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EQUATION SOLUTION FOR THE CURRENT IN RADIAL IMPEDANCE MONOPOLE ON THE PERFECTLY CONDUCTING SPHERE

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Abstract—The problem about the electrical current distribution along thin radial impedance monopole, located on the perfectly conducting sphere, has been solved in a rigorous electrodynamic formulation in the paper. The problem formulation strictness is provided by the use of the Green's function for the Hertz's vector potential for unbounded space outside the perfectly conducting sphere at formulation of the initial integral equation concerning the current in monopole. The approximate analytical solution of the integral equation has been obtained by the method of iterations both for the case of excitation of the monopole by the δ-generator of voltage, located on the finite distance over the spherical scatterer, and at the excitation of the monopole at its basis.

1. INTRODUCTION

Antennas in the form of thin vibrator radiators are used widely on moving objects of different applications, including air and space vehicles, in the range of meter and decimeter waves. If the object has the form, close to the prolate body of rotation, then the vibrator antennas are usually classified due to the method of their location relatively to its longitudinal axis, coinciding with the dominant direction of the object movement, by which we distinguish radial (transverse), oblique and longitudinal vibrator antennas. Of course, such classification of radiators turns out to be unsuitable in the case, when the corpus of the object has a spherical form. So the vibrator

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structures, located near such bodies, characterize them by spatial orientation relatively to the spherical surface, approximating the form of the object corpus in a general case.

Asymmetrical radially oriented vibrator radiators (monopoles) are applied mainly in practice. It is explained by simplicity of the fulfillment of feeding such radiators with the help of coaxial feeders. In those cases, when necessity in the symmetrical vibrator antennas application arises, they can be made of two radiators, each of which is asymmetrical relatively to the object corpus, and it is fed by a separate section of the coaxial feeder, and the connection with the general generator is made with the help of the power dividers.

It is obvious that the creation of the mathematical model of the mentioned kind of surface antennas in a strict electrodynamic formulation, which takes into account both the concrete geometry of the vibrator element and the geometry of the spherical object, has great practical significance. First of all, it concerns numerical-analytical modeling of the radiation characteristics of such antennas. However, because of complexity of realization of such modeling these questions are not described very well in the literary sources [1–4]. What is more, even in these rare papers the simple models (such as electrically short dipoles) are considered as radiators despite that a rather developed theoretical basis exists for the analysis of impedance vibrators [5–14].

We should note that the creation (availability) of the Green's function of the corresponding electrodynamic volume is one of the required conditions of the possible analytical solution for the problem about electromagnetic waves radiation by the thin vibrator with the distributed surface impedance. The Green's tensor function, which was created in the monograph [15] and is represented in Appendix A, will be used for the case in question of unbounded space (material medium) outside the perfectly conducting sphere.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

The characteristics of radiation of the antennas in question were investigated earlier in [16, 17] at the current distribution approximation along the perfectly conducting vibrators by one cosine function. However, such approximation permits to consider only electrically short radiators (of the length of the $\lambda/4$ order) and to obtain the electromagnetic fields, excited by the vibrators outside the perfectly conducting sphere, by means of numerical integration. So it is of great interest to obtain the approximate analytical expression for the current of the asymmetrical vibrator without limits on its length in the class

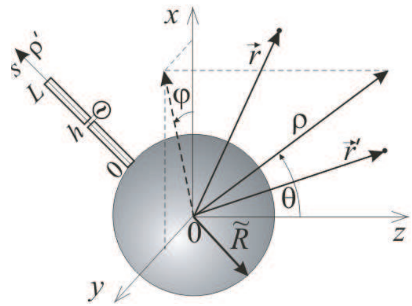

Figure 1. The asymmetrical radial impedance vibrator, located on the sphere (the spherical surface antenna).

of the Bessel's spherical functions, allowing to make integration in the expressions for the vibrator radiation fields due to the known formulas in the $\{\rho;\ \theta;\ \varphi\}$ spherical coordinate system. Such a solution has been obtained for the perfectly conducting monopoles in [15].

Let us consider the perfectly conducting sphere of the radius $\tilde{R}$ (Figure 1). We locate the radially oriented thin cylindrical impedance vibrator with the radius r and length L ($r/L \ll 1$), and the axis of which is coupled with the direction ρ', $\theta' = \theta_0$, $\varphi' = \varphi_0$, on the sphere.

The field of the vibrator surface current is equivalent to the field of the $J(\rho')$ linear current, coming along the vibrator longitudinal axis, owing to the accepted model of the thin conductor. Then, the $\vec{\Pi}(\vec{r})$ Hertz's vector potential will have only radial component in the case in question ($\vec{r}$ is the radius-vector of the observation point):

$$\Pi_\rho\left(\vec{r}\right) = \frac{1}{i\omega\varepsilon_1}\int\limits_{\tilde{R}}^{\tilde{R}+L} J\left(\rho'\right) G_{\rho\rho'}\left(\rho,\theta,\varphi;\rho',\theta_0,\varphi_0\right) d\rho', \tag{1}$$

where ε_1 is the permittivity of environment, and $G_{\rho\rho'}(\rho,\theta,\varphi;\rho',\theta',\varphi')$ is the Green's function of an electrical kind for the space outside the perfectly conducting sphere from Appendix A.

The initial integral equation is represented in the following form for the case of the $z_i = const$ constant impedance (z_i is the internal impedance per unit length ([Ohm/m])) on the vibrator generator, which is approximated by the section of the $\rho \in \left[\tilde{R};\tilde{R}+L\right]$ radial ray in the direction $\theta = \theta_0$ and $\varphi = \varphi_0 + r/(\tilde{R}+L/2)$:

$$\frac{d^2\Pi_\rho(\rho)}{d\rho^2} + \frac{2}{\rho}\frac{d\Pi_\rho(\rho)}{d\rho} + \left(k_1^2 - \frac{2}{\rho^2}\right)\Pi_\rho(\rho) = -E_{0\rho}(\rho) + z_i J(\rho), \tag{2}$$

where $E_{0\rho}(\rho)$ is the radial component of the impressed excitation field, $k_1 = k\sqrt{\varepsilon_1\mu_1}$, $k = 2\pi/\lambda$; λ is the wavelength in free space; μ_1 is the permeability of the medium. Taking into account (1) and the symbol $G_{\rho\rho'}(\rho,\rho') = G_{\rho\rho'}(\rho,\theta_0,\varphi_0 + r/(\tilde{R} + L/2); \rho',\theta_0,\varphi_0)$, this equation can be written in the form:

$$\left[\frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} + \left(k_1^2 - \frac{2}{\rho^2}\right)\right] \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho')G_{\rho\rho'}(\rho,\rho')d\rho' = -i\omega\varepsilon_1 E_{0\rho}(\rho) + i\omega\varepsilon_1 z_i J(\rho). \tag{3}$$

