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ANALYSIS ON THE STEALTH CHARACTERISTIC OF TWO DIMENSIONAL CYLINDER PLASMA ENVELOPES

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Abstract—Stealth characteristic of two dimensional cylinder plasma envelopes is studied. Three cases about plasma refraction effect, reflection characteristic and attenuation by absorbing electromagnetic wave (EMW) are concerned synthetically. As for plasma refraction stealth, EMW traces equation in cylinder plasma is deduced; a novel concept of plasma refraction deviation angle is presented; the relation between refraction deviation angle and incidence angle of EMW is yielded; the relation between refraction deviation angle and plasma density distribution is made out. As for reflection stealth and attenuation stealth, reflection calculation of multi-layer plasma is presented first, and plasma collision frequency as well as corresponding collision absorption is taken into account simultaneously, then EMW reflectivity with double-path attenuation is obtained. It is shown that cylinder plasma envelopes considering the three cases above could make distinct stealth.

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1. INTRODUCTION

Since 1990s, academic institutions and researchers, overseas and domestic, have been studying plasma stealth technology [1–15]. The concept of plasma stealth was proposed first by Vidmar [1], who theoretically studied the reflection, transmission and absorption characteristic of EMW propagation in un-magnetized plasma. The absorption and attenuation characteristics of EMW propagation in un-magnetized plasma were observed experimentally by Hughes Research Laboratories [2]. Laroussi [3] and Hu [4] carried out the studies of the reflection, transmission and absorption characteristics of EMW propagation in magnetized plasma, by using numerical method and scattering matrix method respectively. Pertrin [5–7] studied the transmission of microwave through un-magnetized plasma layer and magnetoactive plasma layer. Liu et al. analyzed the stealth mechanism of sphere plasma [8, 9]. From above references, it can be seen that, when EMW propagated in plasma, the plasma stealth effect caused by plasma refraction, reflection and absorption had not been considered simultaneously in the same document.

In this paper, the stealth characteristic of two dimensional cylinder plasma envelopes is studied. The subject investigated is a model of a cylinder perfect conductor, whose radius is r_0, covered with concentric cylinder plasma envelopes, as shown in Figure 1. When plane EMW propagates in plasma envelopes, two cased are concerned. One case is that the rays among parallel EMW rays with longer distance to the circle center, supposed $r_d > r_0$, have bigger incidence angle, which will be refracted by plasma before they arrive at conductor cylinder. The other case is that the rays among parallel EMW rays with

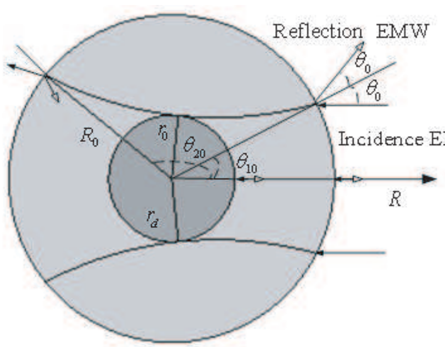


Figure 1. Horizontal section sketch of EMW incidence in cylinder plasma envelopes.

shorter distance to the circle center, supposed $r_d < r_0$, have smaller incidence angle, which may be incidence on the conductor. However, EMW energy will be partially attenuated because of absorption in the processing of propagating in plasma. Thus, efficiency reflection energy only takes a little part of the whole EMW energy. As for the first case, refraction is dominant, and it may be taken as refraction stealth, then EMW tracks equation in cylinder plasma is deduced, and refraction deviation angle is presented. Sequentially, the relation between refraction deviation angle and incidence angle is obtained that the bigger incidence angle is, the bigger refraction deviation angle and the better stealth effect are. In addition, the relation between refraction deviation angle and plasma density distribution is obtained as well, which is that the stronger plasma numerical density is (i.e., the bigger m value is), the smaller refraction deviation angle is. As for the second case, reflection is dominant, when double-path attenuation is concerned, and it can be considered as reflection stealth and absorption stealth. Afterwards, the calculation formula about reflection coefficient of EMW incidence in multi-layer plasma is discussed. Furthermore, the reflectivity with double-path attenuation is worked out as plasma collision is concerned. Moreover, the principle is that the stronger plasma numerical density is, the less reflectivity is. As the cases above, all considered, the stealth function of plasma cylinder will be reached.

In Figure 1, r_0 is the radius of conductor cylinder; R_0 is the radius of plasma cylinder; r_d is the distance between EMW rays to the circle center. Solid arrow denotes incidence EMW; hollow arrow denotes reflection EMW; θ_{10} is EMW incidence angle, θ_0; θ_{20} is EMW emergence angle.

2. THEORY ANALYSIS AND RESULT DISCUSSION

2.1. The Dispersion of EMW in Un-magnetized Collision Plasma

As EMW interacts with collision plasma, the equivalent permittivity is a complex [1–4, 15].

$$\varepsilon_{pr} = 1 - \frac{\omega_p^2}{\omega(\omega - jv_c)} \tag{1}$$

where ω_p is plasma angle frequency; ω is incidence EMW angle frequency; v_c is plasma effective collision frequency.

