Progress In Electromagnetics Research, PIER 100, 55–68, 2010

PLANAR TRANSFORMERS EXCITED BY SQUARE WAVES

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Abstract—This paper presents an analysis of found results in experiments developed with transformers built with planar coils, when they are excited by square waves, in comparison with transformers built with planar coils inner ring coils. The transformer was built joining two planar coils one over the other. In this kind of transformer, similar responses as analysis of transformers built with planar coil inner ring coil are found, as well the results of resonance. Because the low self-inductances and parasitic capacitances obtained in these configurations, although the coils resistance is low, which generates low exponential drops on responses. The resonance is found in higher frequencies, but satisfying conditions of sum of responses in resonance.

1. INTRODUCTION

Developed researches about transformers, in the main cases, concern solenoids with sinusoidal excitation [1–6]. In some cases, several analyses about transformers built with planar coils [7–11] are found in literature, which generally is made in integrated circuits [12–18] and few in large dimensions (in the order of centimeters) with crossing of planar coils and ring coils excited by square waves [19–21]. In other studies, analysis of the responses when the transformer is excited by square waves is generally applied to pulse transformers [22–28], or power electronics [29–31].

The analysis of transformers, built only with planar coils, is found in [32] for integrated circuits. In this case, the application is specific, and the analysis is not used to high energy, although some results may be utilized for applications in induced *emf* for transformers with

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large dimensions, in a similar way to [19–21]. The need to analyze these specific transformers is found in the reduction of electromagnetics devices and their applications, which is much needed today.

The analysis of these kinds of transformers is presented in two ways: the first is the excitation with square waves to see responses and effects, which is applied to several engineering problems as power electronics and others, and the second way is the excitation with sinusoidal voltages, to verify their energy transfer, evaluate possibilities of their use in reduction and applications of the actual transformers. These applications may be seen in several researches on electromagnetics fields [33–38], considering analysis of their characteristics as self and mutual inductances [39–41] or parasitic capacitances [42]. With these feasible applications, an analysis about transformers built with planar coils is presented in this paper, considering the case of square wave excitation, comparing this analysis with the related problems of [19–21].

So, this paper is presented as follows: the coils data and utilized equipments in realized experiments are shown in Section 2; the analysis about responses of the analyzed configurations, transfer function of the system and discussions about found results, comparing with results of transformers built with planar coils inner ring coils are shown in Section 3; Section 4 analyzes the problem of system resonance and compares these results with the cited transformers of [19, 20, 21]; the conclusions of this paper are presented in Section 5.

2. DATA AND EXPERIMENTAL METHODOLOGY

The experiments were realized with 7 planar coils with turn numbers: 10, 20, 50, 200, 500, 800 and 1600. For each coil, the crossing with all others was realized, where a signal based on a square wave of 5 V peak to peak, with frequencies ranging from 1 kHz to 25 MHz, excites the primary, and the other coil placed on the first defines the system

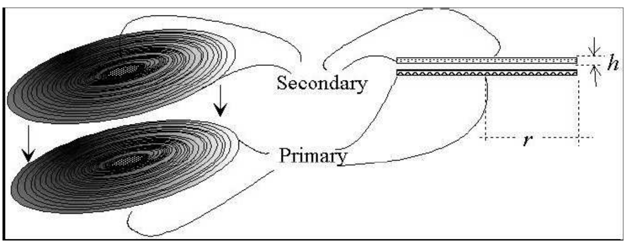


Figure 1. Air core planar transformer.

output, where the induced emf is verified. In this way, the joining of the two planar coils is presented as a planar transformer, and its structure may be seen in Fig. 1.

The coils were built on copper wire with diameter 2.02×10^{-4} m (32 AWG) for turn numbers lower than 500 and 1.80×10^{-4} m (36 AWG) for turn number above 500. The diameter of the coils is $d = 4.01 \times 10^{-2}$ m, and their heights are $h = 1.80 \times 10^{-4}$ m (from 10 to 50 turns) and $h = 5 \times 10^{-4}$ m (≥ 200 turns).