3. EQUATION SOLUTION FOR THE CURRENT BY THE METHOD OF CONSISTENT ITERATIONS

We select the singularity of a quasi-stationary kind, by which the integral-differential equation kernel (3) is characterized, analogically with [9, 15], making the following identical transformations:

$$\int\limits_{\tilde{R}}^{\tilde{R}+L} J\left(\rho'\right) G_{\rho\rho'}\left(\rho,\rho'\right) d\rho' = J(\rho)\Omega(\rho) + \int\limits_{\tilde{R}}^{\tilde{R}+L} \left[J\left(\rho'\right) G_{\rho\rho'}\left(\rho,\rho'\right) - \frac{J(\rho)}{R(\rho,\rho')}\right] d\rho', \tag{4}$$

where $R(\rho,\rho') = \sqrt{(\rho-\rho')^2 + r^2}$, $\Omega(\rho) = \int\limits_{\tilde{R}}^{\tilde{R}+L} \frac{d\rho'}{\sqrt{(\rho-\rho')^2+r^2}} = \ln[\frac{\sqrt{(L-\rho)^2+r^2}+(L-\rho)}{\sqrt{\rho^2+r^2}-\rho}]$, and the integral mean value $\bar{\Omega}(\rho) = 2\ln(L/r) - 0.614$.

Thus, we will introduce the symbol of the functional:

$$F\left[\rho, J(\rho)\right] = \left[\frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} + \left(k_1^2 - \frac{2}{\rho^2}\right)\right] \int\limits_{\tilde{R}}^{\tilde{R}+L} \left[J(\rho')G_{\rho\rho'}(\rho,\rho') - \frac{J(\rho)}{R(\rho,\rho')}\right] d\rho', \tag{5}$$

and also of the small parameter $\alpha = -1/\bar{\Omega}(\rho) \approx \frac{1}{2\ln(r/L)}$. Then, Equation (3) can be written in the following form:

$$\left[\frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} + \left(k_1^2 - \frac{2}{\rho^2}\right)\right] J(\rho) = i\omega\varepsilon_1\alpha E_{0\rho}(\rho) - i\omega\varepsilon_1\alpha z_i J(\rho) - \alpha F\left[\rho, J(\rho)\right]. \tag{6}$$

If $\tilde{\tilde{k}}_1 = k_1\sqrt{1+i\alpha\omega\varepsilon_1 z_i/k_1} = k_1\sqrt{1+i2\alpha\bar{Z}_S/(kr)}$, where $\bar{Z}_S = \bar{R}_S + i\bar{X}_S$ is the normalized on the 120π Ohm surface impedance and symbolized in the expression (6), then the equation for the current in the impedance vibrator will have the form:

$$\left[\frac{d^2}{d\rho^2}+\frac{2}{\rho}\frac{d}{d\rho}+\left(\tilde{\tilde{k}}_1^2-\frac{2}{\rho^2}\right)\right]J(\rho)=\alpha\left\{i\omega\varepsilon_1 E_{0\rho}(\rho)-F\left[\rho,J(\rho)\right]\right\}. \quad (7)$$

The solution of $J^{(0)}(\rho)$ of the homogeneous differential Equation (7) with the null right part is represented in the form in this case:

$$J^{(0)}(\rho)=C_1 j_1\left(\tilde{\tilde{k}}_1\rho\right)+C_2 y_1\left(\tilde{\tilde{k}}_1\rho\right), \quad (8)$$

where C_1 and C_2 are the arbitrary constants; $j_1(\tilde{\tilde{k}}_1\rho)$ and $y_1(\tilde{\tilde{k}}_1\rho)$ are the Bessel's spherical functions of the first order of I and II kinds, correspondingly:

$$j_1\left(\tilde{\tilde{k}}_1\rho\right)=\frac{\sin\left(\tilde{\tilde{k}}_1\rho\right)}{\left(\tilde{\tilde{k}}_1\rho\right)^2}-\frac{\cos\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho},\ y_1\left(\tilde{\tilde{k}}_1\rho\right)=-\frac{\cos\left(\tilde{\tilde{k}}_1\rho\right)}{\left(\tilde{\tilde{k}}_1\rho\right)^2}-\frac{\sin\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho}. \quad (9)$$

It is necessary to add the $J^{(\emptyset)}(\rho)$ arbitrary private solution of the inhomogeneous Equation (7) to the solution of Equation (8) in order to obtain a complete solution of Equation (7). What is more, it is useful that (as parameters) the coordinates of both ends of the monopole will be represented in it. It is not difficult to check by direct substitution that the following expression is such a private solution:

$$J^{(\emptyset)}(\rho)=\frac{3\alpha}{2\tilde{\tilde{k}}_1}\int_{\tilde{R}}^{\rho}\left\{i\omega\varepsilon_1 E_{0\rho}(\rho')-F\left[\rho',J(\rho')\right]\right\}j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right)d\rho'$$
$$-\frac{3\alpha}{2\tilde{\tilde{k}}_1}\int_{\rho}^{\tilde{R}+L}\left\{i\omega\varepsilon_1 E_{0\rho}(\rho')-F\left[\rho',J(\rho')\right]\right\}j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right)d\rho'. \quad (10)$$

The coefficient before the integrals in expression (10) is introduced, taking into account that

$$\lim_{\left(\tilde{\tilde{k}}_1\rho\right)\to 0} j_1\left(\tilde{\tilde{k}}_1\rho\right)=0 \text{ and } \lim_{\left(\tilde{\tilde{k}}_1\rho\right)\to 0}\frac{dj_1\left(\tilde{\tilde{k}}_1\rho\right)}{d\left(\tilde{\tilde{k}}_1\rho\right)}=\frac{1}{3}. \quad (11)$$

Thus, summing expressions (8) and (10), we obtain the general solution for the impedance radial monopole current, located on the perfectly conducting sphere, in the final form:

$$J(\rho) = J^{(0)}(\rho) + J^{(\emptyset)}(\rho) = C_1 j_1\left(\tilde{\tilde{k}}_1\rho\right) + C_2 y_1\left(\tilde{\tilde{k}}_1\rho\right)$$
$$+\frac{3\alpha i\omega\varepsilon_1}{2\tilde{\tilde{k}}_1}\left\{\int\limits_{\tilde{R}}^{\rho} E_{0\rho}(\rho')j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right)d\rho' - \int\limits_{\rho}^{\tilde{R}+L} E_{0\rho}(\rho')j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right)d\rho'\right\}$$
$$-\frac{3\alpha}{2\tilde{\tilde{k}}_1}\left\{\int\limits_{\tilde{R}}^{\rho} F\left[\rho', J(\rho')\right] j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\rho'\right) d\rho'\right.$$
$$\left. - \int\limits_{\rho}^{\tilde{R}+L} F[\rho', J(\rho')] j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\rho'\right) d\rho'\right\}. \tag{12}$$

Let us note that expression (12) has been obtained for the $E_{0\rho}(\rho)$ arbitrary exciting impressed field.

