Progress In Electromagnetics Research B, Vol. 19, 41–63, 2010

DEGREE OF ROUGHNESS OF ROUGH LAYERS: EXTENSIONS OF THE RAYLEIGH ROUGHNESS CRITERION AND SOME APPLICATIONS

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Abstract—In the domain of electromagnetic wave propagation in the presence of rough surfaces, the Rayleigh roughness criterion is a widely-used means to estimate the degree of roughness of considered surface. In this paper, this Rayleigh roughness criterion is extended to the case of rough layers. Thus, it provides an interesting qualitative tool for estimating the degree of electromagnetic roughness of rough layers.

1. INTRODUCTION

The Rayleigh roughness criterion is a common tool for estimating the degree of electromagnetic roughness of a considered rough surface. It was first studied by Lord Rayleigh [1, 2]. He considered the case of a propagating monochromatic plane wave incident on a sinusoidal surface, and for normal incidence [2]. This work led to the establishment of the so-called Rayleigh roughness criterion, which makes it possible to estimate the degree of roughness of a rough surface, related to the coherent field.

Contrary to classical quantitative (asymptotic or rigorous) methods that describe the electromagnetic wave scattering from rough surfaces [3, 4], it is essentially a qualitative tool which makes it possible to fast evaluate the degree of electromagnetic roughness of rough surfaces. Nevertheless, under the tangent plane approximation (usually called Kirchhoff Approximation [18–20]) which assumes large surface curvature radius and gentle surface slopes, it allows one to calculate

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the coherent scattered intensity attenuation owing to the surface roughness.

Moreover, it is used in practice in several simple models to describe the electromagnetic wave scattering from rough surfaces. For instance, in ocean remote sensing, it is used in the Ament model [5, 6, 16, 17] to calculate the grazing incidence forward (i.e., in the specular direction) radar propagation over sea surfaces, in optics to determine optical constants of films [7, 8] and other applications [9–15], or in indoor propagation, in ray-tracing based wave propagation models that take the wall roughness into account by introducing a power attenuation parameter [21–25].

First, Section 2 recalls the Rayleigh roughness parameter associated to the scattering in reflection from a rough surface. Then, the Rayleigh roughness parameter is extended in Section 3 to the case of transmission through a rough surface, where comparisons are led with the case of reflection. Last, it is extended in Section 4 to the reflection from a rough layer. Under the tangent plane approximation, an application to the coherent scattered intensity attenuation owing to the layer roughness is presented.

2. ESTIMATION OF THE ELECTROMAGNETIC ROUGHNESS OF ROUGH SURFACES

In the *electromagnetic* point of view, the roughness of a surface depends, of course, on the surface heights, but we will see in what follows that it also depends on the incident wave frequency as well as on the incidence angle. Indeed, the *electromagnetic* roughness of a surface is related to the phase variations $\delta\phi_r$ of the wave reflected by the surface, owing to the surface height variations. It is obtained under the tangent plane approximation, which is valid for large surface curvature radii and gentle slopes.

2.1. Phase Variation of the Reflected Field from a Rough Surface

For the case of a random rough surface (see Fig. 1), the total reflected field E_r results from the contribution of all reflected fields from the random heights of the rough surface. Then, to quantify the electromagnetic surface roughness, it is the phase *variation* $\delta\phi_r$ of the reflected field around its mean value (which corresponds to the phase of the mean plane surface) that must be considered.

Let us consider an incident plane wave inside a medium Ω_1 of wavenumber k_1 and of incidence angle θ_i. For the case of a rough

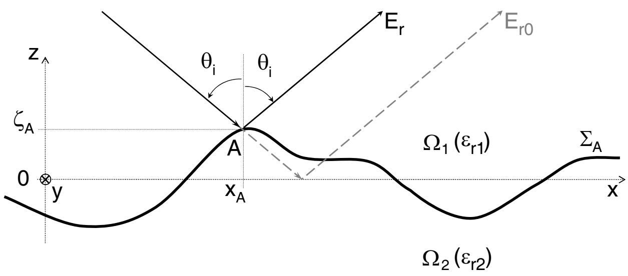

Figure 1. Surface electromagnetic roughness in reflection: Phase variation of the reflected wave owing to the surface roughness.

surface (see Fig. 1), the phase variation $\delta\phi_r$ is given by the relation

$$\delta\phi_r = 2k_1\delta\zeta_A \cos\theta_i, \tag{1}$$

where k_1 is the wavenumber inside the medium Ω_1, $\delta\zeta_A = \zeta_A - \langle\zeta_A\rangle$ the height variation, and θ_i the incidence angle. $\langle\zeta_A\rangle$ is the mean value of the rough surface height, (with $\langle\ldots\rangle$ representing statistical average), which is equal to 0 here in Fig. 1.

Thus, if $|\delta\phi_r| < \pi/2$, the waves interfere constructively, and the surface can be considered as slightly rough, or even nearly flat if $|\delta\phi_r| \ll \pi/2$. In the reverse configuration, if $|\delta\phi_r| > \pi/2$, the waves interfere destructively, and the surface can be considered as rough.

2.2. Coherent and Incoherent Scattered Intensities

For an infinite area surface, the field scattered by a rough surface in reflection inside Ω_1, E_r, can be split up into a mean and a fluctuating component such that [26, 27]

$$E_r = \langle E_r\rangle + \delta E_r, \text{ with } \langle\delta E_r\rangle = 0, \tag{2}$$

where $\langle E_r\rangle$ represents the field statistical average and δE_r the field variations. Indeed, the operator $\langle\ldots\rangle$ is an ensemble mean, and represents here a statistical average. As a consequence, the total intensity scattered by the surface, $\langle|E_r|^2\rangle$, can be expressed as the sum [3, 28]

$$\langle|E_r|^2\rangle = |\langle E_r\rangle|^2 + \langle|\delta E_r|^2\rangle. \tag{3}$$

The term $|\langle E_r\rangle|^2$ represents the coherent intensity, owing to its well-defined phase relation with the incident wave. It corresponds for an infinite area surface to the specular reflection from a perfectly flat surface. By contrast, the term

$$\langle|\delta E_r|^2\rangle = \langle|E_r|^2\rangle - |\langle E_r\rangle|^2 \tag{4}$$

represents the incoherent intensity, owing to its angular spreading and its weak relation (i.e., correlation) with the incident wave. It corresponds to the scattering (in reflection) from a very rough surface.

Thus, when the surface is perfectly flat, the coherent term is maximum and the incoherent term vanishes, as all the incident intensity is reflected in the specular direction for an infinite area surface. When the surface electromagnetic roughness increases, the coherent term is increasingly damped, leading to an increase of the incoherent term $\langle|\delta E_r|^2\rangle$. For a so-called flat surface, the incoherent component can be neglected, and the coherent component can be assimilated to the reflection from a perfectly flat surface. For a so-called slightly rough surface, the coherent term is predominant, whereas for a so-called rough surface, this is the incoherent term which is predominant. Last, for a so-called very rough interface, the coherent term can even be neglected.

