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BLIND DIRECTION OF ANGLE AND TIME DELAY ESTIMATION ALGORITHM FOR UNIFORM LINEAR ARRAY EMPLOYING MULTI-INVARIANCE MUSIC

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Abstract—This paper addresses the problem of direction of arrival and time delay estimation, and derives multi-invariance MUSIC (MI-MUSIC) algorithm therein. The proposed MI-MUSIC, which only requires one-dimension searching, can avoid the high computational cost within two-dimension MUSIC (2D-MUSIC) algorithm. It means that MI-MUSIC algorithm has better performance than that of ESPRIT and MUSIC, and also can be viewed as a generalization of MUSIC. Simulation results verify the usefulness of our algorithm.

1. INTRODUCTION

Direction of arrival (DOA) estimation and time delay estimation are two key problems in the signal processing field. Recently, the problem of joint angle and time delay estimation has also attracted considerable attentions, and it has been used for wireless location and emergency services [1]. A precise DOA and time delay estimation results in a more accurate location estimate. Joint angle and delay estimation have been investigated in [2–9], including maximum likelihood method [2], two dimensional multiple signal classification (MUSIC) algorithm [3, 4], estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [5, 6], Fourth-order cumulant method [7], parallel factor analysis (PARAFAC) algorithm [8], etc. In [3], 2D-MUSIC can be used to estimate the angles of arrival and delays of the multipath signals using a collection of space-time channel estimates. In [5], Van der Veen uses ESPRIT to separate and estimate the phase shifts due to delay and direction-of-incidence. High order

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cumulant method requires the signal statistical properties, and it needs larger snapshots to get good performance. Also it has heavier computation load. ESPRIT is a closed-form eigen structure-based parameter estimation technique which requires data possession of certain "invariance" structures. An alternative eigen-decomposition based method to estimate DOAs is MUSIC algorithm, which uses the noise-subspace eigenvectors of the data correlation matrix to form a null spectrum and yields the corresponding signal parameter estimates. Notably, it has also concerned that MUSIC matches some kind of irregularly-spaced array with high popularity [10]. It has been proved that 2D-MUSIC algorithm represents an implement for DOA and delay estimation. However, the requirement of 2D search renders much higher computational complexity. In this paper, we derive the multi-invariance MUSIC (MI-MUSIC) algorithm which distinctively reduces the complexity for angle and delay estimation. We compare their root mean squared error (RMSE) and costs of computational complexity against those of conventional algorithms. The proposed MI-MUSIC algorithm can have better angle and delay estimation performance than ESPRIT, PARAFAC and MUSIC algorithm. Numerical results for different array antenna manifolds and a variety of data lengths are also presented in the simulations.

Notation: $(.)^*$, $(.)^T$, $(.)^H$, $(.)^\dagger$ and $\|\,\|_F$ denote the complex conjugation, transpose, conjugate-transpose, pseudo-inverse and Forbenius norm, respectively. $\mathbf{I}_P$ is a $P \times P$ identity matrix; $\circ$ is Khatri-Rao product; $\otimes$ is the Kronecker product.

2. DATA MODEL

A total of K rays are transmitting to a base station equipped with a uniform linear array of M antennas with half-wavelength spacing. Then, the received baseband signal at the output of the antenna array can be expressed as

$$x(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k)\beta_k g(t - \tau_k)s_k(t) + n(t) \tag{1}$$

where $n(t)$ is the received noise, which is independent of the signal; β_k is the channel fading of the kth ray; τ_k is the time delay of the kth ray; θ_k is the direction of arrival (DOA) of the kth ray; $\mathbf{a}(\theta_k)$ is the array direction vector for DOA θ_k; $s_k(t)$ is the information-bearing signal of the kth ray. It is assumed that all sources employ a common pulse shape $g(t)$. We take sample $x(t)$ at a rate of P times the symbol rate during L symbol periods and collect samples. Then, construct $MP \times L$

matrix $\mathbf{X}$

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}(0) & \mathbf{x}(1) & \dots & \mathbf{x}(L-1) \\ \mathbf{x}(\frac{1}{P}) & \mathbf{x}(1+\frac{1}{P}) & \dots & \mathbf{x}(L-1+\frac{1}{P}) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}(1-\frac{1}{P}) & \mathbf{x}(2-\frac{1}{P}) & \dots & \mathbf{x}(L-\frac{1}{P}) \end{bmatrix} \tag{2}$$