Universality of the obtained solution (12) for the current of the monopole $J(\rho)$ is that it can be used in the algorithm of the method of consistent iterations along the α small parameter directly. The unknown C_1 and C_2 are defined from the boundary conditions on the monopole ends, and the $E_{0\rho}(\rho)$ exciting field is concretized for the problem in question. It is obvious that the expression must be chosen from the solution (12) as the $J_0(\rho)$ zeroth-order approximation for the monopole current:

$$J_0(\rho) = C_1 j_1\left(\tilde{\tilde{k}}_1\rho\right) + C_2 y_1\left(\tilde{\tilde{k}}_1\rho\right)$$
$$+\frac{3\alpha i\omega\varepsilon_1}{2\tilde{\tilde{k}}_1}\left\{\int\limits_{\tilde{R}}^{\rho} E_{0\rho}(\rho')j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right) - \int\limits_{\rho}^{\tilde{R}+L} E_{0\rho}(\rho')j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\rho'\right)\right\}. \tag{13}$$

Let us consider a general case of excitation of the monopole by means of the δ-generator of voltage, located on the distance h over the spherical object. We take into consideration that:

$$E_{0\rho}(\rho') = V_0\delta[\rho' - (\tilde{R} + h)], \tag{14}$$

where V_0 is the excitation field complex amplitude. Then, the zeroth-order approximation for the vibrator current, due to (13), can be written in the following form:

$$J_0(\rho) = C_1 j_1\left(\tilde{\tilde{k}}_1\rho\right) + C_2 y_1\left(\tilde{\tilde{k}}_1\rho\right) + \frac{3\alpha i\omega\varepsilon_1}{2\tilde{\tilde{k}}_1}\begin{cases} -V_0 j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right), & \tilde{R} \le \rho \le \tilde{R}+h \\ V_0 j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right), & \tilde{R}+h \le \rho \le \tilde{R}+L \end{cases}. \quad (15)$$

The C_2 constant in expression (15) is defined from the boundary condition of the equality to null of the current on the monopole end: $J_0(\tilde{R}+L) = 0$. As a result, we obtain:

$$C_2 = -C_1\frac{j_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)}{y_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)} - \frac{3\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1}\frac{j_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)}{y_1\left(\tilde{\tilde{k}}_1\left[\bar{R}+L\right]\right)}. \quad (16)$$

Taking into account the equality (16), expression (15) is written in the form:

$$J_0(\rho) = C_1\left[j_1\left(\tilde{\tilde{k}}_1\rho\right) - \frac{j_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)}{y_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)}y_1\left(\tilde{\tilde{k}}_1\rho\right)\right] - \frac{3\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1}\frac{j_1\left(\tilde{\tilde{k}}_1\left[L-h\right]\right)}{y_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right)}y_1\left(\tilde{\tilde{k}}_1\rho\right) + \frac{3\alpha i\omega\varepsilon_1}{2\tilde{\tilde{k}}_1}\begin{cases} -V_0 j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right), & \tilde{R} \le \rho \le \tilde{R}+h \\ V_0 j_1\left(\tilde{\tilde{k}}_1\rho - \tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right), & \tilde{R}+h \le \rho \le \tilde{R}+L \end{cases}. \quad (17)$$

We note that the $J_0(\tilde{R}+L) = 0$ boundary condition is performed for expression (17) at the arbitrary values of the C_1 and h parameters. The C_1 unknown constant is defined from the boundary condition in the point of the monopole contact with the conducting sphere $\rho' = \tilde{R}$. The fulfillment of the $\text{div}J(\tilde{R}) = 0$ equality is required in this point because of the current continuity. Taking into account that $\text{div}J(\rho) = \frac{2J(\rho)}{\rho} + \frac{dJ(\rho)}{d\rho}$ in the spherical coordinate system, we obtain after identical transformations:

$$C_1 = \frac{3\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1}\frac{j_0\left(\tilde{\tilde{k}}_1 h\right) y_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right) + j_1\left(\tilde{\tilde{k}}_1\left[L-h\right]\right) y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)}{j_0\left(\tilde{\tilde{k}}_1\tilde{R}\right) y_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right) - j_1\left(\tilde{\tilde{k}}_1\left[\tilde{R}+L\right]\right) y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)}, \quad (18)$$

where $j_0(\tilde{\tilde{k}}_1\rho)$ and $y_0(\tilde{\tilde{k}}_1\rho)$ the Bessel's spherical functions of the zeroth order of I and II kinds, correspondingly. They are defined by the following ratios:

$$j_0\left(\tilde{\tilde{k}}_1\rho\right)=\frac{\sin\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho}\text{ and }y_0\left(\tilde{\tilde{k}}_1\rho\right)=-\frac{\cos\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho}.\tag{19}$$

Thus, the finite expression for the monopole current in the case in question can be written in the following form, more suitable for making calculations:

$$\begin{aligned}J_0(\rho)\;=\;&\frac{3\alpha i\omega\varepsilon_1V_0}{2\tilde{\tilde{k}}_1}\left[\tilde{C}_1j_1\left(\tilde{\tilde{k}}_1\rho\right)+\tilde{C}_2y_1\left(\tilde{\tilde{k}}_1\rho\right)\right.\\&\left.+\left\{\begin{array}{ll}-j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right),\tilde{R}\le\rho\le\tilde{R}+h\\ j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\left[\tilde{R}+h\right]\right),\tilde{R}+h\le\rho\le\tilde{R}+L\end{array}\right\}\right],\end{aligned}\tag{20}$$

where

$$\begin{aligned}\tilde{C}_1&=C_0\left[j_0\left(\tilde{\tilde{k}}_1h\right)y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)+y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)j_1\left(\tilde{\tilde{k}}_1L-\tilde{\tilde{k}}_1h\right)\right],\\ \tilde{C}_2&=C_0\left[j_0\left(\tilde{\tilde{k}}_1h\right)j_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)+j_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)j_1\left(\tilde{\tilde{k}}_1L-\tilde{\tilde{k}}_1h\right)\right],\\ C_0&=\frac{\tilde{\tilde{k}}_1\tilde{R}\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)^2}{\sin\left(\tilde{\tilde{k}}_1L\right)-\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)\cos\left(\tilde{\tilde{k}}_1L\right).}\end{aligned}$$

As it is seen from (20), the obtained solution for the current in the impedance vibrator is for both tuned and untuned vibrators, which are the radiators of the L/λ arbitrary electrical length.