2.3. Rayleigh Roughness Parameter

In this context, the Rayleigh roughness parameter, denoted Ra, is an interesting and simple means for estimating the degree of electromagnetic roughness of a surface, i.e., to determine if a surface can be qualified as flat, slightly rough, rough, or very rough. Indeed, it appears in the expression of the *coherent* scattered intensity $|\langle E_r\rangle|^2$, calculated under the tangent plane approximation (which is usually called the Kirchhoff Approximation, or sometimes the physical optics approximation). For an infinite area surface, it can be shown that the average reflected scattered field $\langle E_r\rangle$ under the tangent plane approximation (TPA, which is valid for surfaces with large mean curvature radius and gentle RMS slope) is expressed as

$$\langle E_r\rangle = E_0 \times \langle\exp(j\delta\phi_r)\rangle, \tag{5}$$

leading to the coherent scattered intensity [3, 29]

$$|\langle E_r\rangle|^2 = |E_0|^2 \times |\langle\exp(j\delta\phi_r)\rangle|^2, \tag{6}$$

where the term $|\langle\exp(j\delta\phi_r)\rangle|^2$, which carries an average over the surface heights, describes the surface electromagnetic roughness. This term quantifies the attenuation of the coherent intensity owing to the surface roughness, the term $|E_0|^2$ corresponding to the reflection from a perfectly flat surface. That is why $|\langle\exp(j\delta\phi_r)\rangle|^2$ is denoted $\mathcal{A}_{coh}$. For a Gaussian height pdf (probability density function), this term is equal to $\mathcal{A}_{coh} = \exp\left[-4\,(Ra_r)^2\right]$, with Ra_r the Rayleigh roughness parameter associated to the reflected wave, given by the relation [3, 29]

$$Ra_r = k_1\sigma_h\cos\theta_i, \tag{7}$$

with σ_h the surface RMS (root mean square) height. Indeed, it can easily be shown that $Ra_r = \sqrt{\langle(\delta\phi_r)^2\rangle}/2$.

Thus, the Rayleigh roughness parameter Ra_r, obtained from the root mean square of the phase variation $\delta\phi_r$, is a useful parameter for estimating the surface electromagnetic roughness. Then, a surface is usually qualified as slightly rough when [23, 27, 30, 31]

$$Ra_r < \pi/16, \tag{8}$$

which corresponds to an attenuation of the coherent intensity $\mathcal{A}_{coh} < \exp(-\pi^2/64) \simeq 0.68 \simeq -0.7\,\text{dB}$. This condition corresponds to a criterion called Fraunhoffer criterion [23, 27, 30, 31], which can be written for normal incidence, $\theta_i = 0$, in the form

$$\sigma_h/\lambda < 0.03. \tag{9}$$

By contrast, a surface is qualified as very rough when the coherent intensity $|\langle\exp(j\delta\phi_r)\rangle|^2$ is negligible, in comparison with the incoherent intensity $\langle|\delta E_r|^2\rangle$. From a qualitative point of view, by using the Rayleigh roughness parameter, this corresponds to the condition [3, 29, 32–34]

$$Ra_r > \pi/C, \tag{10}$$

with C a constant, which is usually taken between 2 and π. Indeed, under the tangent plane approximation (TPA) which is valid for surfaces with large mean curvature radius and gentle RMS slope, for a Gaussian height pdf, the coherent intensity attenuation $\mathcal{A}_{coh} = \exp\left[-4\,(Ra_r)^2\right]$ checks the condition $\mathcal{A}_{coh} < -17\,\text{dB}$ for $C = \pi$ and $\mathcal{A}_{coh} < -43\,\text{dB}$ for $C = 2$.

3. EXTENSION OF THE RAYLEIGH ROUGHNESS PARAMETER TO THE TRANSMISSION THROUGH A ROUGH SURFACE

To our knowledge, the Rayleigh roughness parameter has never been explicitly extended to the case of transmission through a rough interface before [29, 35]. Though, when studying the case of transmission through a rough dielectric interface, it is interesting to know the degree of electromagnetic roughness of this surface. The way of determining it is the same, i.e., by calculating the phase variation of the transmitted wave, $\delta\phi_t$, owing to the surface roughness (see Fig. 2). It is related to the surface height variation $\delta\zeta_A = \zeta_A - \langle\zeta_A\rangle$ ($\langle\zeta_A\rangle$ being equal to 0 here in Fig. 2) by the relation [29, 35]

$$\delta\phi_t = k_0\delta\zeta_A(n_1\cos\theta_i - n_2\cos\theta_t), \tag{11}$$

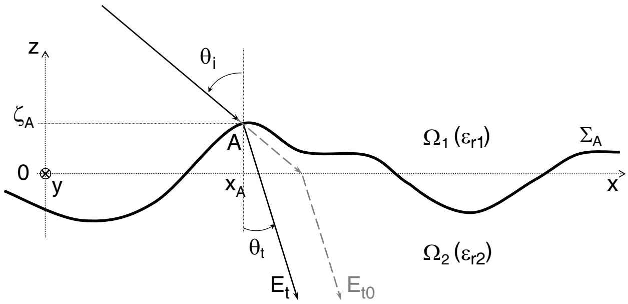

Figure 2. Surface electromagnetic roughness in transmission: Phase variation of the transmitted wave owing to the surface roughness.

with k_0 the wave number inside the vacuum, θ_t the transmission angle, and $n_1 = \sqrt{\epsilon_{r1}}$ and $n_2 = \sqrt{\epsilon_{r2}}$ the refractive indexes of the non-magnetic media Ω_1 and Ω_2, respectively.

Then, using the same way as for the case of reflection from the rough interface, the Rayleigh roughness parameter can be extended to the case of transmission through the rough interface. Denoted as Ra_t, and calculated from the same relation, $Ra_t = \sqrt{\langle(\delta\phi_t)^2\rangle}/2$, it is expressed as [29, 35]

$$Ra_t = k_0 \sigma_h \frac{|n_1 \cos\theta_i - n_2 \cos\theta_t|}{2}. \tag{12}$$

The Rayleigh roughness parameter is defined for calculating the attenuation of the coherent scattered intensity owing to the surface roughness. Then, for a surface of infinite area, the transmission angle θ_t is related to the incidence angle θ_i by the Snell-Descartes law for a perfectly flat interface, $n_1 \sin\theta_i = n_2 \sin\theta_t$ (with n_2 the refractive index of the lower medium Ω_2).

Let us note that the Rayleigh roughness parameter is in general (for media indexes of values of same order) lower for the case of transmission than for the case of reflection. As a result, it must be noted that a surface can be considered as very rough electromagnetically when studying the wave scattered in reflection, but can be considered as only rough or even slightly rough electromagnetically when studying the wave scattered in transmission. Moreover, it can be noticed that contrary to the reflection case, for the transmission case the Rayleigh roughness parameter depends on the refractive index n_2 of the lower medium (through the product term $n_2 \cos\theta_t$). Thus, care must be taken of using the right Rayleigh roughness parameter with respect to the case under study. In what follows, this behavior is studied in more details.

