Progress In Electromagnetics Research M, Vol. 9, 227–241, 2009

BAND STRUCTURE, REFLECTION PROPERTIES AND ABNORMAL BEHAVIOUR OF ONE-DIMENSIONAL PLASMA PHOTONIC CRYSTALS

V. Kumar and K. S. Singh

Department of Physics
Digamber Jain (P.G.) College
Baraut-250611, India

S. P. Ojha

Chaudhary Charan Singh University
Meerut-200005, India

Abstract—In this paper, some studies on one-dimensional plasma photonic crystal (PPC) containing alternate layers of dielectric and micro-plasma have been presented. The band structures, reflectivity, group velocities and effective group index of such photonic crystals have been studied. For the purpose of computation, we have used transfer matrix method. In this study, we take two PPC structures named PPC1 and PPC2. In PPC1, we take SiO_2 as the material for the dielectric layers whereas in PPC2, we take TiO_2 as the material for the dielectric layers. It is found that the forbidden band gap(s) can be increased by increasing the thickness of plasma layers. The ranges of 100% reflection is found to be in the higher normalized frequency region in the case of PPC1 whereas in PPC2 the ranges of 100% reflection is found in the lower normalized frequency region. It is also found that for a certain normalized frequency, the group velocity becomes negative in both PPCs. However, the range of normalized frequency for which the group velocity is negative is larger in the case PPC1 than in PPC2. This abnormal behaviour of group velocities of both PPCs results in superluminal propagation (speed of EM wave in PPC greater than speed of light) of electromagnetic waves. Also, because of the abnormal behaviour of group velocity, effective group index becomes negative and possesses ultra high values. Such structures may be considered as a flip flop as there is positive and negative symmetry of effective group velocity. Also, PPC2 exhibits superluminal propagation

Corresponding author: V. Kumar (vrpcommon@yahoo.com).

for wider range of normalized frequency where there is superluminal propagation inside the structure as compared to that of PPC1.

1. INTRODUCTION

In last two decades, photonic crystals (PCs) have drawn attention of many researchers because of the ability of PCs to control the propagation of electromagnetic waves. In general, photonic crystal structures modify electromagnetic properties of ordinary bulk materials [1–3]. The ability of PCs to control EM radiations has many interesting applications in micro cavities, optical filter, localization of photon, spontaneous emission control and omni-directional reflection [4–8]. Some photonic crystals exhibit complete EM band gap(s); and such PCs are known as photonic band gap (PBG) structures. The modulation of refractive index in all three spatial directions in PCs is required to observe complete band gap in PCs [9–11]. Recently, Ojha et al. have reported that a simple-to-fabricate periodic 1D photonic crystal can have high reflectivity over a broad range of frequencies at all incident angles, i.e., an omnidirectional reflection, if the refractive indices and the thickness of the constituent dielectric layers are properly chosen [12–15]. To better understand the physics of photonic crystals, exact analytical solutions are quite useful. One dimensional photonic crystal with Kronig Penny periodic dielectric structures is the simplest of photonic crystal structures [16, 17].

Chen et al. [18] found an omni-directional photonic crystal in near infra red range using SiO_2 and TiO_2 quarter wave films. Studies on the propagation of electromagnetic wave in dispersive media have been presented in many papers [19]. Garrett and McCumber [20] considered the propagation of Gaussian pulse through an anomalous dispersive medium, and it is found that superluminal or negative group velocity could be obtained without significant distortion in pulse shape.

Japha et al. [21] suggested a universal mechanism responsible for transmission attenuation, superluminal time delays and wave packet narrowing of coherent pulses in non-dissipative media. This mechanism may be employed for the manipulation of various properties of coherent pulses. Also, Romero-Rochin et al. [22] gave the idea of superluminal transmission of light pulse through optically opaque barriers from the consideration of causal electrodynamics.

It is known that from the general theorems given by Bolda et al. [23] (based on the Kramers-Kronig relation), superluminal and infinite or negative group velocities must exist for any dispersive medium at some frequencies. Also at these frequencies, attenuation (or gain) of electromagnetic waves is maximum.

In 1968, Veselago [24] has considered the idea of negative electric permittivity and negative magnetic permeability simultaneously at a certain frequency, thereby introducing the concept of negative index of refraction. An exhaustive review article on the physics of negative refractive index materials by Ramakrishna [25] discusses various aspects of this emerging area of research.

It has been demonstrated by Pendry et al. [26] that artificial plasmas composed of arrays of thin conducting wires can have negative effective permittivity.