A special case, important for practical applications, is the case of the vibrator excitation directly in the point of the monopole contact with the sphere, i.e., when $h=0$ and the δ-generator is located in the $\rho'=\tilde{R}$ point on the surface of the sphere. One would choose expression (17) as the original one, here, rewritten in the following form, taking into account the $h=0$ equality:

$$\begin{aligned}J_0(\rho)=&\frac{C_1}{y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)}\left[j_1\left(\tilde{\tilde{k}}_1\rho\right)y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)-y_1\left(\tilde{\tilde{k}}_1\rho\right)j_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)\right]\\&+\frac{3\alpha i\omega\varepsilon_1V_0}{2\tilde{\tilde{k}}_1y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)}\left[j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\tilde{R}\right)y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1L\right)\right.\\&\left.-y_1\left(\tilde{\tilde{k}}_1\rho\right)j_1\left(\tilde{\tilde{k}}_1L\right)\right].\end{aligned}\tag{21}$$

It is necessary to use a physical requirement of equality of the $\Phi(\rho) = -\mathrm{div}\Pi_\rho(\rho)$ scalar potential to the V_0 complex amplitude in the generator location point in order to define the value of the constant C_1 in this case:

$$-\mathrm{div}\Pi_\rho(\rho)\big|_{\rho=\tilde{R}} = V_0. \tag{22}$$

Because for the zeroth-order approximation in the $\Pi_\rho(\rho)$ integral representation its main value from (4) is used:

$$\Pi_\rho(\rho) = \frac{1}{i\omega\varepsilon_1}\int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho')G_{\rho\rho'}(\rho,\rho')d\rho' \approx -\frac{1}{\alpha i\omega\varepsilon_1}J_0(\rho), \tag{23}$$

then the boundary condition (22) for the $J_0(\rho)$ current will be written in the form:

$$\frac{1}{\alpha i\omega\varepsilon_1}\mathrm{div}J_0(\rho)\bigg|_{\rho=\tilde{R}} = V_0. \tag{24}$$

Using condition (24) for the monopole current (21), it is not difficult to obtain further:

$$C_1 = \frac{\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1}\frac{3j_1\left(\tilde{\tilde{k}}_1 L\right)y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right) - y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)}{j_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)-y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right)j_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)}. \tag{25}$$

This permits to write the finite expression for the current distribution along the monopole in the form similar to (20):

$$J_0(\rho) = \frac{3\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1}\left[\tilde{C}_1 j_1\left(\tilde{\tilde{k}}_1\rho\right)+\tilde{C}_2 y_1\left(\tilde{\tilde{k}}_1\rho\right)+j_1\left(\tilde{\tilde{k}}_1\rho-\tilde{\tilde{k}}_1\tilde{R}\right)\right], \tag{26}$$

where

$$\begin{aligned}
\tilde{C}_1 &= C_0\left[j_1\left(\tilde{\tilde{k}}_1 L\right)y_0\left(\tilde{\tilde{k}}_1\tilde{R}\right) - \frac{1}{3}y_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)\right],\\
\tilde{C}_2 &= -C_0\left[j_1\left(\tilde{\tilde{k}}_1 L\right)j_0\left(\tilde{\tilde{k}}_1\tilde{R}\right) - \frac{1}{3}j_1\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)\right],\\
C_0 &= \frac{\tilde{\tilde{k}}_1\tilde{R}\left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)^2}{\sin\left(\tilde{\tilde{k}}_1 L\right) - \left(\tilde{\tilde{k}}_1\tilde{R}+\tilde{\tilde{k}}_1 L\right)\cos\left(\tilde{\tilde{k}}_1 L\right)}.
\end{aligned}$$

From a fundamental point of view, it is interesting to observe transformation of expression (26) at the fulfillment of the $\tilde{\tilde{k}}_1\tilde{R}\to\infty$ condition. This condition requires "transition" of the surface of the sphere into the infinite perfectly conducting screen formally. To be precise, it is required that the current distribution in the radial monopole over such a limiting surface corresponds to the current distribution in the vertical monopole over the horizontal perfectly conducting screen. In its turn, the current distribution in the monopole over the screen (taking into account its mirror image) must correspond to the current distribution in the symmetrical vibrator radiator, located in the infinite material medium. It turns out that the current in the monopole (26) is represented by the expression as a result of such limiting transition ($\tilde{\tilde{k}}_1\tilde{R} \to \infty$):

$$\lim_{\tilde{\tilde{k}}_1\tilde{R}\to\infty} J_0(s) = \frac{\alpha i\omega\varepsilon_1 V_0}{2\tilde{\tilde{k}}_1 \cos\left(\tilde{\tilde{k}}_1 L\right)} \left\{\sin\left[\tilde{\tilde{k}}_1 (L-s)\right] - 3j_1\left(\tilde{\tilde{k}}_1 L\right)\left[\cos\left(\tilde{\tilde{k}}_1 s\right) - \cos\left(\tilde{\tilde{k}}_1 L\right)\right] + 3\left[j_1\left(\tilde{\tilde{k}}_1 s\right) - j_1\left(\tilde{\tilde{k}}_1 L\right)\right]\cos\left(\tilde{\tilde{k}}_1 L\right)\right\}, \quad (27)$$

where transition to the $s \in [0; L]$ local coordinate has been made for the convenience of comparison. This expression corresponds to the trinomial formula for the current in the thin impedance vibrator due to its structure, obtained in [18]. We should note that the functional multiplier in the curly brackets (27) has a negative value, which provides a positive value for the $J_0(s)$ current, taking into account the negative value of the α magnitude at numerical calculations.