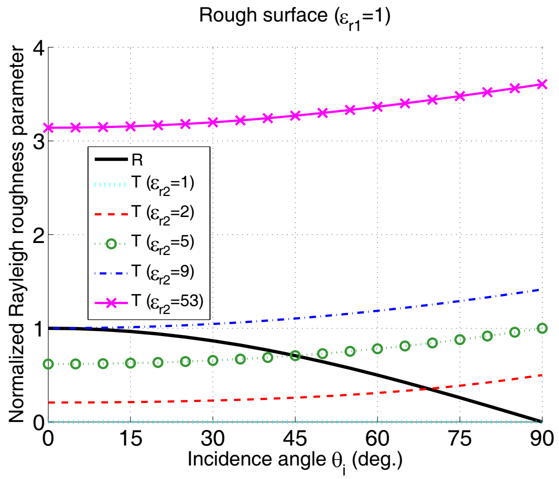

Figure 3. Comparison of the normalized Rayleigh roughness parameters in reflection and in transmission, for different values of the lower relative permittivity ϵ_{r2}, with $\epsilon_{r1} = 1$. The Rayleigh roughness parameters are normalized with respect to the term $k_0\sigma_h$.

3.1. Comparison between the Rayleigh Roughness Parameters in Reflection and in Transmission

The computation of the Rayleigh roughness parameters in reflection Ra_r and in transmission Ra_t makes a comparison possible between the surface electromagnetic roughness between the case of a reflected scattered wave and the case of a transmitted scattered wave. This comparison is led for a surface of infinite area: the Rayleigh roughness parameter is given by Equation (7) for the case of reflection, and by Equation (12) for the case of transmission, where θ_t is given by the relation $n_1 \sin\theta_i = n_2 \sin\theta_t$. Fig. 3 represents the normalized Rayleigh roughness parameters (i.e., for $k_0\sigma_h = 1$) with $\epsilon_{r1} = 1$ and different values of ϵ_{r2} ($\epsilon_{r2} \in \{1; 2; 5; 9; 53\}$), with respect to the incidence angle θ_i.

As a general rule, it can be observed that the normalized Rayleigh roughness parameter in reflection decreases from 1 to 0 when the incidence angle θ_i increases from 0 to 90 degrees, because it is proportional to $\cos\theta_i$. Quite the reverse, the normalized Rayleigh roughness parameter in transmission increases when the incidence angle θ_i increases (except for $\epsilon_{r2} = 1$, where it is constant and equal to 0). It ranges from $k_0\sigma_h(n_2 - n_1)/2$ for $\theta_i = 0$ to $k_0\sigma_h n_2 \cos\theta_t^l/2$ for $\theta_i = 90°$, with $\theta_t^l = \arcsin(n_1/n_2)$. Indeed, it can be shown that $|n_1 \cos\theta_i - n_2 \cos\theta_t|$ increases when θ_i increases. Moreover, the Rayleigh roughness parameter in transmission increases when ϵ_{r2} increases, because of the same reason. It is noticeable that for low values of ϵ_{r2} and for small incidence angles, the Rayleigh roughness parameter in transmission is inferior to the one in reflection; on the contrary, it becomes superior for higher incidence angles. This result is of importance because it means that for a given value of ϵ_{r2} (which checks the condition $\epsilon_{r2} < 9\,\epsilon_{r1}$), for a given incidence angle, the surface can be rougher electromagnetically in reflection than in transmission, whereas for a higher incidence angle, the surface can be less rough electromagnetically in reflection than in transmission.

Thus, it can easily be shown that the incidence angle θ_i^{rug} at which the Rayleigh roughness parameters in reflection Ra_r and in transmission Ra_t are equal is given for $\epsilon_{r2} \geq \epsilon_{r1}$ by the relation

$$\theta_i^{rug} = \arccos\left(\sqrt{\frac{\epsilon_{r2} - \epsilon_{r1}}{8\,\epsilon_{r1}}}\right), \tag{13}$$

under the condition of existence of θ_i^{rug}, $\epsilon_{r2} \leq 9\,\epsilon_{r1}$. This means that for $\epsilon_{r2} > 9\,\epsilon_{r1}$, $Ra_t > Ra_r\ \forall\theta_i$. Then, as illustrated in Fig. 3, for $\epsilon_{r2} = \epsilon_{r1}$, $\theta_i^{rug} = 90°$, for $\epsilon_{r2} = 2\,\epsilon_{r1}$, $\theta_i^{rug} \simeq 69.3°$, for $\epsilon_{r2} = 5\,\epsilon_{r1}$, $\theta_i^{rug} = 45°$, and for $\epsilon_{r2} = 9\,\epsilon_{r1}$, $\theta_i^{rug} = 0°$.

In conclusion, for $\epsilon_{r2} > 9\,\epsilon_{r1}$, $Ra_t > Ra_r\ \forall\theta_i$, and for $\epsilon_{r2} \leq 9\,\epsilon_{r1}$, it is the case only for $\theta_i > \theta_i^{rug}$. Thus, for relative permittivities ϵ_{r2} close to 1 (for $\epsilon_{r1} = 1$) and for low to moderate incidence angles, $Ra_t < Ra_r$: the surface is in this case rougher electromagnetically if the study focuses on the reflected scattered wave than if it focuses on the transmitted scattered wave.

4. EXTENSION OF THE RAYLEIGH ROUGHNESS PARAMETER TO THE REFLECTION FROM A ROUGH LAYER

In this section, the Rayleigh roughness parameters are extended to the case of reflection from rough layers. The case of transmission through rough layers, expressed in [34] for uncorrelated rough surfaces, is not detailed here. In what follows, like previously, the surfaces are assumed to have dimensions much greater than the wavelength, so that they can be considered of infinite area, and that the coherent intensity contributes only in the specular direction.

For the case of rough layers, the incident wave undergoes multiple scattering inside the rough dielectric waveguide (i.e., the medium Ω_2). Consequently, there are multiple reflected scattered fields $E_{r,1}$, $E_{r,2}$, and so on (see Fig. 4). As a result, similarly as for the case of a single rough interface, a Rayleigh roughness parameter $Ra_{r,n}$ can be associated to each order n of average reflected field $\langle E_{r,n}\rangle$ (or its associated coherent scattered intensity $|\langle E_{r,n}\rangle|^2$). To do this, first, the phase variation $\delta\phi_{r,n}$ associated to each average reflected field $\langle E_{r,n}\rangle$ must be calculated, in order to determine its associated Rayleigh roughness parameter $Ra_{r,n}$ from the relation $Ra_{r,n} = \sqrt{\langle(\delta\phi_{r,n})^2\rangle}/2$. We must insist on the following: it implies in general that contrary to a single interface, for a rough layer, several Rayleigh roughness parameters exist, each one being associated to each average scattered field $\langle E_{r,n}\rangle$ (or its associated coherent scattered intensity $|\langle E_{r,n}\rangle|^2$).

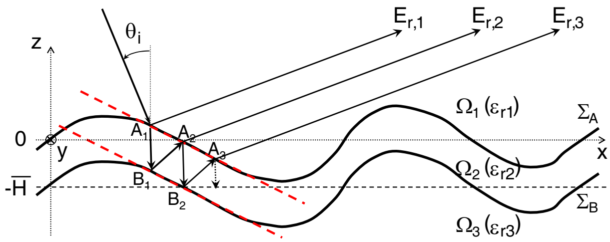

Figure 4. Surface electromagnetic roughness in reflection from a rough layer: Case of thin films with identical surfaces (here, a general configuration of reflected fields away from the specular direction is presented).