On the other hand, Ojha et al. [27] showed the anomalous behaviour of group velocity in PCs. It becomes negative in a certain range of frequencies and approaches to zero at band edges. Because of this anomalous behaviour, effective group index also becomes negative and approaches to infinity at band edges. Ojha et al. [28, 29] also studied superluminal propagation and abnormal behaviour of group velocity in dielectric-plasma and dielectric-negative index material photonic crystals.

Hojo and Mase [30] studied plasma photonic crystal and showed that photonic band gap(s) increase as we increase the width of plasma layer as well as the plasma density. It is also confirmed by Ojha et al. [28].

In this paper, the electromagnetic properties of one dimensional dielectric-plasma photonic crystals (PPCs) namely, band structure, reflection properties, group velocity and effective group index in the microwave region of electromagnetic wave have been presented. Here, two structures PPC1 and PPC2 are considered by choosing SiO_2 and TiO_2 as the materials of dielectric layers of PPC1 and PPC2 respectively.

2. THEORY

We consider one-dimensional plasma photonic crystals (PPCs) having alternate layers of dielectric and micro-plasma as shown in Fig. 1.

The dispersion relation, reflectivity, group velocity and effective group index of such one-dimensional PPCs are computed using transfer matrix method [9, 31]. The Maxwell wave equations for electromagnetic wave propagating along the x-axis in one-dimensional PPCs may be written as

$$\frac{d^2E(x)}{dx^2} + k_0^2\varepsilon(x)E(x) = 0, \tag{1}$$

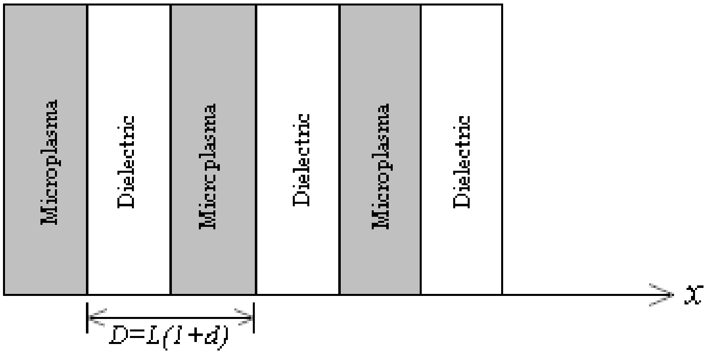

Figure 1. Periodic variation of plasma and dielectric showing 1-D plasma photonic crystals.

with

$$\varepsilon(x) = \begin{cases} 1 - \frac{\omega_p^2}{\omega^2}, & -Ld < x < 0, \\ \varepsilon_m, & 0 < x < L, \end{cases} \tag{2}$$

and

$$\varepsilon(x) = \varepsilon(x + D), \tag{3}$$

where,

$k_0 = \omega/c$ is the wave frequency;

c is the speed of light;

$\omega_p = (e^2 n_p/\varepsilon_0 m)^{1/2}$ is the electron plasma frequency with density n_p;

ε_m is the dielectric constant of the dielectric material.

The schematic diagram of the spatial variation of micro-plasma and dielectric material is given in Fig. 1, where $D = L(1 + d)$ is the lattice period with the widths of dielectric and micro-plasma being L and Ld respectively.

The general solution of Equation (1) is given by

$$E(x) = \begin{cases} a_m e^{ik_d x} + b_m e^{-ik_d x}, & -Ld < x < 0, \\ c_m e^{ik_p x} + d_m e^{-ik_p x}, & 0 < x < L, \end{cases} \tag{4}$$

where, $k_d = \omega/c(e_m)^{1/2}$ and $k_p = \omega/c(\omega_p^2/\omega^2 - 1)^{1/2}$, a_m, b_m, c_m, d_m are constants.

By imposing the continuity of $E(x)$ and $E'(x)$ at the interfaces of different materials of the structures, we obtained the following matrix equation

$$\begin{bmatrix} a_{m-1} \\ b_{m-1} \end{bmatrix} = M \begin{bmatrix} a_m \\ b_m \end{bmatrix} \tag{5}$$

Here, the translation matrix is given by

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{6}$$

where the elements of the matrix in Equation (6) are

$$m_{11} = e^{ik_dL}\left[\cosh(k_pLd) - \frac{i}{2}\left(\frac{k_p}{k_d} - \frac{k_d}{k_p}\right)\sinh(k_pLd)\right], \tag{7a}$$

$$m_{12} = e^{-ik_dL}\left[-\frac{i}{2}\left(\frac{k_p}{k_d} + \frac{k_d}{k_p}\right)\sinh(k_pLd)\right], \tag{7b}$$

$$m_{21} = e^{ik_dL}\left[\frac{i}{2}\left(\frac{k_p}{k_d} + \frac{k_d}{k_p}\right)\sinh(k_pLd)\right], \tag{7c}$$

$$m_{22} = e^{-ik_dL}\left[\cosh(k_pLd) + \frac{i}{2}\left(\frac{k_p}{k_d} - \frac{k_d}{k_p}\right)\sinh(k_pLd)\right]. \tag{7d}$$