4. RADIATION FIELDS OF THE RADIAL IMPEDANCE VIBRATOR ON THE PERFECTLY CONDUCTING SPHERE

The full radiation field will be defined by the radial component (1) of the Hertz's vector potential $\Pi_\rho(\vec{r})$ owing to the spherical surface antenna model, considered earlier in Figure 1, representing itself the system, which consists of the impedance vibrator and the metallic scatterer of a spherical form. One needs to substitute the obtained current distribution $J(\rho') = J_0(\rho')$ along the impedance monopole in the form (20) for a general case of location of the δ-generator on the vibrator or in the form (26) for the case of its excitation in the point of contact with the sphere into expression (1).

Due to the formulas [13]

$$\begin{aligned} \vec{E}(\vec{r}) &= \vec{E}_0(\vec{r}) + \left(\mathrm{grad}\,\mathrm{div} + k^2\varepsilon_1\mu_1\right)\vec{\Pi}(\vec{r}), \\ \vec{H}(\vec{r}) &= \vec{H}_0(\vec{r}) + ik\varepsilon_1 \mathrm{rot}\vec{\Pi}(\vec{r}), \end{aligned} \quad (28)$$

in which we must except $\vec{E}_0(\vec{r}) = \vec{H}_0(\vec{r}) = 0$, and the expression for the components of full radiation field of the spherical surface antenna is obtained:

$$\begin{aligned}
E_\rho(\vec{r}) &= \frac{\partial^2\Pi_\rho(\vec{r})}{\partial\rho^2} + \frac{2}{\rho}\frac{\partial\Pi_\rho(\vec{r})}{\partial\rho} + \left(k^2\varepsilon_1\mu_1 - \frac{2}{\rho^2}\right)\Pi_\rho(\vec{r}),\\
E_\theta(\vec{r}) &= \frac{1}{\rho}\frac{\partial^2\Pi_\rho(\vec{r})}{\partial\rho\partial\theta} + \frac{2}{\rho^2}\frac{\partial\Pi_\rho(\vec{r})}{\partial\theta},\\
E_\varphi(\vec{r}) &= \frac{1}{\rho\sin\theta}\frac{\partial^2\Pi_\rho(\vec{r})}{\partial\rho\partial\theta} + \frac{2}{\rho^2\sin\theta}\frac{\partial\Pi_\rho(\vec{r})}{\partial\varphi},\\
H_\rho(\vec{r}) &= 0,\\
H_\theta(\vec{r}) &= \frac{ik\varepsilon_1}{\rho\sin\theta}\frac{\partial\Pi_\rho(\vec{r})}{\partial\varphi},\\
H_\varphi(\vec{r}) &= -\frac{ik\varepsilon_1}{\rho}\frac{\partial\Pi_\rho(\vec{r})}{\partial\theta}.
\end{aligned} \tag{29}$$

Let us note that formulas (29) permits to obtain the electromagnetic radiation fields at any distance from the antenna, that is, at arbitrary $\rho \geq \tilde{R}$.

If the outer homogeneous medium is not characterized by losses, that is, ε_1 is a purely real value, then formulas (29) will be simplified for the antenna far zone ($\rho \gg \lambda$), because the addendums, proportional to the coefficient $1/\rho^2$, can be omitted in them.

Let us represent the expressions for the components of the magnetic radiation field of the spherical antenna in the case of the radial impedance monopole excitation in the point of its contact with the sphere in an explicit form as an example:

$$\begin{aligned}
H_\theta(\vec{r}) =& \frac{3\alpha ik\varepsilon_1 V_0}{4\tilde{\tilde{k}}_1\rho\sin\theta}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m\varepsilon_m}{C_{mn}}\mathrm{P}_n^m(\cos\theta)\,\mathrm{P}_n^m(\cos\theta_0)\sin m(\varphi-\varphi_0)\\
&\times\int_{\tilde{R}}^{\tilde{R}+L} h_n^{(2)}(\rho,\rho')\left[\tilde{C}_1 j_1\left(\tilde{\tilde{k}}_1\rho'\right)+\tilde{C}_2 y_1\left(\tilde{\tilde{k}}_1\rho'\right)+j_1\left(\tilde{\tilde{k}}_1\rho'-\tilde{\tilde{k}}_1\tilde{R}\right)\right]d\rho',\\
H_\varphi(\vec{r}) =& \frac{3\alpha ik\varepsilon_1 V_0}{4\tilde{\tilde{k}}_1\rho}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\varepsilon_m}{C_{mn}}\frac{d\mathrm{P}_n^m(\cos\theta)}{d\theta}\mathrm{P}_n^m(\cos\theta_0)\cos m(\varphi-\varphi_0)\\
&\times\int_{\tilde{R}}^{\tilde{R}+L} h_n^{(2)}(\rho,\rho')\left[\tilde{C}_1 j_1\left(\tilde{\tilde{k}}_1\rho'\right)+\tilde{C}_2 y_1\left(\tilde{\tilde{k}}_1\rho'\right)+j_1\left(\tilde{\tilde{k}}_1\rho'-\tilde{\tilde{k}}_1\tilde{R}\right)\right]d\rho'.
\end{aligned} \tag{30}$$

The symbols, accepted earlier, are used in formulas (30), including the ones from Appendix A. The expressions (30) are transformed easily, taking into account that the Hankel spherical functions of the second kind have the known asymptotic representation at $\tilde{\tilde{k}}_1\rho \to \infty$ and $|\tilde{\tilde{k}}_1\rho| \gg m$ for the antenna far zone:

$$h_n^{(2)}(kr) \approx (i)^{n+1}\frac{e^{-ikr}}{kr}. \tag{31}$$

We should note that integration in expressions (30) can be made on the ground of the formula from [19] for most addendums analytically:

$$\int \frac{1}{x} Z_\mu^{(1)}(ax) Z_\nu^{(2)}(ax)\mathrm{d}x = \frac{ax}{\mu^2 - \nu^2}\left[Z_{\mu+1}^{(1)}(ax) Z_\nu^{(2)}(ax) - Z_\mu^{(1)}(ax) Z_{\nu+1}^{(2)}(ax)\right] + \frac{1}{\mu+\nu} Z_\mu^{(1)}(ax) Z_\nu^{(2)}(ax), \tag{32}$$

where $Z_\mu^{(1)}(ax)$ and $Z_\nu^{(2)}(ax)$ are the different combinations of the Bessel, Neumann and Hankel functions.