We take the discrete Fourier transform of the oversampled antenna output, and obtain a matrix which satisfies the model [3],

$$\bar{\mathbf{X}} = [\mathbf{F}_\tau \circ \mathbf{A}_\theta]\mathbf{\Gamma}\mathbf{S} + \bar{\mathbf{E}} = \begin{bmatrix} \mathbf{A}_\theta D_1(\mathbf{F}_\tau) \\ \mathbf{A}_\theta D_2(\mathbf{F}_\tau) \\ \vdots \\ \mathbf{A}_\theta D_P(\mathbf{F}_\tau) \end{bmatrix} \mathbf{S}_\mathbf{E} + \bar{\mathbf{E}} \tag{3}$$

where $\mathbf{A}_\theta = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is the direction matrix; $\mathbf{\Gamma} = \mathrm{diag}[\beta_1 \dots \beta_K] \in \mathbb{C}^{K\times K}$ is the channel fading matrix; $\mathbf{F}_\tau \circ \mathbf{A}_\theta$ is Khatri-Rao product; $\mathbf{S} \in \mathbb{C}^{K\times L}$ is the source matrix; $\mathbf{S}_\mathbf{E} = \mathbf{\Gamma}\mathbf{S}$; $D_p(.)$ is to extract the pth row of its matrix argument and construct a diagonal matrix out of it. $\bar{\mathbf{E}}$ is the noise component; $\mathbf{F}_\tau \in \mathbb{C}^{P\times K}$ is a delay matrix, which element $[\mathbf{F}_\tau]_{p,k} = e^{-j2\pi\tau_k(p-1)/P}$. For the signal model in (3), the covariance matrix $\hat{\mathbf{R}}_x$ can be estimated by $\hat{\mathbf{R}}_x = \bar{\mathbf{X}}\bar{\mathbf{X}}^H/L$, which is denoted by

$$\hat{\mathbf{R}}_x = \mathbf{E}_s\mathbf{D}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{D}_n\mathbf{E}_n^H \tag{4}$$

where $\mathbf{D}_s$ stands for a $K \times K$ diagonal matrix whose diagonal elements contain the largest K eigenvalues, and $\mathbf{D}_n$ stands for a diagonal matrix whose diagonal entries contain the smallest $MP-K$ eigenvalues. $\mathbf{E}_s$ is the matrix composed of the eigenvectors corresponding to the largest K eigenvalues of $\hat{\mathbf{R}}_x$, while $\mathbf{E}_n$ represents the matrix including the rest eigenvectors.

3. MUSIC-LIKE ALGORITHMS FOR ANGLE AND DELAY ESTIMATION

According to Eq. (4), we construct the 2D-MUSIC spatial spectrum function

$$f_{2dmusic}(\tau, \theta) = \frac{1}{[\mathbf{f}(\tau) \otimes \mathbf{a}(\theta)]^H \mathbf{E}_n\mathbf{E}_n^H [\mathbf{f}(\tau) \otimes \mathbf{a}(\theta)]}$$

where

$$\mathbf{f}(\tau) = [1 \quad \exp(-j2\pi\tau/P) \quad \dots \quad \exp(-j2\pi(P-1)\tau/P))]^T \tag{5}$$

$$\mathbf{a}(\theta) = [1 \quad \exp(-j\pi\sin\theta) \quad \dots \quad \exp(-j\pi(M-1)\sin\theta)]^T \tag{6}$$

here we have the K largest peaks of $f_{2dmusic}(\tau,\theta)$ taken as the estimates of the angles and delays. Since 2D-MUSIC requires an exhaustive two-dimension searching, its approach is normally inefficient due to high computational cost. In the following subsections, we present MI-MUSIC algorithm, which qualifies for the angle and delay estimation just through one-dimension searching.