The used equipments for analysis in experimental work were a digital storage oscilloscope Agilent Technologies DSO3202A with passive probe N2862A (input resistance $=$ 10 MΩ and input capacitance $\simeq$ 12 pF), a function generator Rigol DG2021A and a digital multimeter Agilent Technologies U1252A.

Considering the crossing of the coils to realize the experiments, the total number of analyzed systems was 42 transformers, where more than 2400 measurements are realized. The measurements and system analysis are compared with system described in [19–21] which is a transformer built with planar coil inner ring coil, to verify conditions of resonance and energy transfer, including effects of parasitic capacitances on these coils [42], to future applications of these devices.

3. EXPERIMENTAL ANALYSIS AND SYSTEM RESULTS

Using results of [39–41, 43] the self inductances of the coils may be calculated, and using results of [42] the parasitic capacitances of the coils are evaluated. In the case of the coils with the turn number greater than 200, in which the wires are presented with more than one layer, the evaluation of the parasitic capacitances is verified based on frequency of the oscillatory response to step voltage and calculated self-inductance. All these data are presented in Table 1, with respective coils resistance measured with digital multimeter.

The mutual inductances are presented in Table 2, in accordance with [39–41, 43].

The values in Table 2 referring to transformers in opposite turn number (e.g., 10×20 and 20×10) are due to the relations of the radius of each one, in accordance to [39–41, 43].

Based on equivalent circuit of the system described in [19, 20], since the transformer elements are the same (existence of parasitic capacitances, resistances, self and mutual inductances), similar analysis may be realized for this case. Consequently, the system response for input step voltage is given by

$$v_0 = A\left[\sin\left(\omega_1 t\right)\sin\left(\omega_2 t\right)e^{-bt} + c\right]e^{-dt} \tag{1}$$

and the system transfer function may be found as [20]:

$$G(s) = \frac{z_5 s^5 + z_4 s^4 + z_3 s^3 + z_2 s^2 + z_1 s}{s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0} \tag{2}$$

where:

- $z_5 = c$;
- $z_4 = 4xc$;
- $z_3 = \left(\left(6x^2 + y_2 + y_1\right)c + 2\left(y_2 - y_1\right)a\right)$;
- $z_2 = \left(\left(4x^3 + 2x\left(y_1 + y_2\right)\right)c + \left(2\left(y_2 - y_1\right)\left(x + d\right)a\right)\right)$;
- $z_1 = \left(\left(x^4 + x^2\left(y_1 + y_2\right) + y_1 y_2\right)c + \left(2\left(y_2 - y_1\right)dx\right)a\right)$;
- $p_4 = 4x + d$;
- $p_3 = \left(6x^2 + y_2 + y_1 + 4xd\right)$;

Table 1. Coils data.

Turn Number	Resistance (Ω)	Internal Radius (mm)	External Radius (cm)	Self-Inductance (µH)
10	0.50	1.0	1.90	0.433
20	0.89	1.0	2.05	1.85
50	2.39	0.7	2.05	11.0
200	7.87	0.5	2.00	167.0
500	54.22	0.5	1.80	948.0
800	59.20	0.5	2.00	2,670.0
1600	72.55	0.5	2.00	10,700.0

Table 2. Mutual inductances (µH).

Turns	10	20	50	200	500	800	1600
10	-	2.774	6.832	26.99	64.60	107.9	215.9
20	2.801	-	14.43	56.48	130.2	225.9	451.8
50	6.891	14.42	-	138.9	320.4	555.9	1,112.0
200	27.13	56.15	138.3	-	1,266.0	2,175.0	4,350.0
500	64.02	128.0	315.2	1,250.0	-	4,999.0	9,999.0
800	108.5	224.6	553.2	2,175.0	5,064.0	-	17,400.0
1600	217.1	449.2	1,106.0	4,350.0	10,130.0	17,400.0	-