We should like to write a few words about the radiation pattern (RP) of such a spherical surface antenna in free space. The principles of its formation are analogous to the case of spherical surface antennas with the perfectly conducting monopole, considered in the monograph [15]. We shall give only general tendencies of these analogues briefly here.

The RP of the spherical surface antenna, excited by the radial monopole, has the cut, lobe behavior at the increase of the diffraction radius of the sphere $k\tilde{R}$. The oscillations of the radiation fields amplitudes generally take place in the region of geometrical shadow ($\theta > \pi/2$ at $\theta_0 = 0$). From physical point of view, these oscillations are explained by that, that the waves come into the shadow zone, propagating along the surface of the spherical scatterer along the meridians in forward and back directions. The result of interference of the waves is the oscillations of the amplitudes of the antenna radiation fields. When the $k\tilde{R}$ value is larger, the number of standing waves, "put" on the surface of the sphere, is greater, and the number of the side lobes, which will be in the RP, is greater.

The deepest oscillation is observed near the "dark pole" ($\theta = \frac{\pi}{2} - \theta_0$, $\varphi = \varphi_0 + \pi$), where the amplitudes of the interfering waves are similar approximately. When the observation point is more remote from the "dark pole", the difference in the lengths of the propagation paths of the straight and back waves will increase. As a result, the difference in the degree of damping of these waves will increase, and the amplitude of the oscillations will decrease.

As expected, the screening influence of the sphere increases at the increase of its diffraction radius $k\tilde{R}$. So, the form of the antenna RP in the front half-space ($\varphi_0 - \pi/2 \leq \varphi \leq \varphi_0 + \pi/2$) approaches the form of the RP of the vertical monopole of the same geometry over the infinite perfectly conducting screen at a considerable increase of sizes of the sphere. The radiation field amplitude in the back half-space decreases considerably because of the increasing screening influence of the spherical scatterer.

It is obvious that availability of the impedance monopole on the spherical scatterer will influence the form of the RP of the spherical surface antenna at the change of its electrical length. More considerable influence will occur in the cases of rather small diffraction radiuses of the sphere $k\tilde{R}$ of the order of some wavelengths. However, the investigation of the back influence of the spherical scatterer on the kind of the current distribution along the impedance monopole, the results of which will be represented in the following section, is more essential for practical applications.

5. NUMERICAL RESULTS

In spite of proximity of the sinusoidal distribution function of the current along cylindrical vibrators ($\sin \tilde{\tilde{k}}_1(L - |s|)$), this approximation is used for the calculation of different characteristics of vibrators such as the radiation field in all zones of observation in practice widely and successfully. We must note that the searched values are the integral characteristics from the distribution function of the current, and the small errors in its approximation do not give considerable contribution to the finite result. This also concerns the monopoles, located near metallic bodies of different configurations as a whole. However, one must take into consideration that the vector potential in each point of the monopole is defined by the currents summarized operation, induced on both the rest parts of the monopole and the spherical scatterer, but not by the current local value in this point of the radiator in the case of the spherical scatterer in question. This is the very circumstance, which allows us to consider the "monopole-perfectly conducting sphere" system in the form of a single antenna. Thus, the current actual (true) distribution along the monopole (26) differs from the sinusoidal distribution in this case, and it is more considerable, when the diffraction radius of the sphere $k\tilde{R}$ is less (that is, when the difference in the degree of interaction with the monopole of the scatterer of concrete sizes or the infinite screen is larger). We note that this circumstance stays just both for perfectly conducting and impedance monopoles.

The results of calculations of the currents normalized amplitude distributions in the radial perfectly conducting monopoles, located on the perfectly conducting sphere in Figure 1 are represented in Figure 2. The calculations have been made according to formula (26) under the condition of the choice of the real impedance value $\bar{R}_S = 0.0001$ and $r/\lambda = 0.0033$ for the local longitudinal coordinate $s = \rho - \tilde{R}$. As seen from the plots, the most considerable difference of the current distribution from the sinusoidal one is observed for the smallest value from the considered ones of the spherical scatterer diffraction radius $k\tilde{R} = 0.2\pi$ ($\tilde{R}/\lambda = 0.1$) even in the case of the quarter-wave monopoles with $L = 0.25\lambda$ (Figure 2(a)). When the radius of the sphere is larger, the calculated distributions approach the sinusoidal dependence more strongly. What is more, the current dependence on the longitudinal coordinate in the quarter-wave monopole is close to the linear one analogically with the case of symmetrical electrically short vibrators in free space for the sizes of the sphere $k\tilde{R} = 3.0$.

Small efficiency of excitation of the radiator is observed at the increase of the monopole length for the $k\tilde{R}$ small values. The current distributions have the behavior of damping exponential dependencies

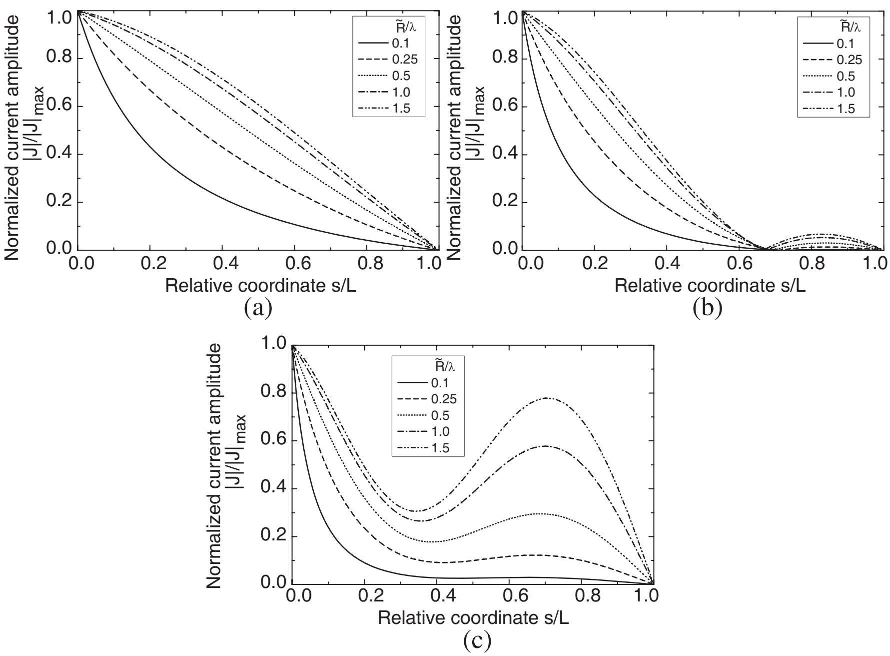

Figure 2. The current normalized amplitude distribution along the radial perfectly conducting monopole on the spherical scatterer at $\bar{R}_S = 0.0001$, $r/\lambda = 0.0033$: (a) $L = 0.25\lambda$, (b) $L = 0.5\lambda$, (c) $L = 1.0\lambda$.