3.1. Multi-invariance MUSIC Algorithm for Angle and Delay Estimation

MI-MUSIC algorithm can be viewed as a generalization of MUSIC and has been proposed for DOA estimation with the exploitation of array invariance [11]. In this subsection, the idea of MI-MUSIC has been adopted to estimate DOA and delay. Assuming that no noise is presented, the signal subspace $\mathbf{E}_s$ in Eq. (4) can be denoted as

$$\begin{aligned}\mathbf{E}_s &= \begin{bmatrix}\mathbf{A}_\theta D_1(\mathbf{F}_\tau)\\ \mathbf{A}_\theta D_2(\mathbf{F}_\tau)\\ \vdots \\ \mathbf{A}_\theta D_P(\mathbf{F}_\tau)\end{bmatrix}\mathbf{T} = \begin{bmatrix}\mathbf{A}_\theta\\ \mathbf{A}_\theta\mathbf{\Phi}\\ \vdots \\ \mathbf{A}_\theta\mathbf{\Phi}^{P-1}\end{bmatrix}\mathbf{T} = \mathbf{\Lambda}\mathbf{T}\\ &= [\mathbf{f}(\tau_1)\otimes\mathbf{a}(\theta_1),\ldots,\mathbf{f}(\tau_K)\otimes\mathbf{a}(\theta_K)]\mathbf{T}\end{aligned} \tag{7}$$

where $\mathbf{T}$ is a $K\times K$ full-rank matrix, $\mathbf{\Phi} = \mathrm{diag}\{[\exp(-j2\pi\tau_1/P)\ \ldots\ \exp(-j2\pi\tau_K/P)]\}$ is the rotational matrix, $\mathbf{\Lambda} = [\mathbf{f}(\tau_1)\otimes\mathbf{a}(\theta_1),\ldots,\mathbf{f}(\tau_K)\otimes\mathbf{a}(\theta_K)]$. The signal in Eq. (7) is with multi-invariance characteristic, and we can use MI-MUSIC algorithm [11] for angle and delay estimation. According to (7), $\mathbf{\Lambda} = \mathbf{E}_s\mathbf{T}^{-1}$ can be easily obtained, then the signal subspace fitting is given in this form $\hat{\mathbf{T}},\hat{\mathbf{\Lambda}} = \arg\min_{\mathbf{T},\mathbf{\Lambda}}\|\mathbf{\Lambda}-\hat{\mathbf{E}}_s\mathbf{T}^{-1}\|_F^2$, where $\hat{\mathbf{E}}_s$ is the estimate of $\mathbf{E}_s$. The subspace fitting can be also denoted as

$$\hat{\mathbf{T}},\hat{\mathbf{\Lambda}} = \arg\min tr\left(\mathbf{\Lambda}^H\mathbf{\Pi}_{\hat{\mathbf{E}}_s}^{\perp}\mathbf{\Lambda}\right) \tag{8}$$

where $\mathbf{\Pi}_{\hat{\mathbf{E}}_s}^{\perp} = \mathbf{I}_{MP} - \hat{\mathbf{E}}_s(\hat{\mathbf{E}}_s^H\hat{\mathbf{E}}_s)^{-1}\hat{\mathbf{E}}_s^H$, $tr(.)$ denotes the sum of the elements of the principal diagonal of the matrix. Consider now the minimization of Eq. (8), which becomes,

$$\mathbf{f}(\tau_k),\mathbf{a}(\theta_k) = \arg\min\sum\nolimits_{k=1}^{K}[\mathbf{f}(\tau_k)\otimes\mathbf{a}(\theta_k)]^H\mathbf{\Pi}_{\hat{\mathbf{E}}_s}^{\perp}[\mathbf{f}(\tau_k)\otimes\mathbf{a}(\theta_k)] \tag{9}$$