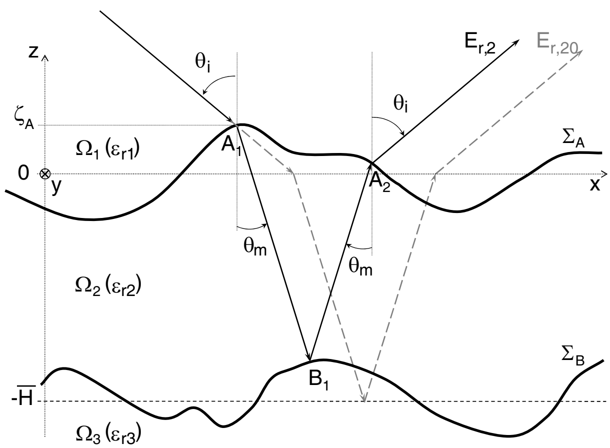

Figure 5. Surface electromagnetic roughness in reflection from a rough layer: Case of uncorrelated surfaces with second-order reflected field $E_{r,2}$.

The phase variation $\delta\phi_{r,1}$ associated to the first-order reflected field $E_{r,1}$ corresponds to the one of a single interface

$$\delta\phi_{r,1} = 2k_1 \delta\zeta_{A_1} \cos\theta_i. \tag{14}$$

For the study of the Rayleigh roughness parameter $Ra_{r,n}$ associated to each average reflected field $\langle E_{r,n}\rangle$, different cases can be encountered. Indeed, for the case where the two surfaces are identical and form a thin film (see Fig. 4), the roughness is very different from the case where the two surfaces are totally uncorrelated (see Fig. 5). Then, both cases are studied here. Nevertheless, the Rayleigh roughness parameter $Ra_{r,1}$ associated to the first-order average reflected field $\langle E_{r,1}\rangle$ (or its associated coherent scattered intensity $|\langle E_{r,1}\rangle|^2$) is equal for both cases to the one corresponding to the reflection from the upper surface such that

$$Ra_{r,1} = k_1 \sigma_{hA} \cos\theta_i, \tag{15}$$

with σ_{hA} the RMS height of the upper surface Σ_A.

4.1. Case of Thin Film with Identical Surfaces

For the case of a thin film with identical parallel rough surfaces (see Fig. 4), it can be observed that comparatively to the first-order reflected field $E_{r,1}$, the higher-order reflected fields $E_{r,n}$ (with $n \geq 2$) are related to $E_{r,1}$ by a well-defined phase difference. Indeed, in this configuration, the two interfaces can be considered as locally flat parallel interfaces, leading to a local Fabry-Pérot interferometer (or local flat layer). This means that all reflected fields $E_{r,n}$ are totally correlated, and have the same phase variation $\delta\phi_{r,n}$ owing to the surface roughness. Then, Equation (22) giving this phase deviation $\delta\phi_{r,n}$ can be simplified, and it can be seen that it is equal to the one obtained in Equation (14) for a single rough interface such that

$$\forall n \in \mathbb{N}^*,\ \delta\phi_{r,n} = \delta\phi_{r,1} = 2k_1 \delta\zeta_{A_1} \cos\theta_i, \tag{16}$$

with $\delta\zeta_{A_1}$ the height deviation of a point A_1 of the surface Σ_A from the mean plane $\langle\zeta_{A_1}\rangle = 0$. Then, the Rayleigh roughness parameter $Ra_{r,n}$ associated to each average reflected field $\langle E_{r,n}\rangle$ (or its associated coherent scattered intensity $|\langle E_{r,n}\rangle|^2$) is given by

$$\forall n \in \mathbb{N}^*,\ Ra_{r,n} = \sqrt{\langle(\delta\phi_{r,n})^2\rangle}/2 = k_1 \sigma_{hA} \cos\theta_i. \tag{17}$$

This is valid if the points of successive reflections are totally correlated, so that the film can be considered as locally flat and can be seen as a local Fabry-Pérot interferometer. Then, it is valid if the horizontal distance l_{hor} between two points of successive reflections

A_k and A_{k+1} (with $k \in \{1 \ldots n\}$) is much smaller than the surface correlation length L_c, $l_{hor} \ll L_c$. We can show that l_{hor} is equal for specular direction $\theta_r = \theta_i$ to

$$l_{hor} = 2H \frac{n_1 \sin\theta_i}{\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}}, \tag{18}$$

which implies that $l_{hor} \in [0; 2Hn_1/\sqrt{n_2^2 - n_1^2}]$.

4.2. Case of Uncorrelated Surfaces

For uncorrelated rough surfaces (see Fig. 5), in the calculation of the Rayleigh roughness parameter $Ra_{r,n}$ associated to the nth-order average reflected field $\langle E_{r,n} \rangle$ (or its associated coherent scattered intensity $|\langle E_{r,n} \rangle|^2$), the surface points ζ_{A_k} and ζ_{B_k} (with $k \in \{1 \ldots n\}$) can be considered as uncorrelated between one another. Moreover, for the following demonstration to be true, it is necessary that all the points of successive reflections at the same rough interface, A_k and $A_{k'}$ as well as B_k and $B_{k'}$, are uncorrelated between one another. This is studied in more details in Appendix A (Section 7), in which it is shown that the following simple approach which considers only specular propagation angles inside the rough layer is valid for small RMS slopes σ_{sA} and σ_{sB} (checking $\{\sigma_{sA}, \sigma_{sB}\} \lesssim 0.3$), as well as for moderate incidence angles θ_i and for the condition $n_2 \gtrsim 1.4 n_1$.

Then, for the second-order reflected field $E_{r,2}$, it is easy to demonstrate that its associated phase variation $\delta\phi_{r,2}$ (see Fig. 5) is given in the specular direction of reflection $\theta_r = \theta_i$ by the relation [29]

$$\delta\phi_{r,2} = k_0(\delta\zeta_{A_1} + \delta\zeta_{A_2})\,(n_1 \cos\theta_i - n_2 \cos\theta_m) + 2k_2 \delta\zeta_{B_1} \cos\theta_m, \tag{19}$$

with $\delta\zeta_{B_1} = \zeta_{B_1} + \bar{H}$ the height deviation of the point B_1 from the mean plane $z = -\bar{H}$ of the lower surface Σ_B, and θ_m the angle of propagation inside Ω_2. θ_m is given by the Snell-Descartes law for a perfectly flat interface, $n_1 \sin\theta_i = n_2 \sin\theta_m$. It must be highlighted that, $\delta\phi_{r,2}$ being a phase variation, it is relative to the variations of the surface heights ζ_{A_1}, ζ_{B_1}, and ζ_{A_2}, and it is independent of the mean layer thickness $\bar{H}$ (this remark also holds for each phase variation $\delta\phi_{r,n}$). Using the same way, the phase variation $\delta\phi_{r,3}$ associated to the third-order reflected field $E_{r,3}$ is given by

$$\delta\phi_{r,3} = k_0(\delta\zeta_{A_1} + \delta\zeta_{A_3})(n_1 \cos\theta_i - n_2 \cos\theta_m) + 2k_2(\delta\zeta_{B_1} - \delta\zeta_{A_2} + \delta\zeta_{B_2})\cos\theta_m. \tag{20}$$