Now, because of the periodicity in the structure of the PPCs, the electric field vector can be expressed in the form $E(x) = E_K(x)e^{-iKx}$, (according to Bloch's theorem) where $E_K(x)$ is periodic with the lattice period of D. The constant K is known as wave number and given by

$$K(\omega) = \frac{1}{D}\cos^{-1}\left[\tfrac{1}{2}\left(M_{11} + M_{22}\right)\right] \tag{8}$$

The solution of Equation (8) leads to the dispersion relation for the PPC structures containing the alternate layers of dielectric, and microplasma is given by

$$K(\omega) = \frac{1}{D}\cos^{-1}\left[\cos(k_dL)\cdot\cosh(k_pLd) + \frac{1}{2}\left(\frac{k_p}{k_d} - \frac{k_d}{k_p}\right)\sin(k_dL)\cdot\sinh(k_pLd)\right] \tag{9}$$

The group velocity $[V_g(\omega)]$ can be calculated by using formula [32]

$$V_g = \left(\frac{dK}{d\omega}\right)^{-1} \tag{10}$$

And the effective group index of refraction $n_{eff}(g)$ can be calculated by using the following formula

$$n_{eff}(g) = \frac{c}{V_g} \tag{11}$$

Also, the coefficient of reflectivity of the PPC structure with N unit cells can be calculated by using the following relation [31]

$$R_N = \left(\frac{b_0}{a_0}\right)_{b_N=0} \tag{12}$$

where, a_0 and b_0 are the complex amplitudes of incident and reflected plane waves, and the condition $b_N = 0$ implies the boundary condition that to the right of periodic structure there is electromagnetic plane wave incident on the structure considered.

By solving the above equation up to the Nth power of a unimodular matrix, we find the expressions for the reflectivity of the PPC structure, which is given by

$$|R_N|^2 = \frac{|M_{21}|^2}{|M_{21}|^2 + (|\sin(KD)/\sin(NKD)|)^2} \tag{13}$$

3. RESULT AND DISCUSSION

In this section, we compute the band structure, group velocity (V_g), effective group index [$n_{eff}(g)$] and reflectivity on the scale of normalized frequency ($\omega L/2\pi c$) of one-dimensional PPCs.

In this study, we consider two PPC structures with alternate layers of dielectric and micro-plasma as

(i) PPC1: SiO_2-Plasma
(ii) PPC2: TiO_2-Plasma

For both the PPCs, the thickness of each plasma layer is taken as Ld with plasma frequency 5.6×10^{11} Hz. The thickness of dielectric layer is taken as L with refractive index 1.5 for SiO_2 (in PPC1) and 2.3 for TiO_2 (in PPC2). Two cases of different thickness ratios ($Ld/L = d$) for both structures are considered for which the ratios are 0.01 and 0.10 for the PPC1 and PPC2 respectively.

The band structure of one-dimensional PPCs, in terms of normalized frequency and wave vector obtained from dispersion relation given by Equation (9), is shown in Fig. 2 and Fig. 3 respectively.

For PPC1, if we take the thickness ratio d to be equal to 0.01, we get only one photonic band gap that lies in the normalized frequency range from 1.322 to 1.358 centered at 1.340 with a forbidden band width of 0.036. On the other hand, for $d = 0.10$ (10 times the width of plasma layer of the previous case), the forbidden band gap lies in the normalized frequency range 1.240 to 1.482 centered at 1.360 with a band width of 0.242. It is clear that if we increase the thickness of plasma layer by 10 times, the forbidden band width increases by 7 times approximately. Also, the mid point of the band gap shifts slightly towards the higher values of the normalized frequency when the thickness of the plasma layers is increased.

For PPC2, if we take the thickness ratio (d) to be equal to 0.01, we obtain two band gaps in the same normalized frequency range as

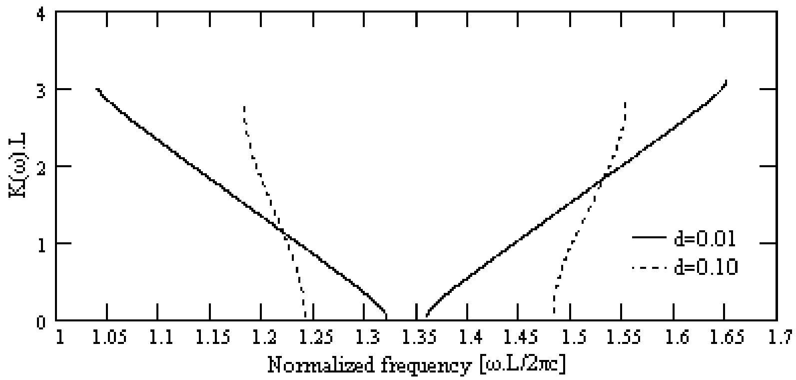

Figure 2. Variation of normalized wave vector $[K(\omega) \cdot L]$ versus normalized frequency showing the dispersion relation for PPC1.