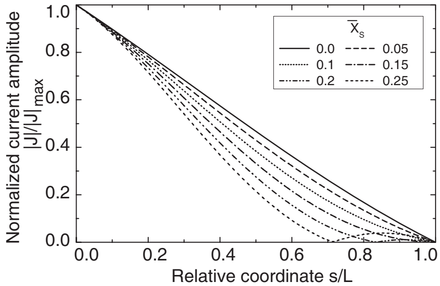

Figure 3. The current normalized amplitude distribution along the radial impedance monopole, located on the sphere at $\bar{R}_S = 0.0001$, $L = 0.25\lambda$, $r/\lambda = 0.0033$, $k\tilde{R} = \pi$.

in this case. The efficiency of the monopole excitation increases sufficiently at the increase of the diffraction radius of the sphere. As one would expect, additional antinodes and nodes in the current distributions, coupled with the creation of sections with the antiphase currents on the monopole, are formed in this case.

Thus, we can state that availability of the spherical scatterer with small and resonant diffraction radiuses influences the kind of the current distribution in the monopole considerably. It is more considerable than the change of the proper length of the radiator. Therefore, when introducing the distributed surface impedance, it is impossible to expect a similar kind of the effect of its electrical length change as in the case of the impedance vibrator in free space. And it is natural, because the monopole impedance covering is only a part of the spherical antenna general surface, which, we emphasize once more, represents itself a uniform system of the monopole and the sphere.

The results of calculations of the normalized amplitude distributions of the currents along the quarter-wave monopole ($L = 0.25\lambda$), which is characterized by the surface impedance of the $\bar{Z}_S = i\bar{X}_S$ inductive kind, on the sphere with the $k\tilde{R} = \pi$ diffraction radius are represented in Figure 3 in order to prove everything written above. As seen from the plots, a formal lengthening of the monopole electrical size on the account of the impedance of an inductive kind is only a required condition for realization of this effect. The impedance condition must be added by a sufficient one, which is required by the choice of the size of the spherical scatterer for its display. These conditions agree weakly in the case in question (Figure 3), but such examples prove the necessity of making optimization simulation at the creation of a concrete antenna system of the investigated kind in the case in question.

6. CONCLUSION

The approximate analytical solution for the current in radial impedance monopole, located on the perfectly conducting sphere, has been obtained in this paper. This solution has been obtained in the form of the zeroth approximation of the method of iterations. The solution's correctness is provided by the strict electrodynamic formulation of the problem in the frames of conventional approximations of the theory of thin wire antennas, and the Green's function for the Hertz's vector potential for unbounded space outside the perfectly conducting sphere has been used in the formulation of the original integral equation for the current in monopole. The solutions have been obtained both for the case of excitation of monopole of the δ-generator of voltage, located on the finite distance h over the spherical scatterer, and for the case of excitation of monopole at its basis, when $h = 0$. The possibilities of the investigations of the radiation fields of the spherical antenna in question, representing itself a uniform system of impedance monopole and a spherical scatterer, have been analyzed. General tendencies of their formation in the antenna far zone are given. The influence of the spherical scatterer on the kind of the current distribution along the impedance monopole has been investigated on concrete examples.

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APPENDIX A. ELECTRICAL DYADIC GREEN'S FUNCTION FOR THE UNBOUNDED SPACE OUTSIDE THE PERFECTLY CONDUCTING SPHERE

For unbounded space outside the perfectly conducting sphere of radius $\tilde{R}$ with the permittivity and permeability of the medium ε_1 and μ_1 (Figure 1) it has:

$$\hat{G}^e\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right)=\begin{vmatrix} G^e_{\rho\rho'} & 0 & 0 \\ 0 & G^e_{\theta\theta'} & G^e_{\theta\varphi'} \\ 0 & G^e_{\varphi\theta'} & G^e_{\varphi\varphi'} \end{vmatrix},$$

$$G^e_{\rho\rho'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = -\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\varepsilon_m h_n(\rho,\rho')}{2C_{nm}}P_n^m(\cos\theta)P_n^m\left(\cos\theta'\right)\cos m(\varphi-\varphi'),$$

$$G^e_{\theta\theta'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = -\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\varepsilon_m u_n(\rho,\rho')\cos m(\varphi-\varphi')}{2n\,(n+1)\,C_{nm}\sin\theta\sin\theta'} \times\left[m^2 P_n^m(\cos\theta)\,P_n^m\left(\cos\theta'\right)+\sin\theta\sin\theta'\frac{dP_n^m(\cos\theta)}{d\theta}\frac{dP_n^m(\cos\theta')}{d\theta'}\right],$$

$$G^e_{\theta\varphi'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{m u_n(\rho,\rho')\sin m(\varphi-\varphi')}{n\,(n+1)\,C_{nm}} \times\left[\frac{dP_n^m(\cos\theta)}{d\theta}\frac{P_n^m(\cos\theta')}{\sin\theta'}+\frac{P_n^m(\cos\theta)}{\sin\theta}\frac{dP_n^m(\cos\theta')}{d\theta'}\right],$$

$$G^e_{\varphi\theta'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = -G^e_{\theta\varphi'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right),$$

$$G^e_{\varphi\varphi'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = G^e_{\theta\theta'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right).$$