Similarly, the phase variation $\delta\phi_{r,4}$ associated to the fourth-order reflected field $E_{r,4}$ is given by [29]

$$\begin{aligned} \delta\phi_{r,4} \; = \; & k_0(\delta\zeta_{A_1} + \delta\zeta_{A_4})(n_1 \cos\theta_i - n_2 \cos\theta_m) \\ & + 2k_2(\delta\zeta_{B_1} - \delta\zeta_{A_2} + \delta\zeta_{B_2} - \delta\zeta_{A_3} + \delta\zeta_{B_3}) \cos\theta_m. \end{aligned} \tag{21}$$

Thus, it can be shown $\forall n \geq 3$ that the phase variation $\delta\phi_{r,n}$ associated to the nth-order reflected field $E_{r,n}$ is given by

$$\begin{aligned}\delta\phi_{r,n} &= k_0(\delta\zeta_{A_1}+\delta\zeta_{A_n})(n_1\cos\theta_i - n_2\cos\theta_m)\\ &\quad +2k_2\left[\delta\zeta_{B_1}+\sum_{k=2}^{n-1}\big(-\delta\zeta_{A_k}+\delta\zeta_{B_k}\big)\right]\cos\theta_m. \end{aligned} \quad (22)$$

As a result, $\forall n \geq 2$, the calculation of $Ra_{r,n} = \sqrt{\langle(\delta\phi_{r,n})^2\rangle}/2$ becomes easy and equals the square root of the quadratic summation of the elementary Rayleigh roughness parameters, associated to each order of scattering inside the rough layer [29, 34]

$$Ra_{r,n} = \sqrt{2\big(Ra^A_{t12}\big)^2 + (n-1)\big(Ra^B_{r23}\big)^2 + (n-2)\big(Ra^A_{r21}\big)^2}, \quad (23)$$

with Ra^A_{t12} the Rayleigh roughness parameter in transmission from the medium Ω_1 into the medium Ω_2 through the upper surface Σ_A, Ra^B_{r23} the Rayleigh roughness parameter in reflection inside the medium Ω_2 onto the lower surface Σ_B separating the medium Ω_3, and Ra^A_{r21} the Rayleigh roughness parameter in reflection inside the medium Ω_2 onto the upper surface Σ_A separating the medium Ω_1.

One can notice for $\epsilon_{r2} > \epsilon_{r1}$ and for equal RMS heights $\sigma_{hB} = \sigma_{hA}$ that $Ra^B_{r23} > Ra^A_{r12}$. As a consequence, $Ra_{r,2} > Ra_{r,1}$, and the higher orders being superior to $Ra_{r,2}$, one has

$$\forall n \geq 2, Ra_{r,n+1} > Ra_{r,n} > Ra_{r,1}. \quad (24)$$

4.3. Comparison between the Different Rayleigh Roughness Parameters

Similarly as for the single interface case, a comparison is led between the different Rayleigh parameters $Ra_{r,n}$ associated to each average reflected field $\langle E_{r,n}\rangle$, for the case of uncorrelated surfaces. The RMS roughnesses of the lower and upper interfaces are taken identical, $\sigma_{hB} = \sigma_{hA}$. In the numerical simulations, we will focus on the first two contributions $Ra_{r,1}$ and $Ra_{r,2}$. Fig. 6 represents the normalized Rayleigh roughness parameters of the first two contributions, with the same parameters as in Fig. 3.

It can be observed that for this configuration, the second-order (normalized) Rayleigh roughness parameter is always superior to the first-order one (except for the case $\epsilon_{r2} = \epsilon_{r1} = 1$, where they are equal). Moreover, keeping in mind that for the case of a thin film with identical surfaces, all the Rayleigh roughness parameters are equal, it means that the case of uncorrelated surfaces (with $\sigma_{hB} = \sigma_{hA}$) is always rougher than the case of a thin film with identical surfaces. For $\epsilon_{r2} > \epsilon_{r1} = 1$, it can be observed that the behavior of $Ra_{r,2}$ with respect to the incidence angle θ_i differs from the case of a single interface. Indeed, $Ra_{r,2}$ is a combination of a Rayleigh roughness parameter in transmission, Ra^A_{t12}, which increases when θ_i increases, and a Rayleigh roughness parameter in reflection, Ra^B_{r23}, which decreases when θ_i increases. As a consequence, it is observed that for rather low values of ϵ_{r2}, $Ra_{r,2}$ first decreases slightly when θ_i increases from 0 to a given angle. Then, it increases slightly when θ_i increases from this given angle to 90 degrees.

The numerical results (not presented) for the higher-order Rayleigh roughness parameters $Ra_{r,n}$, which check the condition in Equation (24), led to the same general behaviors and comments.

4.4. Case of a Flat Lower Interface

For the case of a flat lower interface, the Rayleigh roughness parameters in reflection from the lower interface $Ra^B_{r23} = 0$ in Equation (23). Like in the previous subsection, the behavior of the first two Rayleigh roughness parameters $Ra_{r,1}$ and $Ra_{r,2}$ are plotted in Fig. 7.

In this case of a flat lower interface, the general behavior of the second-order Rayleigh roughness parameter $Ra_{r,2}$ is the same as for the single interface case. In fact, it can be seen that $Ra_{r,2} = \sqrt{2}Ra^A_{t12}$.

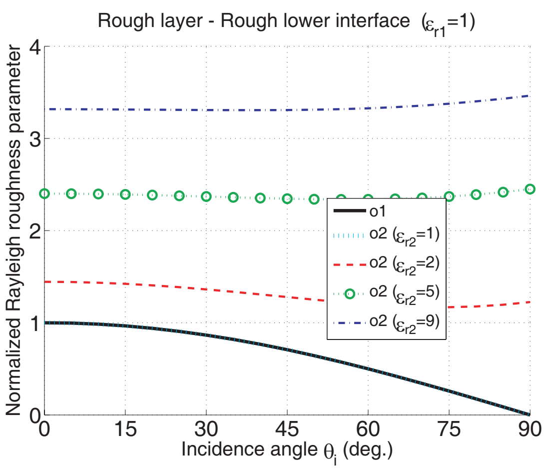

Figure 6. Comparison of the first two normalized Rayleigh roughness parameters in reflection from a rough layer $Ra_{r,1}$ and $Ra_{r,2}$ with a rough lower interface (with same RMS heights as for the upper interface), for different values of the layer relative permittivity ϵ_{r2}, with $\epsilon_{r1} = 1$. The Rayleigh roughness parameters are normalized with respect to the term $k_0\sigma_h$.

Similarly, it can be shown that the third-order contribution $Ra_{r,3}$ is equal to the second-order contribution of the case of a rough lower interface with identical RMS heights, which is presented in Fig. 6.

This approach can easily be extended to the case of transmission through the rough layer, like in [34] for uncorrelated rough surfaces. Similarly, the extension to the case of several rough layers can easily be done. In what follows, the Rayleigh roughness parameters are applied to the calculation of the coherent scattered intensity attenuation under the tangent plane approximation (TPA).