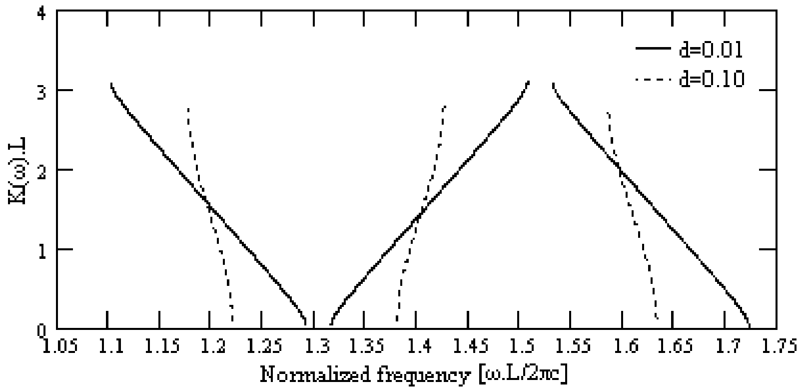

Figure 3. Variation of normalized wave vector $[K(\omega) \cdot L]$ versus normalized frequency showing the dispersion relation for PPC2.

for PPC1. The first band gap lies in the normalized frequency range from 1.291 to 1.316 centered at 1.303 with a band width of 0.025. The second band gap lies between 1.507 and 1.531 of normalized frequency centered at 1.520 with a band width of 0.024. For the both band gaps the band widths are almost equal. However, for $d = 0.10$, the first band gap in PPC2 lies in the normalized frequency range from 1.209 to 1.371 centered at 1.290 with a band width of 0.162, and the second band gap lies from 1.415 to 1.578 centered at 1.497 with a band-width of 0.163. Thus, if we increase the width of plasma layers by 10 folds, the widths of the forbidden band gaps increase by 6.5 times. But unlike the case of PPC1, the mid points of the band-gaps shift towards the lower end of the normalized frequency. It is clear from Fig. 2 and Fig. 3 that if we increase the thickness of the plasma layers, the width of photonic band gap also increases same as what Hojo and Mase [30] and also Ojha et al. [28] reported earlier. Comparing the two structures of one-dimensional plasma photonic crystals PPC1 and PPC2, the widths of allowed and forbidden bands of PPC with low refractive index dielectric material are more than the corresponding bands of PPC with high refractive index dielectric material.

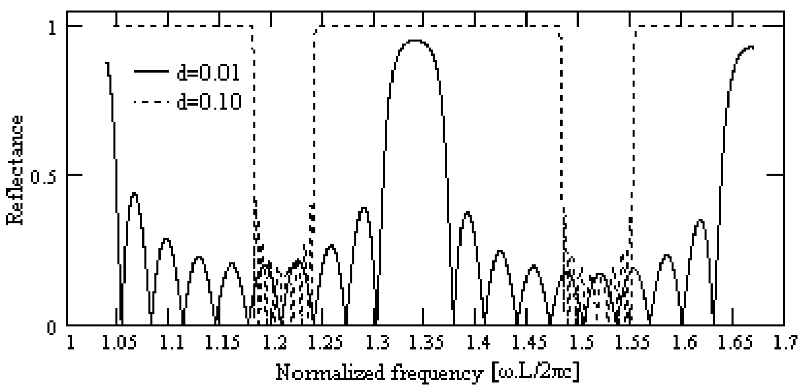

Figure 4. Variation of reflectance versus normalized frequency for PPC1.