Here $P_n^m(\cos\theta)$ is the associated Legendre functions of the first sort,

$$C_{nm} = \frac{2\pi\,(n+m)!}{(2n+1)\;(n-m)!},$$

$$h_n(\rho,\rho') = \begin{cases} 4\pi k_1 h_n^{(2)}(k_1\rho')\begin{bmatrix} j_n(k_1\rho)\,Q_n\left(y_n\left(k_1\tilde{R}\right)\right) \\ -y_n(k_1\rho)\,Q_n\left(j_n\left(k_1\tilde{R}\right)\right)\end{bmatrix}, \tilde{R}\le\rho<\rho', \\ 4\pi k_1 h_n^{(2)}(k_1\rho)\begin{bmatrix} j_n(k_1\rho')\,Q_n\left(y_n\left(k_1\tilde{R}\right)\right) \\ -y_n(k_1\rho')\,Q_n\left(j_n\left(k_1\tilde{R}\right)\right)\end{bmatrix}, \rho>\rho', \end{cases}$$

$$Q_n\left(f_n(k_1R)\right) = \frac{n\,f_n\left(k_1\tilde{R}\right)-k_1R\,f_{n+1}\left(k_1\tilde{R}\right)}{n\,h_n^{(2)}\left(k_1\tilde{R}\right)-k_1Rh_{n+1}^{(2)}\left(k_1\tilde{R}\right)},$$

$$u_n(\rho,\rho') = \begin{cases} 4\pi k_1\frac{h_n^{(2)}(k_1\rho')}{h_n^{(2)}(k_1\tilde{R})}\left[j_n(k_1\rho)\,y_n\left(k_1\tilde{R}\right)-y_n(k_1\rho)\,j_n\left(k_1\tilde{R}\right)\right], \\ \qquad \tilde{R}\le\rho<\rho', \\ 4\pi k_1\frac{h_n^{(2)}(k_1\rho)}{h_n^{(2)}(k_1\tilde{R})}\left[j_n(k_1\rho')\,y_n\left(k_1\tilde{R}\right)-y_n(k_1\rho')\,j_n\left(k_1\tilde{R}\right)\right], \\ \qquad \rho>\rho', \end{cases}$$

$$h_n^{(2)}(k_1\rho) = j_n(k_1\rho)-iy_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}}H^{(2)}_{n+1/2}(k_1\rho)$$

is the Hankel spherical function of the second sort; $j_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}} J_{n+1/2}(k_1\rho)$ and $y_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}} N_{n+1/2}(k_1\rho)$ are the Bessel spherical function and the Neumann one; correspondingly, $J_{n+1/2}(k_1\rho)$ is the Bessel function; $N_{n+1/2}(k_1\rho)$ is the Neumann function; $H^{(2)}_{n+1/2}(k_1\rho)$ is the Hankel function of the second sort with the half-integral index [19].

Let us note that it turns out to be possible to represent the expression for the component of the Green's function $G^e_{\rho\rho'}(\rho,\theta,\varphi;\rho',\theta',\varphi')$ in a more suitable form for numerical realization, having made the transition from the double series to the single one with the help of the summation theorem for the Legendre polynomials:

$$G^e_{\rho\rho'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = -\sum_{n=0}^{\infty} \frac{n+1/2}{2\pi}\, h_n(\rho,\rho')\, P_n\left(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi')\right).$$

REFERENCES

1. Yin, W. Y., G. H. Nan, and I. Wolff, "The near and far field distributions of a thin circular loop antenna in a radially multilayered biisotropic sphere," *Progress In Electromagnetics Research*, PIER 21, 103–135, 1999.
2. Vafeas, P., G. Perrusson, and D. Lesselier, "Low-frequency solution for a perfectly conducting sphere in a conductive medium with dipolar excitation," *Progress In Electromagnetics Research*, PIER 49, 87–111, 2004.
3. Valagiannopoulos, C. A., "Single-series solution to the radiation of loop antenna in the presence conducting sphere," *Progress In Electromagnetics Research*, PIER 71, 277–294, 2007.
4. Kouveliotis, N. K. and C. N. Capsalis, "Prediction of the SAR level induced in a dielectric sphere by a thin wire dipole antenna," *Progress In Electromagnetics Research*, PIER 80, 321–336, 2008.
5. Inagaki, N., O. Kukino, and T. Sekiguchi, "Integrated equation analysis of cylindrical antennas characterized by arbitrary surface impedance," *IEICE Trans. Commun.*, Vol. 55-B, 683–690, 1972.
6. Andersen, L. S., O. Breinbjerg, and J. T. Moore, "The standard impedance boundary condition model for coated conductors with edges: A numerical investigation of the accuracy for transverse magnetic polarization," *Journal of Electromagnetic Waves and Applications*, Vol. 12, No. 4, 415–446, 1998.

7. Galdi, V. and I. M. Pinto, "SDRA approach for higher-order impedance boundary conditions for complex multi-layer coatings on curved conducting bodies," *Journal of Electromagnetic Waves and Applications*, Vol. 13, No. 12, 1629–1630, 1999.
8. Ikiz, T., S. Koshikawa, K. Kobayashi, E. I. Veliev, and A. H. Serbest, "Solution of the plane wave diffraction problem by an impedance strip using a numerical-analytical method: E-polarized case," *Journal of Electromagnetic Waves and Applications*, Vol. 15, No. 3, 315–340, 2001.
9. Nesterenko, M. V., "The electomagnetic wave radiation from a thin impedance dipole in a lossy homogeneous isotropic medium," *Telecommunications and Radio Engineering*, Vol. 61, 840–853, 2004.
10. Arnold, M. D., "An efficient solution for scattering by a perfectly conducting strip grating," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 7, 891–900, 2006.
11. Collard, B., M. B. Fares, and B. Souny, "A new formulation for scattering by impedant 3D bodies," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 10, 1291–1298, 2006.
12. Ruppin, R., "Scattering of electromagnetic radiation by a perfect electromagnetic conductor cylinder," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 13, 1853–1860, 2006.
13. Nesterenko, M. V., V. A. Katrich, V. M. Dakhov, and S. L. Berdnik, "Impedance vibrator with arbitrary point of excitation," *Progress In Electromagnetics Research B*, Vol. 5, 275–290, 2008.
14. Wu, J.-J. and T. J. Yang, "Subwavelength microwave guiding by a periodically corrugated metal wire," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 10, 11–19, 2009.
15. Penkin, Y. M. and V. A. Katrich, *Excitation of Electromagnetic Waves in the Volumes with Coordinate Boundaries*, Fakt, Kharkov, 2003 (in Russian).
16. Belkina, M. G. and L. A. Weinstein, *The Characteristics of Radiation of Spherical Surface Antennas. Diffraction of Electromagnetic Waves on Some Bodies of Rotation*, Soviet radio, Moscow, 1957 (in Russian).
17. Resnikov, G. B., *Antennas of Flying Vehicles*, Soviet radio, Moscow, 1967 (in Russian).
18. King, R. W. P. and T. Wu, "The imperfectly conducting cylindrical transmitting antenna," *IEEE Trans. Antennas and Propagat.*, Vol. 14, 524–534, 1966.

19. Abramowits, M. and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series-55, 1964.