Also, the minimization for Eq. (9) can be attained by searching for the deepest K minimum in the following criterion,

$$\begin{aligned}\mathbf{V}(\phi,\theta) &= [\mathbf{f}(\tau)\otimes\mathbf{a}(\theta)]^H\mathbf{\Pi}_{\hat{\mathbf{E}}_s}^{\perp}[\mathbf{f}(\tau)\otimes\mathbf{a}(\theta)]\\ &= \mathbf{a}(\theta)^H[\mathbf{f}(\tau)\otimes\mathbf{I}_M]^H\mathbf{\Pi}_{\hat{\mathbf{E}}_s}^{\perp}[\mathbf{f}(\tau)\otimes\mathbf{I}_M]\mathbf{a}(\theta)\\ &= \mathbf{a}(\theta)^H\mathbf{Q}(\tau)\mathbf{a}(\theta)\end{aligned} \tag{10}$$

where $\mathbf{Q}(\tau) = [\mathbf{f}(\tau) \otimes \mathbf{I}_M]^H \mathbf{\Pi}^{\perp}_{\hat{\mathbf{E}}_s} [\mathbf{f}(\tau) \otimes \mathbf{I}_M]$. Eq. (10) is the problem of quadratic optimization. We also consider the constraint of $\mathbf{e}_1^T \mathbf{a}(\theta) = 1$, where $\mathbf{e}_1 = [1, 0, \ldots, 0]^T \in \mathbb{R}^{M\times 1}$ has been added to eliminate the trivial solution $\mathbf{a}(\theta) = \mathbf{0}_M$. The optimization problem can be reconstructed with the linear constraint minimum variance solution, for which we have

$$\min_{\tau} \mathbf{a}(\theta)^H \mathbf{Q}(\tau)\mathbf{a}(\theta), \text{ s.t. } \mathbf{e}_1^T \mathbf{a}(\theta) = 1 \tag{11}$$

Make solution to Eq. (11), which is shown as

$$\hat{\tau} = \arg\min_{\tau} \frac{1}{\mathbf{e}_1^T \mathbf{Q}(\tau)^{-1}\mathbf{e}_1} = \arg\max_{\tau} \mathbf{e}_1^T \mathbf{Q}(\tau)^{-1}\mathbf{e}_1 \tag{12}$$

Searching τ, we find the K largest peaks of the $(1, 1)$ element of $\mathbf{Q}(\tau)^{-1}$. Note that the K largest peak should correspond to the delays. With respect to Eq. (12), another denotation can be given by

$$\begin{aligned}\mathbf{V}(\phi, \theta) &= [\mathbf{f}(\tau) \otimes \mathbf{a}(\theta)]^H \mathbf{\Pi}^{\perp}_{\hat{\mathbf{E}}_s} [\mathbf{f}(\tau) \otimes \mathbf{a}(\theta)] \\ &= \mathbf{f}(\tau)^H [\mathbf{I}_P \otimes \mathbf{a}(\theta)]^H \mathbf{\Pi}^{\perp}_{\hat{\mathbf{E}}_s} [\mathbf{I}_P \otimes \mathbf{a}(\theta)]\mathbf{f}(\tau) \\ &= \mathbf{f}(\tau)^H \mathbf{P}(\theta)\mathbf{f}(\tau)\end{aligned} \tag{13}$$

where $\mathbf{P}(\theta) = [\mathbf{I}_P \otimes \mathbf{a}(\theta)]^H \mathbf{\Pi}^{\perp}_{\hat{\mathbf{E}}_s} [\mathbf{I}_P \otimes \mathbf{a}(\theta)]$. Similarly, the solution for θ is

$$\hat{\theta} = \arg\max_{\theta} \mathbf{e}_2^T \mathbf{P}(\theta)^{-1}\mathbf{e}_2 \tag{14}$$

where $\mathbf{e}_2 = [1, 0, \ldots, 0]^T \in \mathbb{R}^{P\times 1}$. Searching $\theta \in [-90°, 90°]$, we find the K largest peaks of the $(1, 1)$ element of $\mathbf{P}(\theta)^{-1}$. Note also that the K largest peak should correspond to DOAs.