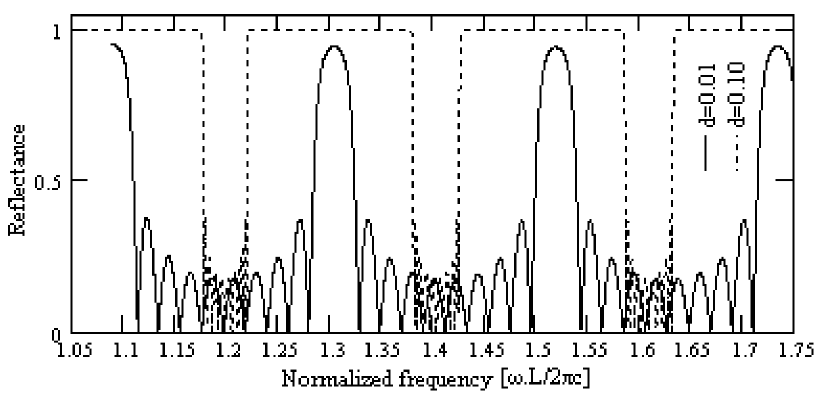

Figure 5. Variation of reflectance versus normalized frequency for PPC2.

Figures 4 and 5 show the variation of the reflectivity (obtained from Equation (13)) as a function of normalized frequency parameter for the two structures discussed here, namely, PPC1 and PPC2 respectively. It is quite clear from the study of the graphs that for $d = 0.01$, the reflectivity of both PPCs is not very large in the entire range of the normalized frequency, but it is appreciable for the regions where there is a forbidden band. In PPC1, the region with substantial reflectivity occurs in very narrow range, i.e., 1.322 to 1.358 of the normalized frequency parameter centered at 1.340, and in PPC2, it occurs in two regions, i.e., from 1.291 to 1.316 centered at 1.303 and 1.507 to 1.531 centered at 1.520.

If we increase the width of the plasma layers by 10 times, we get 100% reflectivity for both PPCs over certain ranges of normalized frequency parameter. For PPC1, the 100% reflectivity occurs from 1.240 to 1.482 of the normalized frequency parameter centered at 1.360, and for PPC2 the 100% reflectivity occurs in regions from 1.209 to 1.371 and from 1.415 to 1.578 of the normalized frequency parameter centered at 1.290 and 1.497 respectively.

Thus, in the case of PPC1, as the width of plasma layers is increased, the centre of high reflectivity ranges shifts towards the higher normalized frequency side, and for PPC2, the centre of high reflectivity ranges shifts towards the lower normalized frequency side as one increases the thickness of the plasma layers. Hence, by adjusting the thickness of plasma layers and choosing proper material for dielectric layers, we can shift the centre of high reflectivity towards lower or higher frequency range.

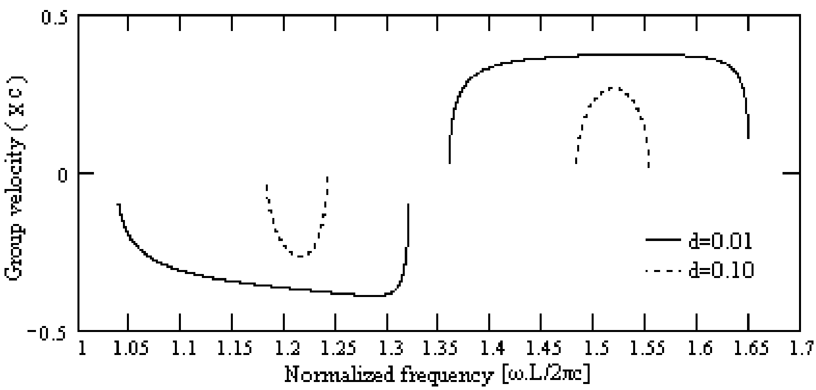

Figure 6. Variation of group velocity versus normalized frequency for PPC1.

Plots of Equation (10) with respect to normalized frequency give an idea how the effective group velocity (V_g) in the two structures

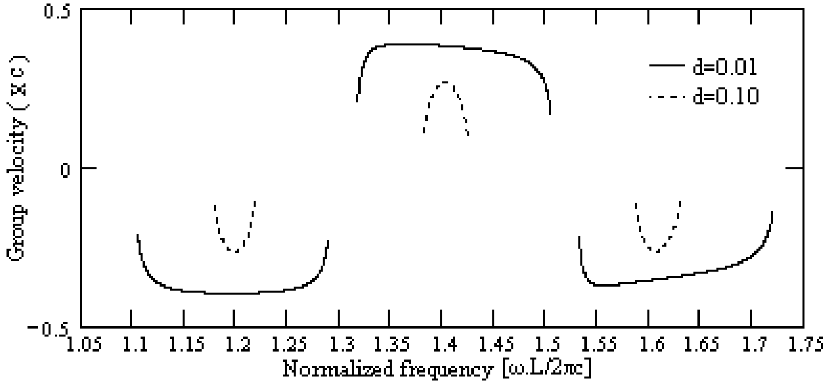

Figure 7. Variation of group velocity versus normalized frequency for PPC2.

changes as we change various parameters of the PPC structures. The resulting graphs shown in Fig. 6 and Fig. 7 corresponding to PPC1 and PPC2 respectively show that the group velocity in each of the PPC structures becomes negative in certain range(s) of normalized frequency, and it tends to zero at the band edges. Thus, negative group velocity ($V_g < 0$) is an anomalous behaviour of PPCs.