- $p_2 = 4x^3 + 6dx^2 + (2x + d)(y_1 + y_2)$;
- $p_1 = x^4 + 4dx^3 + (y_1 + y_2)\left(x^2 + 2dx\right) + y_1 y_2$;
- $p_0 = \left(x^4 + x^2(y_1 + y_2) + y_1 y_2\right) d$

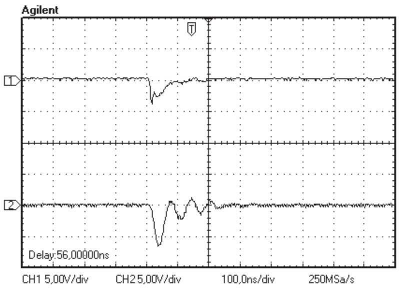
being

- $x = b + d$;
- $y_1 = (\omega_1 + \omega_2)^2$;
- $y_2 = (\omega_1 - \omega_2)^2$.

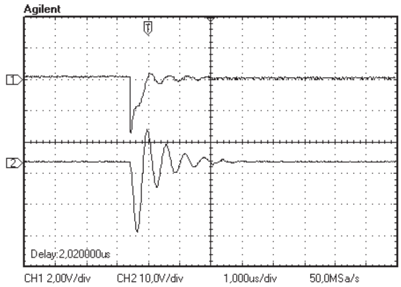
This result may be seen in Fig. 2 for some of the analyzed cases, which may be compared with responses of transformers built with planar coils inner ring coils [19, 20]. In this figure, all responses are obtained for input square wave frequency $f = 1\,\text{kHz}$. Specifically, observing response as the configuration 800×10 turns in this figure, effect known as *beat* [44], is warranted the presented transfer function.

Also, it is observed that in the cases where the primary is built with small turn number, the low resistance defines almost an effect of short-circuit on input signal, and the graph seen in oscilloscope is not presented as a square wave, but as RL circuit with influence of the secondary induced emf (double resonance, i.e., resonance on primary and secondary, as seen in Tesla transformer [25–27]).

In the case of transformers built with planar coils, the relation defined in [19, 20] on turn ratio for system response defined as modulated sine wave, i.e., $15 < n_p/n_s < 25$, being n_p the turn number of the primary coil and n_s the turn number of the secondary coil, is not satisfied. In this case, because of the magnetic flux distribution throughout the disc, where the turns are distributed, this effect is visible when $n_s = 10$. As the turn number of the secondary coil increases, the lower frequency (modulated sine wave) decreases. When turn ratio decreases, both frequencies decreases, which become almost equal, as seen in Fig. 2(h). This is because the changes on parameters (resistances, parasitic capacitances, self and mutual inductances) in the equivalent circuit causes changes in these frequencies, and consequently, values in the transfer function changes the system response. In this way, although these changes are verified in comparison with the system described in [19, 20], the problem is seen similarly, such that the increase in the frequency excitation (input square wave) results in a sum of responses and resonance effect with output high voltage, as seen in [21, 28]. This analysis is presented in the next section.



(a)



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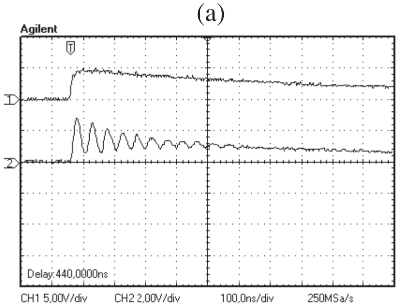

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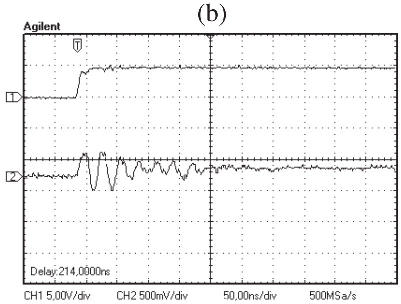