Till now, we have achieved the proposal for multi-invariance MUSIC-based algorithm for DOA and delay estimation. We show the major steps of MI-MUSIC as follows: Step 1. Perform the eigen-decomposition operation for the covariance matrix $\hat{\mathbf{R}}_x$ to $\hat{\mathbf{E}}_s$, and calculate $\mathbf{\Pi}^{\perp}_{\hat{\mathbf{E}}_s}$; Step 2. Searching τ, we find the K largest peaks of the $(1, 1)$ element of $\mathbf{Q}(\tau)^{-1}$ with respect to Eq. (12) to get the estimate of delays; Step 3. Searching θ, we find the K largest peaks of the $(1, 1)$ element of $\mathbf{P}(\theta)^{-1}$ with respect to Eq. (14) to get the estimate of DOAs. It is pointed out that when the coherent signals impinge the antenna array, the source matrix is not full rank, and we can use the smoothing technique to solve this problem.

3.2. Complexity Analysis

In contrast, our algorithm can have heavier computational load than ESPRIT and MUSIC and lower complexity than 2D-MUSIC. ESPRIT requires $O(LM^2P^2 + M^3P^3 + 2K^2(M-1)P + 2K^2(P-1)M + 6K^3)$, while 2D-MUSIC costs $O(LM^2P^2 + M^3P^3 + n^2[2MP(MP-K) + MP])$; MUSIC algorithm needs $O(LMP^2 + LM^2P + M^3 + P^3 + n[2P(P-K) + 2M(M-K) + P + M])$, and our algorithm (MI-MUSIC) urges $O(LM^2P^2 + M^3P^3 + M^2P^2K + n[M^3P^2 + M^2P^3 + M^3P + MP^3 + P^2 + M^2])$. Among the three methods stated above, n can be the total times for their searching steps in each algorithm. In the PARAFAC algorithm [8], the complexity of each trilinear alternating least squares (TALS) iteration is $O(3K^3 + 2MPLK + 3K^2(MP + ML + PL + M + P + L))$, and the number of iterations depends on the three way data to be decomposed.

4. SIMULATION RESULTS

We present Monte Carlo simulations that assess the performance of our algorithm. We assume binary phase shift keying (BPSK) modulated signal. Note that L, K, M and P are the number of snapshots, rays, antennas and oversampling rate, respectively. In the following simulations, we assume that there are three noncoherent rays to arrive at array antenna. Their DOAs are 10°, 20°, 30°, and their corresponding delays are 0.1 chip, 0.3 chip, 0.5 chip, respectively. The number of sources can be estimated based on Akaike's information criterion and Rissanen's Minimum description length principle. We define $RMSE = \sqrt{\frac{1}{500}\sum_{n=1}^{500}(\hat{a}_{n,k} - a_k)^2}$, where $\hat{a}_{n,k}$ is the estimated angle/delay for kth ray of the nth trial; a_k is the prefect angle/delay of the kth ray.

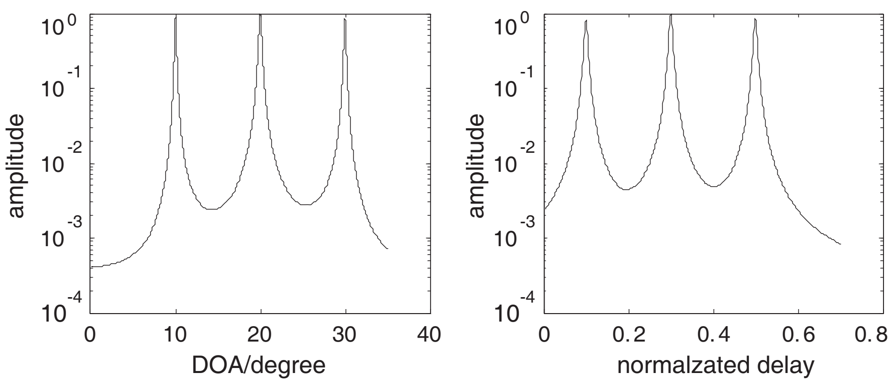

Figure 1. The angle and delay estimation performance with MI-MUSIC at SNR = 25 dB.