For PPC1, if we take $d = 0.01$, the group velocity (V_g) is negative only in the first allowed band, i.e., 1.038 to 1.322 (the width of the range of frequency being 0.292) of the normalized frequency. But the group velocity (V_g) is positive corresponding to second allowed band, i.e., 1.358 to 1.650 (width being 0.292) of the normalized frequency. If we take $d = 0.10$, the range in first allowed band over which the group velocity (V_g) is negative reduces to a great extent, i.e., from 1.180 to 1.240 (width being 0.060), and the group velocity (V_g) is found to be positive in the second allowed band i.e. 1.482 to 1.552 (width being 0.070).

If we take similar value of thickness of the plasma layers in the case of PPC2, the group velocity is found to be negative corresponding to first and third allowed bands and positive corresponding to second allowed band. If we choose $d = 0.01$, the group velocity has negative value corresponding to first allowed band from 1.100 to 1.291 of the normalized frequency (width being 0.191) and also corresponding to third allowed band from 1.531 to 1.722 (width being 0.191) of the normalized frequency. However, the group velocity is positive corresponding to second allowed band from 1.316 to 1.507 of the normalized frequency. If we choose $d = 0.10$ group velocity has negative values corresponding to first allowed band from 1.176 to 1.209 (with total width of 0.043) and the third allowed band 1.578 to 1.630 (with total width of 0.045). However, the group velocity has positive values corresponding to second allowed band from 1.371 to 1.415 (with total width of 0.045). Thus, for PPC1, we can obtain one region of negative group velocity and one region of positive group velocity, but for PPC2, we can obtain two regions of negative group velocity and one region of positive group velocity for the same range of normalized frequency.

For both the PPC structures with different d values, the group velocity tends to zero at the band edges. The magnitude of group velocity for a particular value of normalized frequency is larger for $d = 0.01$ than that of $d = 0.10$ in each of the structures.

Thus, if we decrease the thickness of plasma layers of PPCs, the magnitude of group velocity increases. Thus, the plasma width is the controlling parameter of group velocity. The negative group velocity ($V_g < 0$) of electromagnetic waves in PPCs is an abnormal behaviour, and it is responsible for superluminal behaviour in PPCs. From Fig. 6, it is clear that there are left-right and up-down symmetries, and it supports two different states. These states of symmetry may be used to make devices involving flip-flops like logic gates, optical switches (for optical computing) etc. PPC1 has two different states of flip-flop for a large normalized frequency range (1.0–1.7), whereas PPC2 works as a flip-flop for a short normalized frequency range (1.05–1.51).

The effective group index $n_{eff}(g)$ is computed from Equation (11). It is found that the effective group index becomes negative within PPCs

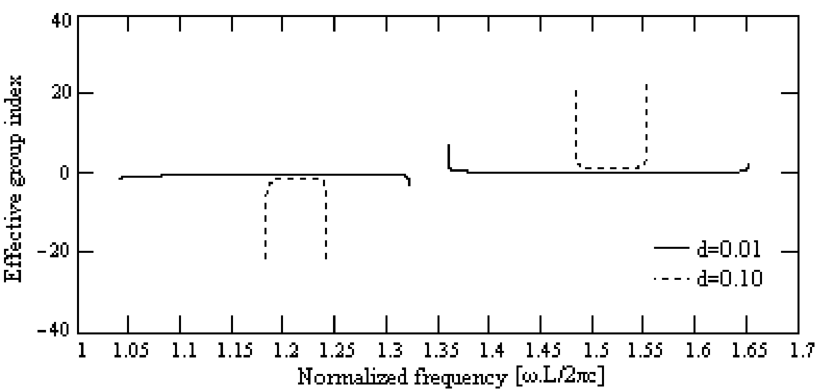

Figure 8. Variation of effective group index of refraction versus normalized frequency for PPC1.