5. APPLICATION TO THE COHERENT SCATTERED INTENSITIES ATTENUATION UNDER THE TPA

Under the tangent plane approximation (TPA) corresponding to locally smooth rough interfaces having gentle slopes, following Subsections 2.2 and 2.3 for a single interface, the coherent total scattered intensity $|\langle E_r^{tot} \rangle|^2$ for a rough layer can be calculated, by using Rayleigh roughness parameters [34]. Then, under this assumption, it must be noted that it is also a quantitative tool for describing the coherent electromagnetic scattering from rough surfaces or layers. In what follows, like previously, we will consider only the case of surfaces with dimensions much greater than the wavelength (called of infinite area), so that the coherent scattered intensity contributes only in the specular direction. That is why only the specular direction is considered here.

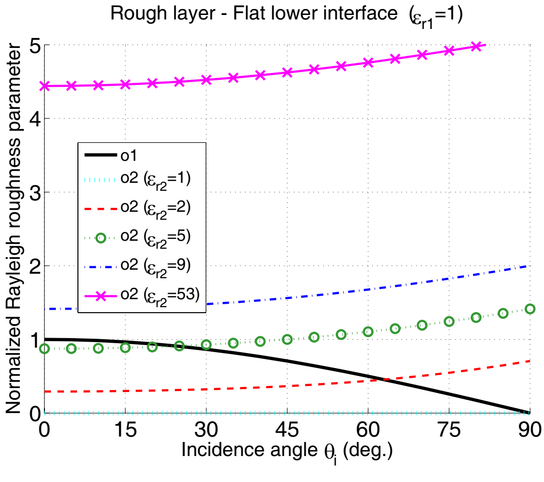

Figure 7. Same simulation parameters as in Fig. 6, but for a flat lower interface.

5.1. Attenuation under the Case of a Thin Layer with Identical Surfaces

For the case of a thin layer with identical surfaces, the reflected field modulus $|E_r^{tot}|$ is equal to $|r^{eq}(\theta_i) \times E_i|$, with E_i the incident wave amplitude and r^{eq} the equivalent Fresnel reflection coefficient given by Equation (17) in [29]. From Equation (6), it can be seen that $|E_0| = |r_{12}(\theta_i)E_i|$. Moreover, as explained in Subsection 4.1, the phase variation of each reflected field $E_{r,n}$ is equal and given in Equation (16). As a consequence, the average total reflected field $\langle E_r^{tot}\rangle$ is given by

$$\langle E_r^{tot}\rangle = r^{eq}(\theta_i)E_i \times \langle \exp(j\delta\phi_{r,1})\rangle, \tag{25}$$

with $\delta\phi_{r,1}$ the phase variation of the wave reflected by the upper interface, given by Equation (14). The term $\langle \exp(j\delta\phi_{r,1})\rangle$ describes the layer electromagnetic roughness, and the term $r^{eq}(\theta_i)E_i$ corresponds to the reflection from a flat layer. Thus, the coherent scattered intensity attenuation for a thin layer with identical surfaces, $\mathcal{A}_{coh} \equiv \mathcal{A}_{coh}^{id}$, is given by the relation

$$\mathcal{A}_{coh}^{id} = |\langle \exp(j\delta\phi_{r,1})\rangle|^2 . \tag{26}$$

Then, for a Gaussian height pdf, $\mathcal{A}_{coh}^{id} = \exp\left[- 4(Ra_{r,n})^2\right]$, with $Ra_{r,n} \equiv Ra_{r,1}$ given by Equation (17).

5.2. Attenuation under the Case of Uncorrelated Surfaces

For the case of uncorrelated surfaces, the average total reflected field $\langle E_r^{tot}\rangle$ can be written with respect to each average reflected field $\langle E_{r,n}\rangle$ by the relation

$$\langle E_r^{tot}\rangle = \sum_{n=1}^{+\infty}\langle E_{r,n}\rangle, \tag{27}$$

with (see Equation (18) of [29])

$$\begin{aligned}\langle E_{r,1}\rangle &= r_{12}(\theta_i)E_i \times \langle \exp(j\delta\phi_{r,1})\rangle , && (28)\\ \langle E_{r,n}\rangle &= t_{12}(\theta_i)t_{21}(\theta_m){r_{23}}^{n+1}(\theta_m){r_{21}}^{n}(\theta_m)e^{j(n+1)\phi_{flat}}E_i \\ &\quad \times \langle \exp(j\delta\phi_{r,n})\rangle , \ \forall n \geq 2, && (29)\end{aligned}$$

r_{ij} and t_{ij} being the Fresnel reflection and transmission coefficients from the medium Ω_i to the medium Ω_j, respectively [36], and $\phi_{flat} = 2k_2\bar{H}\cos\theta_m$ the phase difference between two successive reflected fields $E_{r,n-1}$ and $E_{r,n}$ [29]. As a consequence, the coherent scattered intensity $\left|\langle E_r^{tot}\rangle\right|^2$ can be written in the form

$$\left|\langle E_r^{tot}\rangle\right|^2 = \left|r^{eq,un}(\theta_i)\, E_i\right|^2 , \tag{30}$$

with $r^{eq,un}$ the equivalent Fresnel reflection coefficient from a rough layer, given by Equation (18) of [29].

For a Gaussian height pdf, the term $\langle \exp(j\delta\phi_{r,n}) \rangle$ can be written as $\exp\left[-2(Ra_{r,n})^2\right]$, with $Ra_{r,n}$ given by Equation (15) for $n = 1$ and by Equation (23) $\forall n \geq 2$. Thus, a coherent scattered intensity attenuation $\mathcal{A}_{coh} \equiv \mathcal{A}^{un}_{coh,n}$ can be defined for each average scattered field contribution $\langle E_{r,n} \rangle$ and equals for Gaussian statistics

$$\mathcal{A}^{un}_{coh,n} = |\langle \exp(j\delta\phi_{r,n}) \rangle|^2 = \exp\left[-4\,(Ra_{r,n})^2\right]. \tag{31}$$

5.3. Numerical Results

The numerical results of the attenuation of the coherent scattered intensity owing to the layer roughness are presented for the case of a layer of sand over a granite surface [37]. This configuration corresponds to the case of two uncorrelated rough surfaces.

The relative permittivities of sand and granite are taken as $\epsilon_{r2} = 2.5$ and $\epsilon_{r3} = 8$, respectively. The frequency is $f = 300\,\text{MHz}$, ($\lambda = 1\,\text{m}$) and the incidence angle $\theta_i = 30$ degrees. The mean layer thickness is $\bar{H} = 1.5\lambda$, and the surface RMS heights are $\sigma_{hA} = 0.01\lambda$ and $\sigma_{hB} = 0.35\lambda$.

In this context, the attenuation of the first two coherent scattered field contributions are

$$\mathcal{A}^{un}_{coh,1} = 0.988 \simeq -0.05\,\text{dB}, \tag{32}$$

$$\mathcal{A}^{un}_{coh,2} = 1.25 \times 10^{-19} \simeq -189\,\text{dB}, \tag{33}$$

the third-order one, $\mathcal{A}^{un}_{coh,3}$, and the higher contributions being much inferior to $\mathcal{A}^{un}_{coh,2}$. This means that for this typical configuration, the first-order average scattered field $\langle E_{r,1} \rangle$ is almost not attenuated by the layer roughness, contrary to the higher-order average scattered fields $\langle E_{r,n} \rangle$, with $n \geq 2$, which are very strongly attenuated and can be neglected, as the attenuation is at least $-189\,\text{dB}$, or even stronger.