Figure 1 presents angle and delay estimation results of MI-MUSIC algorithm with $M = 8$, $P = 10$, $L = 100$ and $\text{SNR} = 25\,\text{dB}$, where the spectrum peaks at the angles/delays can be clearly observed. From Fig. 1, we find that MI-MUSIC algorithm works well. Fig. 2 shows the angle and delay estimation performance for ray 2 with $M = 8$, $P = 10$ and $L = 50$, where we compare MI-MUSIC algorithm with ESPRIT, PARAFAC and MUSIC methods. It is indicated in Fig. 2 that among the four algorithms, MI-MUSIC algorithm that we presented has better angle/delay estimation performance than ESPRIT, PARAFAC and

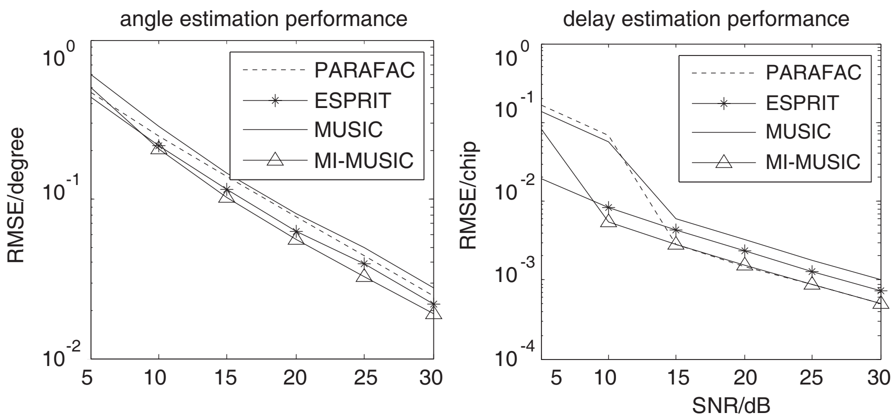

Figure 2. The angle and delay estimation performance comparison with $L = 50$.

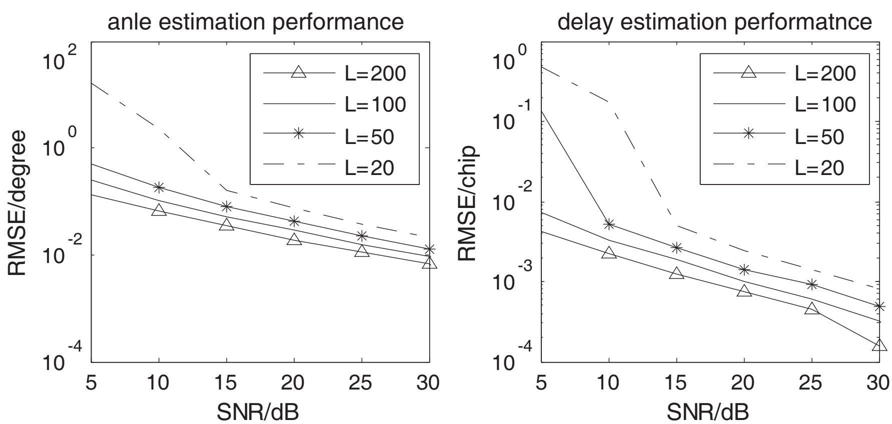

Figure 3. Angle and delay estimation with different L.

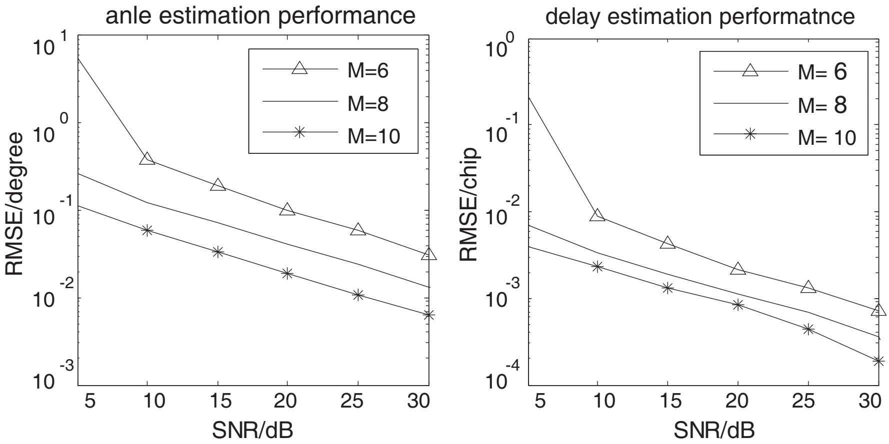

Figure 4. Angle and delay estimation with different M.

