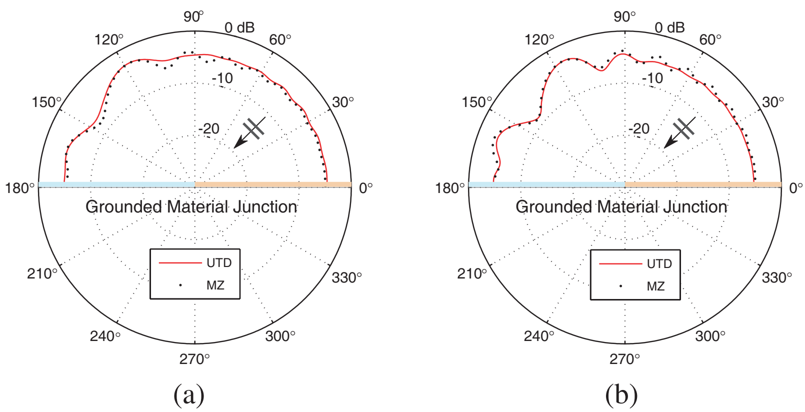

Figure 2. Comparison of scattered fields of UTD and MZ solutions for a DPS material junction excited by a uniform skew incident plane wave shown in Fig. 1(a) where (a) TE and (b) TM at $\phi' = 45°$ and $\beta_o' = 65°$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{ro} = 4$, $\mu_{ro} = 2$) and ($\epsilon_{rn} = 5$, $\mu_{rn} = 1$).

that the excitation for the problem in Fig. 4 is a current moment ($d\bar{p}_e = \hat{z}dp_{ez}$ or $d\bar{p}_m = \hat{z}dp_{mz}$), which produces a spherical wave, whereas the excitation is a skew incident plane wave for Figs. 2 and 3. It is important to note that the surface wave effects are neglected in these plots in order to clearly test if the boundary conditions on the first order UTD diffracted fields are properly satisfied as compared to reference MZ solutions; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries. Note that it is enough to compare the present result only with MZ solution because the MZ and W-H solutions provide the same numerical results.

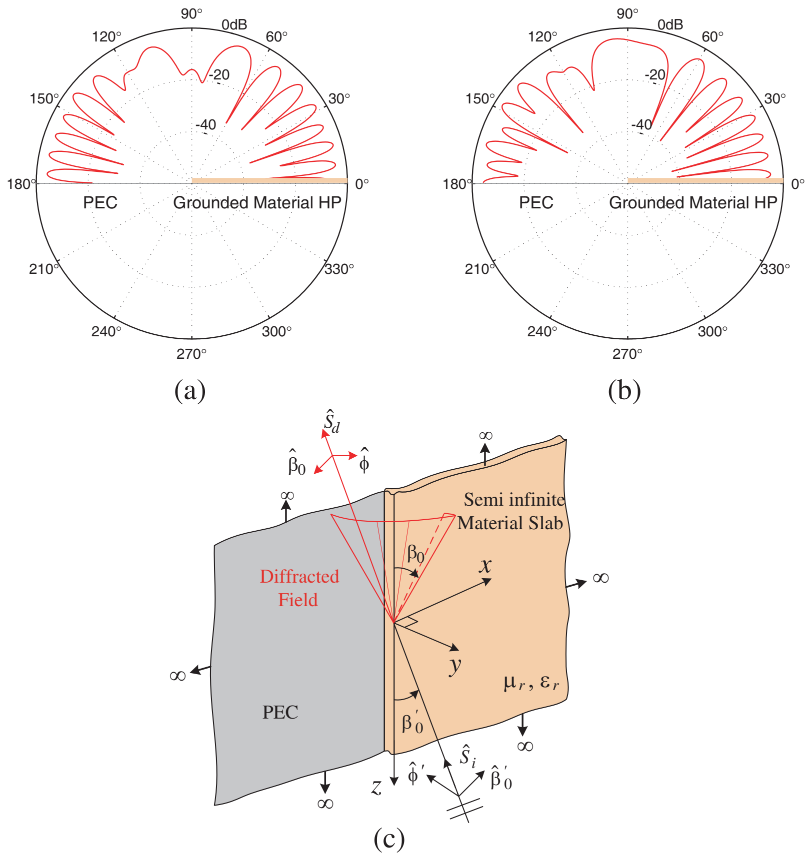

Figure 3. Total field of UTD solution for a grounded DPS material half plane with PEC ground plane excited by a uniform skew incident plane wave (a) TE and (b) TM at $\phi' = 60°$ and $\beta_o' = 120°$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/10$ thick with ($\epsilon_{ro} = 4,\ \mu_{ro} = 2$).