As a consequence, in this configuration, only the first-order average scattered field $\langle E_{r,1} \rangle$ contributes to the total coherent reflected intensity $|\langle E^{tot}_r \rangle|^2$, which means that the only upper air/sand interface can be considered to compute $|\langle E^{tot}_r \rangle|^2$. Moreover, the attenuation coefficient $\mathcal{A}^{un}_{coh,1} \simeq -0.05\,\text{dB}$ being negligible, $|\langle E^{tot}_r \rangle|^2$ can be assimilated to the reflected intensity from the air/sand interface. This result is in full agreement with the results from [37] (see the second paragraph of page 1356).

Similarly, the reverse configuration where the first-order contribution is negligible in comparison with the higher ones was studied in

Section 3 of [34] (see Fig. 3 and Fig. 7), leading as well to good agreement with the numerical results. Thus, this allows us to validate the extension of the Rayleigh roughness parameter to the case of rough layers.

6. CONCLUSION

The Rayleigh roughness parameter is an interesting means for evaluating the degree of roughness of a rough surface. In other words, it allows one to evaluate the attenuation of the coherent scattered intensity owing to the surface roughness, in the case of gentle surface slopes. Its extension to the case of rough layers led in this paper provides us an interesting means for evaluating the degree of roughness of a rough layer, and more precisely the attenuation of each average field contribution $\langle E_{r,n} \rangle$ owing to the layer roughness. Rigorously, this general qualitative tool is valid under the tangent plane approximation (or Kirchhoff Approximation), i.e., for locally smooth rough surfaces having gentle slopes (so that the multiple scattering phenomenon can be neglected). Thus, under the TPA, the Rayleigh roughness parameters can be used for quantifying the attenuation of $\langle E_{r,n} \rangle$. Then, for a Gaussian height pdf, they are easily calculated. These developments can be extended to transmission through rough layers, as well as to reflection form or transmission through rough multilayers.

APPENDIX A. RIGOROUS CALCULATION FOR UNCORRELATED ROUGH SURFACES

For uncorrelated rough surfaces, a rigorous calculation of the phase variation $\delta\phi_{r,2}$ of the second-order reflected field $E_{r,2}$ (owing to the layer roughness) must be calculated from considering random angles of propagation inside the rough layer. That is to say, the angle of propagation from A_1 to B_1 and denoted θ_m in Fig. 6 does not necessarily correspond to specular transmission from the incidence angle θ_i, and the angle of propagation from A_1 to B_1 is different from θ_m, and will be denoted θ_p here. Then, the phase variation $\delta\phi_{r,2}$ is given by the relation

$$\begin{aligned}\delta\phi_{r,2} = \; & k_0 \delta\zeta_{A_1} (n_1 \cos\theta_i - n_2 \cos\theta_m) + k_2 \delta\zeta_{B_1} (\cos\theta_m + \cos\theta_p) \\ & + k_0 \delta\zeta_{A_2} (n_1 \cos\theta_i - n_2 \cos\theta_p),\end{aligned} \tag{A1}$$

Then, to obtain the associated second-order Rayleigh roughness parameter $Ra_{r,2}$, the statistical average over the square of $(\delta\phi_{r,2})^2$,

$\langle(\delta\phi_{r,2})^2\rangle$, must be derived to obtain $Ra_{r,2} = \sqrt{\langle(\delta\phi_{r,2})^2\rangle}/2$. We get

$$\begin{aligned}&\langle(\delta\phi_{r,2})^2\rangle = k_0^2\,\langle(\delta\zeta_{A_1})^2\ (n_1\cos\theta_i - n_2\cos\theta_m)^2\rangle\\&+ k_2^2\,\langle(\delta\zeta_{B_1})^2\ (\cos\theta_m + \cos\theta_p)^2\rangle\\&+ k_0^2\,\langle(\delta\zeta_{A_2})^2\ (n_1\cos\theta_i - n_2\cos\theta_p)^2\rangle\\&+ 2k_0k_2\,\langle\delta\zeta_{A_1}\delta\zeta_{B_1}\,(n_1\cos\theta_i - n_2\cos\theta_m)(\cos\theta_m + \cos\theta_p)\rangle\\&+ 2k_0^2\,\langle\delta\zeta_{A_1}\delta\zeta_{A_2}\,(n_1\cos\theta_i - n_2\cos\theta_m)(n_1\cos\theta_i - n_2\cos\theta_p)\rangle\\&+ 2k_2k_0\,\langle\delta\zeta_{B_1}\delta\zeta_{A_2}\,(\cos\theta_m + \cos\theta_p)(n_1\cos\theta_i - n_2\cos\theta_p)\rangle.\end{aligned}\tag{A2}$$

The statistical average is over the heights $\delta\zeta_{A_1}$, $\delta\zeta_{B_1}$, $\delta\zeta_{A_2}$ of the surface points, and over the propagation angles θ_m and θ_p, which can be transformed into over the surface slopes γ_{A_1} and γ_{A_1}, γ_{B_1}, respectively. As it can be shown that the heights $\delta\zeta_M$ and slopes γ_M of a surface point M are uncorrelated for even height autocorrelation (indeed, it is related to $W'(0)$, where W' is the first derivative of the height autocorrelation function), the first term in Equation (A2) can be simplified as $k_0^2\sigma_{hA}^2\langle(n_1\cos\theta_i - n_2\cos\theta_m)^2\rangle$.

Moreover, the two rough surfaces being assumed to be uncorrelated, this means that the surface points are uncorrelated, implying that the points of successive reflections are such that A_1 and B_1, as well as B_1 and A_2, are uncorrelated between each another. It is valid for both their heights and slopes (the case of the correlation between A_1 and A_2, and more precisely between their heights, will be discussed further). Thus, Equation (A2) simplifies as

$$\begin{aligned}&\langle(\delta\phi_{r,2})^2\rangle = k_0^2\,\sigma_{hA}^2\,\langle(n_1\cos\theta_i - n_2\cos\theta_m)^2\rangle\\&+ k_2^2\,\sigma_{hB}^2\,\langle(\cos\theta_m + \cos\theta_p)^2\rangle\\&+ k_0^2\,\sigma_{hA}^2\,\langle(n_1\cos\theta_i - n_2\cos\theta_p)^2\rangle\\&+ 2k_0k_2\,\langle\delta\zeta_{A_1}\rangle\langle\delta\zeta_{B_1}\rangle\,\langle(n_1\cos\theta_i - n_2\cos\theta_m)(\cos\theta_m + \cos\theta_p)\rangle\\&+ 2k_0^2\,\langle\delta\zeta_{A_1}\delta\zeta_{A_2}\rangle\,\langle(n_1\cos\theta_i - n_2\cos\theta_m)(n_1\cos\theta_i - n_2\cos\theta_p)\rangle\\&+ 2k_2k_0\,\langle\delta\zeta_{B_1}\rangle\langle\delta\zeta_{A_2}\rangle\,\langle(\cos\theta_m + \cos\theta_p)(n_1\cos\theta_i - n_2\cos\theta_p)\rangle.\end{aligned}\tag{A3}$$