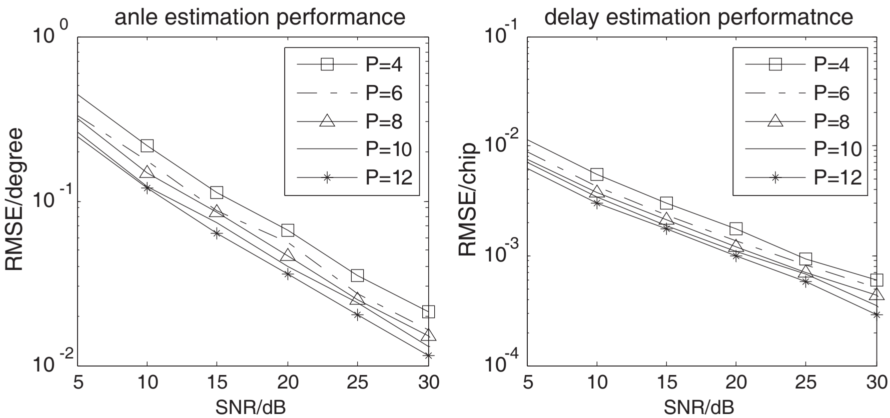

Figure 5. Angle and delay estimation with different P.

MUSIC algorithms. MUSIC has the worst angle/delay estimation performance among the four algorithms. Fig. 3 depicts the algorithmic performance comparisons where MI-MUSIC has been adopted, and the simulation is shown for ray 3 with different L (the same $M = 8$, $P = 10$ as Fig. 2). It is indicated that the performance of angle and delay estimation becomes better with L increasing.

Figure 4 illustrates the angle and delay estimation performance by MI-MUSIC algorithm for ray 2 with different M. It is clearly shown that the estimation performance of MI-MUSIC is gradually improved with the number of antennas increasing. Multiple antennas

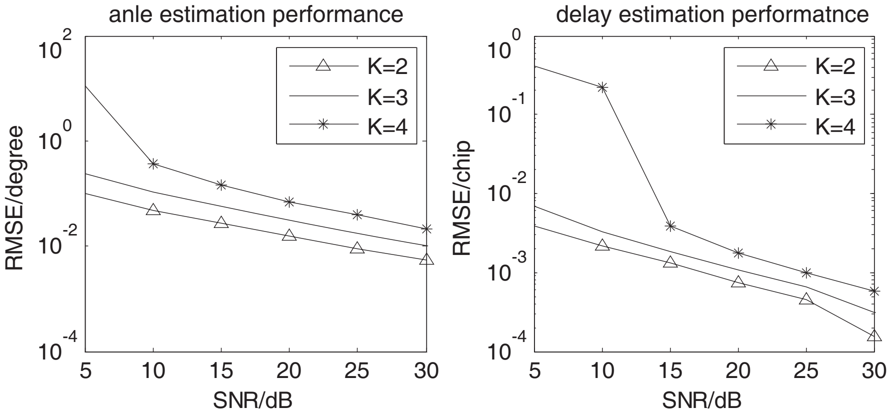

Figure 6. Angle and delay estimation with different K.

improve angle and delay estimation performance because of diversity gain. Fig. 5 presents the angle and delay estimation performance by MI-MUSIC algorithm for ray 2 with different P. It is clearly shown that the estimation performance of MI-MUSIC is gradually improved with increasing P. Fig. 6 displays the algorithmic performance of MI-MUSIC under different K when $M = 8$, $P = 10$, and $L = 100$. From Fig. 6, we conclude that angle and delay estimation performance levels are down with the increment of source numbers.

5. CONCLUSION

In this paper, we have derived the MI-MUSIC algorithm, which avoids the high computational cost within 2D-MUSIC, for blind joint angle and delay estimation. We demonstrate that MI-MUSIC can have much better performance for angle and delay estimation in contrast to ESPRIT, PARAFAC and MUSIC algorithms. Our algorithm can work well in other array manifolds and expand the adoptions, and it can be regarded as a generalization of MUSIC.

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