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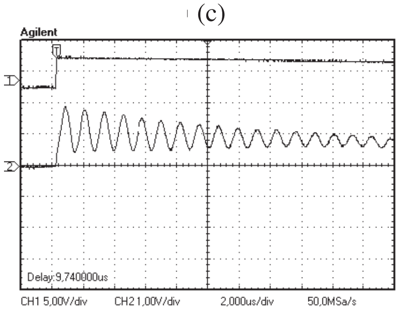

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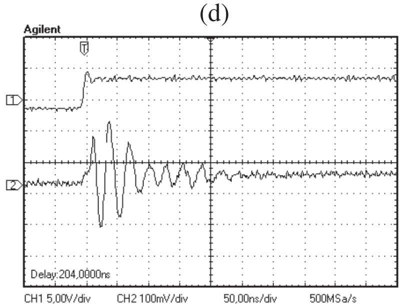

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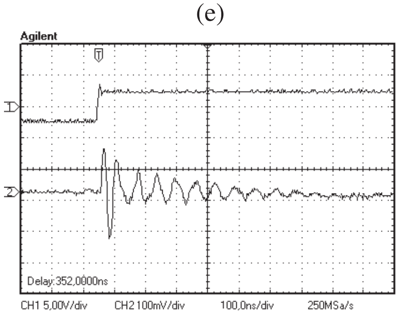
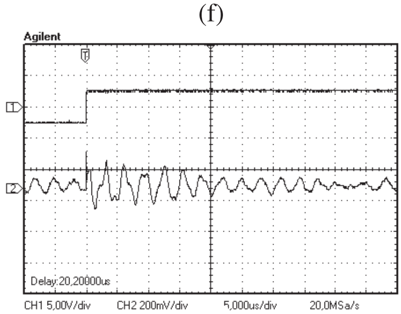


(g)



(h)

Figure 2. System responses in oscilloscope: (a) 10×20; (b) 20×200; (c) 50×20; (d) 200×10; (e) 500×1600; (f) 800×10; (g) 1600×20; (h) 1600×800.

4. SUM OF RESPONSES AND RESONANCE EFFECT

In the same way of the system seen in [19–21], the sum of responses as the input square wave frequency increases, which is verified in the case of the transformers built only with planar coils.

Because of the similarity with the induced *emf* on transformer built with planar coil inner ring coil, in each rise and fall of the square wave, as the input frequency increases, the responses are overlapped. Thus, analyzing these effects in oscilloscope for this case, we see equal conditions as [21], i.e., when the relation

$$f_s = f_r/n, \tag{3}$$

is satisfied, where f_s is the input frequency of the square wave; f_r is the output frequency of the response; n is the cycles number of sine wave between one rise and one fall of the square wave; the values of responses of the system (their peak voltages) are added, reaching maximum values, as shown in Fig. 3. In this case, we can see that for the frequency $f_s = 2.73\,\text{MHz}$, condition (3) is verified, such that