As we have $\langle\delta\zeta_{A_1}\rangle = \langle\delta\zeta_{B_1}\rangle = \langle\delta\zeta_{A_2}\rangle = 0$, the fourth term and the last term of Equation (A3) equal 0. Moreover, $\langle\delta\zeta_{A_1}\delta\zeta_{A_2}\rangle$ is the upper surface height auto-correlation function and can be denoted $W(x_{A_{12}})$, which checks the condition $-\sigma_{hA}^2 \leq W(x_{A_{12}}) \leq +\sigma_{hA}^2$. Then, $\langle(\delta\phi_{r,2})^2\rangle$ simplifies as

$$\begin{aligned}&\langle(\delta\phi_{r,2})^2\rangle = k_0^2\sigma_{hA}^2\left[\langle(n_1\cos\theta_i - n_2\cos\theta_m)^2\rangle + \langle(n_1\cos\theta_i - n_2\cos\theta_p)^2\rangle\right]\\&+ k_2^2\,\sigma_{hB}^2\,\langle(\cos\theta_m + \cos\theta_p)^2\rangle\\&+ 2k_0^2\,W(x_{A_{12}})\,\langle(n_1\cos\theta_i - n_2\cos\theta_m)(n_1\cos\theta_i - n_2\cos\theta_p)\rangle.\end{aligned}\tag{A4}$$

Then, to resolve Equation (A4), the following statistical averages must be solved:

$$I_1 = \langle (n_1 \cos\theta_i - n_2 \cos\theta_m)^2 \rangle \tag{A5}$$

$$I_2 = \langle (\cos\theta_m + \cos\theta_p)^2 \rangle \tag{A6}$$

$$I_3 = \langle (n_1 \cos\theta_i - n_2 \cos\theta_p)^2 \rangle \tag{A7}$$

$$I_4 = \langle (n_1 \cos\theta_i - n_2 \cos\theta_m)(n_1 \cos\theta_i - n_2 \cos\theta_p) \rangle \tag{A8}$$

Let us first focus on I_1.

First, the angle θ_m must be expressed. The local incidence angle χ_i can be expressed by $\chi_i = \theta_i + \arctan\gamma_{A_1}$, and the local transmission angle χ_m by $\chi_i = \theta_m + \arctan\gamma_{A_1}$. Thus, by using the transmission Snell-Descartes law $n_1 \sin\chi_i = n_2 \sin\chi_m$, the angle θ_m can be expressed by

$$\theta_m = \arcsin\left\{\frac{n_1}{n_2}\sin\left[\theta_i + \arctan\gamma_{A_1}\right]\right\} - \arctan\gamma_{A_1}. \tag{A9}$$

Then, $\cos\theta_m$ can easily be obtained from the calculation of $\sin\theta_m$ and with the relation $\cos^2\theta_m = 1 - \sin^2\theta_m$. The surface slopes being gentle, this is the case for γ_{A_1}, and we can make a Taylor series development of $(n_1 \cos\theta_i - n_2 \cos\theta_m)^2$ around $\gamma_{A_1} = 0$ and can restrict ourselves to the second-order in γ_{A_1}. We get

$$(n_1 \cos\theta_i - n_2 \cos\theta_m)^2 = a_0 + a_1\gamma_{A_1} + a_2\gamma_{A_1}^2 + \circ(\gamma_{A_1}^2), \tag{A10}$$

with

$$a_0 = \alpha_1^2, \tag{A11}$$

$$a_1 = a_0, \tag{A12}$$

$$a_2 = \alpha_1^3 \frac{1}{n_2 \cos\theta_m^{spec}} + \alpha_1^2 \tan^2\theta_m^{spec} - \alpha_1\left(n_2^2 - n_1^2\right)\frac{\tan^2\theta_m^{spec}}{n_2\cos\theta_m^{spec}}, \tag{A13}$$

with $\alpha_1 = n_1 \cos\theta_i - n_2 \cos\theta_m^{spec}$, θ_m^{spec} the angle of specular transmission corresponding to a flat surface $\gamma_{A_1} = 0$, and then given by $n_1 \sin\theta_i = n_2 \sin\theta_m^{spec}$.

Thus, the statistical average $I_1 = \langle (n_1 \cos\theta_i - n_2 \cos\theta_m)^2 \rangle$ can be done, the only random variable being here γ_{A_1}. Then, the approximation of considering only the angle of specular transmission θ_m^{spec} in I_1, $\langle (n_1 \cos\theta_i - n_2 \cos\theta_m)^2 \rangle \approx n_1 \cos\theta_i - n_2 \cos\theta_m^{spec} = a_0$, can be quantified by

$$\langle (n_1 \cos\theta_i - n_2 \cos\theta_m)^2 \rangle - a_0 \simeq a_2\sigma_{sA}^2. \tag{A14}$$

For small upper surface RMS slope σ_{sA}, it can be shown that this approximation induces small relative errors in general: it is valid for moderate incidence angles and values of n_2 such that $n_2 \gtrsim 1.4\, n_1$. The same developments and general conclusions can be drawn for the other statistical averages I_2, I_3, and I_4.

Thus, a Taylor series development up to the second order of the first three terms of Equation (A4) over the random variables γ_{A_1} and γ_{B_1} can be led to check the general approximation of considering specular angles θ_m^{spec} and $\theta_p^{spec} = -\theta_m^{spec}$ in $\langle(\delta\phi_{r,2})^2\rangle$. After statistical average, this leads to the relation

$$\langle(\delta\phi_{r,2})^2\rangle = b_0 + b_{1A}\sigma_{sA}^2 + b_{1B}\sigma_{sB}^2, \tag{A15}$$

with

$$b_0 = 2k_0^2\left[(n_1-n_2)^2\sigma_{hA}^2 + 2n_2^2\sigma_{hB}^2\right] + 2k_0^2\left[\frac{n_1}{n_2}(n_1-n_2)^2\sigma_{hA}^2 - 2n_1^2\sigma_{hB}^2\right]\theta_i^2, \tag{A16}$$

$$b_{1A} = 2k_0^2(n_1 - n_2)^2\,\beta_1, \tag{A17}$$

$$b_{1B} = 4k_0^2 n_2^2\,\beta_1, \tag{A18}$$

with $\beta_1 = \left[\frac{n_1-n_2}{n_2}\,\sigma_{hA}^2 - 2\sigma_{hB}^2\right]$. This means that the approximation of specular angles implies the condition

$$\frac{b_{1A}\sigma_{sA}^2 + b_{1B}\sigma_{sB}^2}{b_0} \ll 1. \tag{A19}$$

For small upper and lower RMS slopes σ_{sA} and σ_{sB}, respectively (of the order of $\{\sigma_{sA},\} \lesssim 0.3$), it can be shown that this condition is valid for moderate incidence angles and values of n_2 such that $n_2 \gtrsim 1.4\,n_1$.

Up to now, we did not take the fourth term inside Equation (A4) into account. In fact, a similar derivation would show that this term does not significantly contribute for moderate incidence angles and values of n_2 such that $n_2 \gtrsim 1.4\,n_1$. In conclusion, under these conditions and with gentle RMS slopes (of the order of $\{\sigma_{sA}, \sigma_{sB}\} \lesssim 0.3$), the approximation of taking specular propagation angles inside the rough layer is valid.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their comments that help improve the manuscript.

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