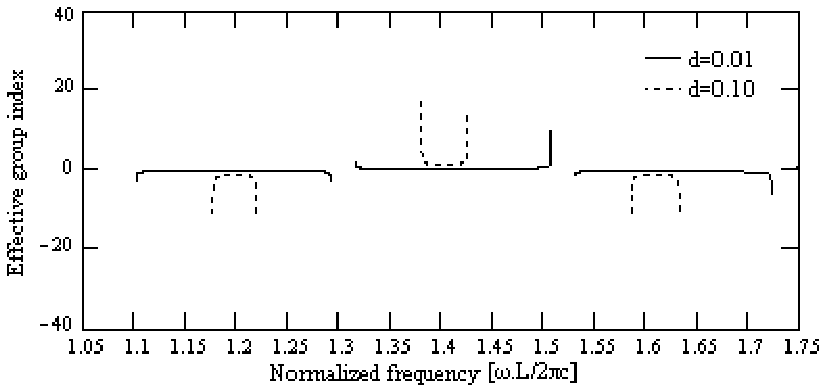

Figure 9. Variation of effective group index of refraction versus normalized frequency for PPC2.

in certain regions of the normalized frequency parameter because of the abnormal behaviour of group velocity. For both PPCs, if we decrease the thickness of plasma layers, there is an increase in both the ultra low and ultra high values of effective group index.

For PPC1, the ultra low and ultra high values of $n_{eff}(g)$ are large. But for PPC2, these values are smaller than the corresponding values of PPC1. The group velocity as well as $n_{eff}(g)$ becomes negative corresponding to the allowed bands for which the value of normalized wave vector $[K(\omega) \cdot L]$ decreases as the normalized frequency increases. On the other hand, the group velocity as well as $n_{eff}(g)$ becomes positive corresponding to the allowed band for which the value wave vector $K(\omega)$ increases as normalized frequency increases.

4. CONCLUSION

This study shows that the thickness of plasma layers in PPCs controls the forbidden band gaps. So we can use a PPC as a broad band reflector and frequency selector by choosing appropriate values of plasma thickness and suitable material for dielectric layers. For PPC1, the mid point of a band gap shifts towards the higher normalized frequency range, whereas for PPC2, the mid point of a band gap shifts towards the lower normalized frequency range by increasing the thickness of the plasma layers. For certain normalized frequency range, the group velocity becomes negative. Because of this abnormal behaviour ($V_g < 0$) superluminal flow of photon occurs in the PPCs. Such structures may be considered as a flip flop as there is positive and negative symmetry of effective group velocity. This symmetry is exhibited by PPC1 for a wide normalized frequency range, and hence such a structure may be considered for the design of a flip flop working over a wide range of frequency. On the other hand, the structure PPC2 may be considered for the design of a flip flop working over narrower range of normalized frequency. The structure PPC2 has three distinct states with two critical values of normalized frequency where change of states takes place; it may be used in the design of double layer memory devices in optical circuits. It also exhibits superluminal propagation both for the lower and higher values of normalized frequency. Thus, out of the entire range of the normalized frequency, PPC2 has wider range of normalized frequency where there is superluminal propagation inside the structure as compared to PPC1 structure and the structure considered by Ojha et al. [28] earlier.

ACKNOWLEDGMENT

One of the authors, Vipin Kumar, wishes to express sincere gratitude to Dr. K. B. Thapa and Dr. S. K. Singh for their valuable suggestions and critical comments.

REFERENCES

1. Yablonovitch, E., "Inhibited spontaneous emission in solid-state physics and electronics," *Phys. Rev. Lett.*, Vol. 58, 2059–2062, 1987.
2. Pendry, J. B., "Photonic band structures," *J. Mod Opt.*, Vol. 41, 209, 1994.
3. Joannopoulos, J. D., R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light*, Princeton Univ. Press, NJ, 1995.
4. Jonopoulos, J. D., P. Villeneuve, and S. Fan, "Photonic crystals: Putting a new twist on light," *Nature*, Vol. 386, 143–149, 1997.
5. Brooks, D. and S. Ruschin, "Integrated electrooptic multielectrode tunable filter," *J. Lightwave Technol.*, Vol. 13, 1508–1513, 1995.
6. John, S., "Strong localization of photon in certain disordered dielectric superlattice," *Phys. Rev. Lett.*, Vol. 58, 2486–2489, 1987.
7. Russell, P. S. J., S. Tredwell, and P. J. Roberts, "Full photonic bandgapes and spontaneous emission control in 1D multilayer dielectric structures," *Opt. Commun.*, Vol. 160, 66–71, 1999.
8. Chigrin, D. N., A. V. Lavrinenko, D. A. Yarotsky, and S. V. Gaponenko, "Observation of total omnidirectional reflection

from a one-dimensional dielectric lattice," *Appl. Phys. A: Mater. Sci. Process.*, Vol. 68, 25–28, 1999.