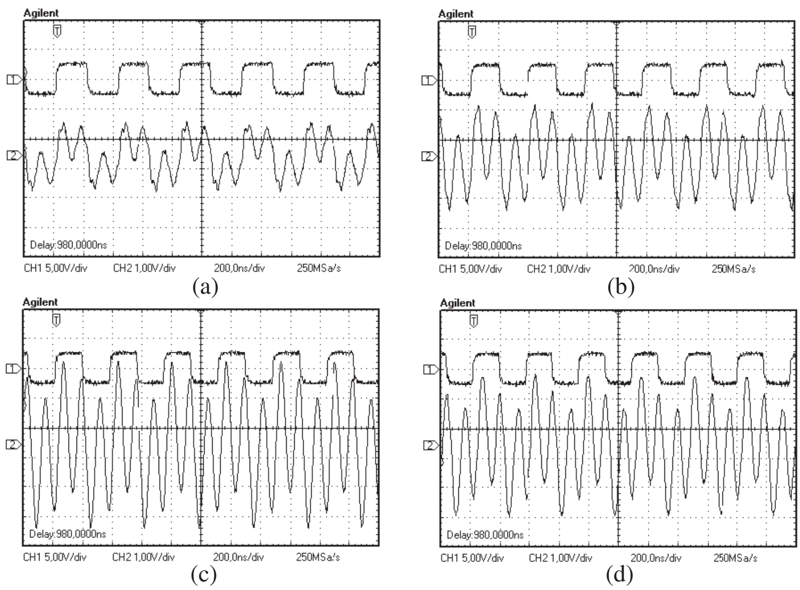


Figure 3. Planar × planar transformer (configuration 200 × 50) and sum of responses in phase in accordance to frequency of the square wave: (a) $f = 2.4\,\text{MHz}$; (b) 2.6 MHz; (c) 2.73 MHz ($f_s = f_r/n$) and (d) 2.8 MHz.

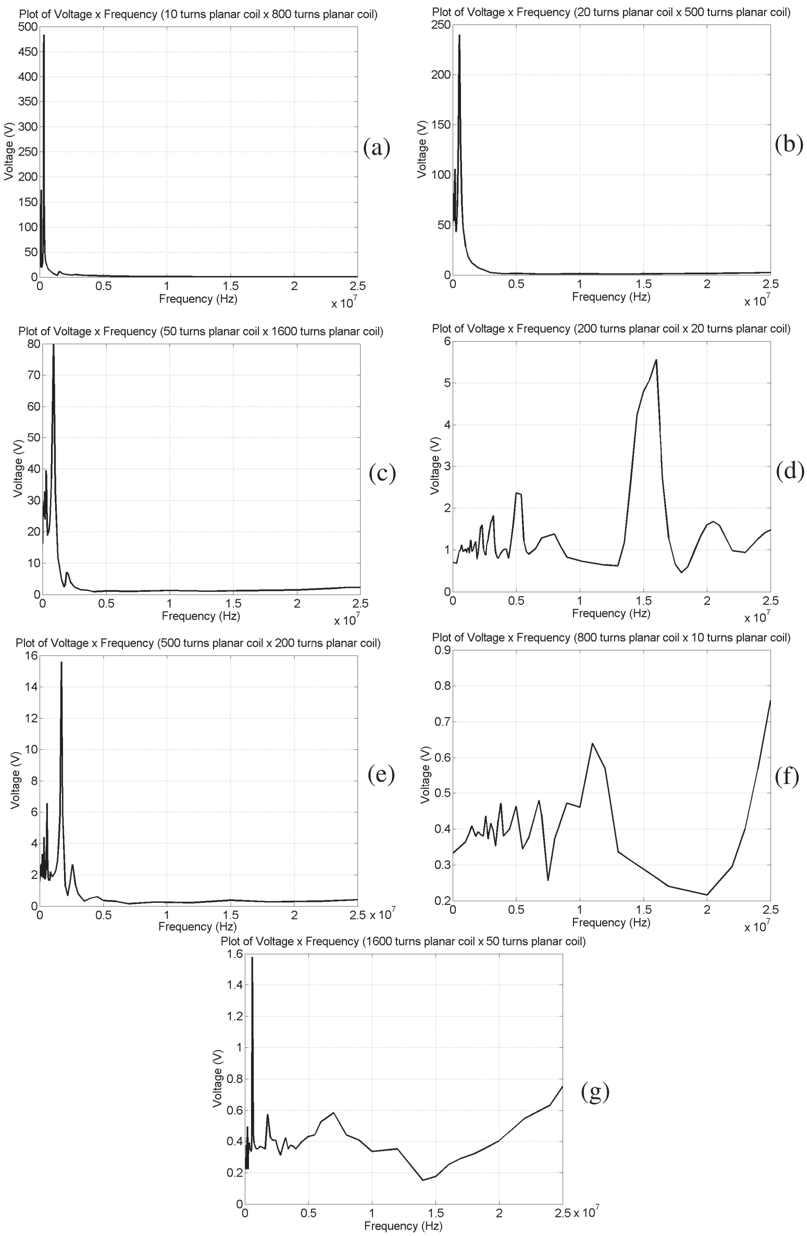


Figure 4. System Resonance for configurations: (a) 10×800; (b) 20×500; (c) 50×1600; (d) 200×20; (e) 500×200; (f) 800×10 and (g) 1600×50.

in frequencies lower and higher than 2.73 MHz, the amplitude of the response is lower.