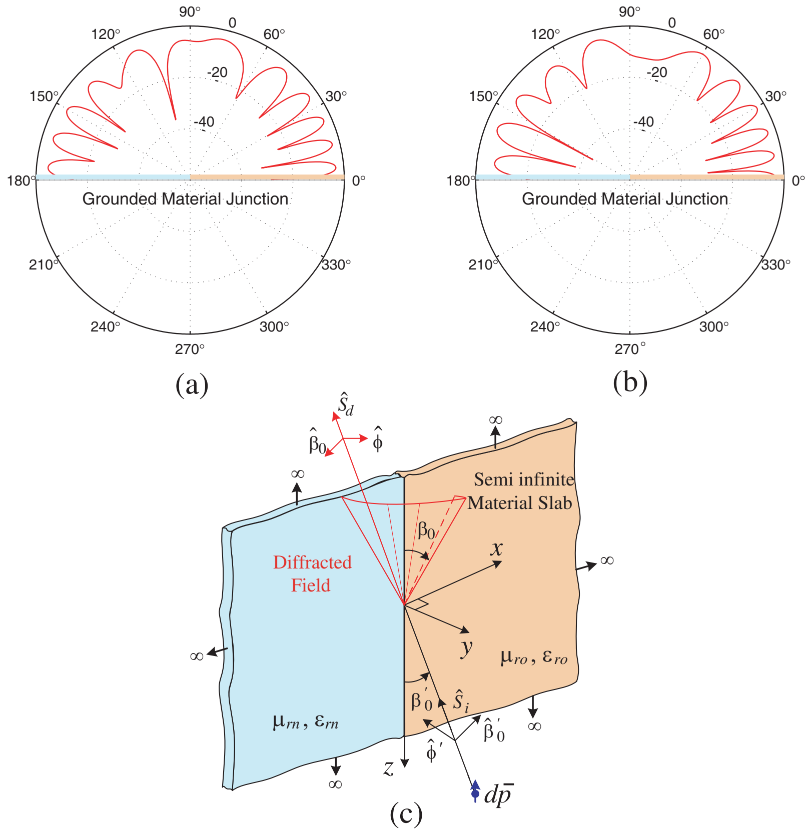

Figure 4. Total field of UTD solution for a DPS material junction excited by (a) a $\hat{z}$-directed electric current moment dp_{ez} and (b) a $\hat{z}$-directed magnetic current moment dp_{mz} at $r' = 7\lambda$, $\phi' = 45°$ and $\theta' = 55°$. The fields are observed at $r = 15\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{ro} = 12,\ \mu_{ro} = 8$) and ($\epsilon_{rn} = 1,\ \mu_{rn} = 4$).

6. CONCLUSION

A promising approximate UTD ray solution for EM diffraction from a junction between two different thin planar material slabs on ground plane is presented. Unlike W-H and MZ solutions, the solution developed in this work recovers the proper local plane wave Fresnel reflection and transmission coefficients (FRTCs), and surface wave constants, respectively, for the actual material, and the present solution also allows the material to be both DPS or DNG. Furthermore, the present works provides solutions for finite sources on or near such structures. In addition, it is important to note that the expressions present in this paper are appropriately approximated via physical reasoning so that they can be made free of the complicated integral forms of the W-H split (or factorization) and MZ functions. This work is useful for analyzing the radiation and scattering from edges on electrically large complex platforms [19]. Platforms involving modern naval ships often contain material treatments over their otherwise metallic surfaces to control their scattering.

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