9. Shadrivov, I. V., A. A. Sukhorukov, and Y. S. Kivshar, "Complete band gaps in one dimensional left handed periodic structure," *Phys. Rev. Lett.*, Vol. 95, 193903-4, 2005.
10. Wu, R. X., P. Chen, F. Yang, and T. E. Zhao, "Wave polarization and left-handed materials in metallic magnetic thin films," *PIERS Online*, Vol. 1, No. 4, 459–463, 2005.
11. Shu, W. and J. M. Song, "Complete mode spectrum of a grounded dielectric slab with double negative metamaterials," *Progress In Electromagnetics Research*, PIER 65, 103–123, 2006.
12. Singh, S. K., J. P. Pandey, K. B. Thapa, and S. P. Ojha, "Structural parameters in the formation of omnidirectional high reflectors," *Progress In Electromagnetics Research*, PIER 70, 53–78, 2007.
13. Srivastava, S. K. and S. P. Ojha, "Omnidirectional reflection bands in one-dimensional photonic crystals with left-handed materials," *Progress In Electromagnetics Research*, PIER 68, 91–111, 2007.
14. Srivastava, S. K. and S. P. Ojha, "Omnidirectional reflection bands in one-dimensional photonic crystal structure using fluorescence films," *Progress In Electromagnetics Research*, PIER 74, 181–194, 2007.
15. Srivastava, R., S. Pati, and S. P. Ojha, "Enhancement of omnidirectional reflection in photonic crystal heterostructures," *Progress In Electromagnetics Research B*, Vol. 1, 197–208, 2008.
16. Banerjee, A., S. K. Awasthi, U. Malaviya, and S. P. Ojha, "Design of a nano-layered tunable optical filter," *J. of Modern Optics*, Vol. 53, 1739–1752, 2006.
17. Aissaoui, M., J. Zaghdoudi, M. Kanzari, and B. Rezig,"Optical properties of the quasi-periodic one-dimensional generalized multilayer fibonacci structures," *Progress In Electromagnetics Research*, PIER 59, 69–83, 2006.
18. Chen, K. M., A. W. Sparks, H.-C. Luan, D. R. Lim, K. Wada, and L. C. Kimerling, "SiO_2/TiO_2 omnidirectional reflector and microcavity resonator via the sol-gel method," *Appl. Phys. Lett.*, Vol. 75, 3805–3807, 1999.
19. Brillouin, L., *Wave Propagation and Group Velocity*, Academic, New York, 1960.
20. Garrett, C. G. B. and D. E. McCumber, "Propagation of Gaussian pulse through an Anomalous Dispersive Medium," *Phys. Rev. A*,

Vol. 1, 305–313, 1970.

21. Japha, Y. and G. Kurizki, "Superluminal delays of coherent pulses in nondessipative media: A universal mechanism," *Phys. Rev. A*, Vol. 53, 586–590, 1996.
22. Romero-Rochin, V., R. P. Duarte-Zamorano, S. Nilsen-Hofseth, and R. G. Barrera, "Superluminal transmission of lights through optical opaque barriers," *Phys. Rev. E*, Vol. 63, 027601–027604, 2001.
23. Bolda, E. L. and R. Y. Chiao, "Two theorems for the group velocity in dispersive media," *Phys. Rev. A*, Vol. 48, 3890–3894, 1993.
24. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics Uspekh1*, Vol. 10, 509–514, 1968.
25. Ramakrishna, S. A., "Physics of negative refractive index materials," *Rep. Prog. Phys.*, Vol. 68, 449–521, 2005.
26. Pendry, J. B., A. J. Holden, W. J. Stewart, and I. Youngs, "Extremely low frequency plasmons in metallic mesostructures," *Phys. Rev. Lett.*, Vol. 76, 4773–4776, 1996.
27. Ojha, S. P. and S. K. Srivastava, "Group velocity, negative and ultra-high index of refraction in photonic band gap materials," *Microwave Opt. Technol. Lett.*, Vol. 42, 82–86, 2004.
28. Ojha, S. P., K. B. Thapa, and S. K. Singh, "Superluminal propagation in plasma Photonic band gap materials," *Optik — International Journal for Light and Electron Optics*, Vol. 119, No. 2, 81–85, 2008.
29. Pandey, G. N., K. B. Thapa, S. K. Srivastava, and S. P. Ojha, "Band structures and abnormal behaviour of one dimensional photonic crystal containing negative index materials," *Progress In Electromagnetics Research M*, Vol. 2, 15–36, 2008.
30. Hojo, H. and A. Mase, "Dispersion relation of electromagnetic waves in one dimensional plasma photonic crystals," *J. Plasma Fusion Res.*, Vol. 80, 89–92, 2004.
31. Yeh, P., *Optical Waves in Layered Media*, Wiley, New York, 1988.
32. Sakoda, K., *Optical Properties of Photonic Crystals*, Springer, Germany, 2001.