Ranging input square waves frequencies from 1 kHz to 25 MHz, we find similar graphs of [21], as may be seen in Fig. 4, for some analyzed configurations. The peaks in these graphs are the frequencies where condition (3) is verified.

Other comparative analysis in these transformers may be realized with the graphs of output voltage as function of the turn number of the secondary coil, when the turn number of the primary coil is maintained constant in resonance frequency, i.e., $n = 1$, or $f_s = f_r$, such that the maximum output voltage is

$$v_0 = \sum_{p=0}^{n} (-1)^p \left(\alpha \sin\left(\omega_1 (t-p)\right) e^{-b(t-p)} + a\right) \tag{4}$$

for $t - p > 0$, where p is the square wave period; α is the sine wave amplitude; a is the DC response [21].

For all configurations, this analysis is shown in Fig. 5, where it is seen that the curves have found a maximum peak when secondary coil presents 800 turns, and this gain falls as the turn number of the secondary coil is increased. Moreover, for analyzed configurations, it is verified that the maximum gain found for this transformer was in configuration 10×800. It is because in the resonance, the high input square wave frequency defines a great sum of responses. Also, in accordance to [21], this resonance peak is a sum of the responses and a value due turn ratio. The reduction in the gain for secondary coil with turn number higher than 800 is verified because the parasitic capacitance, resistance and inductance are higher (built in the same disc of the others, but with small diameter wire), such that the

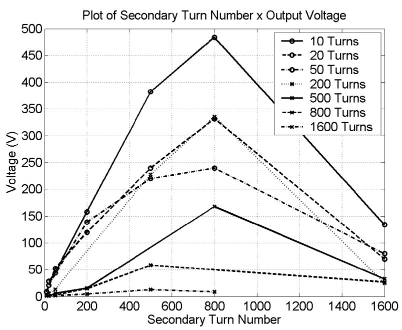


Figure 5. Graph showing peak resonance on transformers built with planar coils.

proximity of the frequencies (ω_1 and ω_2) on response, seen in Figs. 2(g) and (h), generates the effect of not finding a frequency that satisfies the perfect crossing of peaks in accordance to (3).

5. CONCLUSIONS

This paper have presented an analysis of induced *emf* in transformers built with planar coils, comparing the results with transformers built with planar coils inner ring coils presented in [19–21]. In accordance to found results, system responses are similar to analyzed case of [19–21] when excited by step voltage, following the rise and fall of this input signal. However, the *beat* signal is presented in different conditions, being visible as the secondary coil which has small turn number, as the case of 10 turns. In this case, the change observed is because of the magnetic flux distribution throughout the coil which has its turns distributed throughout the disc. Considering the increasing of input square wave frequency, the sum of responses is seen in the same way as in [21], following similar relation between frequencies f_r and f_s. Moreover, the maximum resonance peak is found when $n = 1$ in this relation between frequencies f_r and f_s, defined as $f_r = f_s$. Also, it can be seen that the higher resonance peak voltage is found for transformer in configuration 10×800, this secondary turn number as the better gain performance in the transformer. Analyzing the worked configurations, we can see that when the secondary turn number increases beyond 800, the output voltage decreases quickly, because of the changes on coil parameters (parasitic capacitance, resistance and inductance), such that this step voltage response makes the frequencies ω_1 and ω_2 almost equal, which determines the difficulty in finding a frequency that satisfies the perfect crossing of peaks in accordance to (3). The aim of the analysis of these systems is to verify the possibilities of compact transformers in systems and new future applications of pulsed systems and air core transformers.

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