Consequently, the propagation constant is also a complex [15].

$$k = k_0\sqrt{\varepsilon_{pr}} = k_r + i \cdot k_i \tag{2}$$

where $k_0 = \omega/c$ is the free-space wave number, and k_r and k_i are the real and imaginary parts of k respectively, corresponding to the phase shift constant and attenuation constant. Combining Equations (1) and (2), there yields

$$k_r = k_0 p \cos(\theta/2) \tag{3}$$

$$k_i = k_0 p \sin(\theta/2) \tag{4}$$

where p and θ are defined as

$$p = \left[1 - \frac{\omega_p^2}{\omega^2 + v_c^2}\left(2 - \frac{\omega_p^2}{\omega^2}\right)\right]^{1/4} \tag{5}$$

$$\theta = \begin{cases} \theta_c = \tan^{-1}\left[-\frac{v_c \omega_p^2}{\omega(\omega^2 + v_c^2 - \omega_p^2)}\right] : \ \mathrm{Re}(\varepsilon_r) > 0 \\ \theta_c + \pi : \ \mathrm{Re}(\varepsilon_r) < 0 \end{cases} \tag{6}$$

2.2. Non-uniform Un-magnetized Plasma Cylinder EMW Tracks and Refraction Deviation Angle

EMW tracks in non-uniform un-magnetized plasma depend on plasma refraction index. When plasma collision is omitted, its refraction index is expressed approximately as [8, 9, 11]:

$$n_p^2 = \varepsilon_{pr} = 1 - \frac{\omega_p^2}{\omega^2} \tag{7}$$

Supposed plasma density changes gradually along R direction, that is, plasma density is just a function of radius R, and R changes from r_0 to R_0 and the same bellow. As for the ideal case, plasma density is zero at the position of outer radius of plasma circle, that is $n(R_0) = 1$. The refraction index in plasma circle is defined as [8, 9]:

$$n(R) = (R)^m / R_0^m \tag{8}$$

According to Fermat principle and variation method and combining with formula (8), the equation in polar coordinates is

$$\frac{d}{d\theta}\frac{\partial}{\partial \dot{R}}\left(R^m\sqrt{R^2 + \dot{R}^2}\right) - \frac{\partial}{\partial R}\left(R^m\sqrt{R^2 + \dot{R}^2}\right) = 0 \tag{9}$$

where $\dot{R} = dR/d\theta$, let $\ddot{R} = d^2R/d\theta^2$, then combining formula (8) we get

$$R\ddot{R} - (m+2)\dot{R}^2 - (m+1)R^2 = 0 \tag{10}$$

Make tracks parameters of EMW (R, θ) and solve Equation (10), then four solutions are respectively

$$\theta_1 = -\text{arcsec}\left(\sqrt{C_1}R^{m+1}\right)/(m+1) + C_2 \tag{11a}$$

$$\theta_2 = \text{arcsec}\left(\sqrt{C_1}R^{m+1}\right)/(m+1) + C_2 \tag{11b}$$

$$\theta_3 = -\text{arcsec}\left(\sqrt{C_1}R^{m+1}\right)/(m+1) - C_2 \tag{11c}$$

$$\theta_4 = \text{arcsec}\left(\sqrt{C_1}R^{m+1}\right)/(m+1) - C_2 \tag{11d}$$

Choosing the coordinate of incidence EMW (R_0, θ_0), where θ_0 is EMW incidence angle, which is an angle between EMW incidence ray and plasma circle normal, integral constants are expressed as

$$C_1 = \frac{\cot^2\theta_0 + 1}{R_0^{2m+2}} \tag{12}$$

$$C_2 = \text{arcsec}\left(\sqrt{C_1}R^{m+1}\right)/(m+1) + \theta_0 \tag{13}$$

From formulae (11), we can obtain EMW tracks in cylinder plasma as shown in Figure 2.

Based on Snell's law and combined with Figure 1, we get

$$R_0^m \sin\theta_0 = R^m \sin\theta = r_d^m \sin(\pi/2) = r_d^m \tag{14}$$

Suppose $r_d = r_0$, and we will get the minimal EMW rays incidence angle

$$\theta_{\min} = \arcsin(r_0/R_0)^m \tag{15}$$

According to formula (15), we can obtain $\theta_{\min}$ in the case of r_0. R_0 and m are sure.

Due to the refraction of plasma cylinder envelopes, EMW rays in plasma will deviate greatly to original direction. Combing Figures 1, 2 and formulae (11), we define EMW refraction deviation angle, which is the difference between incidence angle and emergence angle that return to air, as

$$\theta_d = \theta_{20} - \theta_{10} = 2\text{arcsec}\left(\sqrt{C_1}R_0^{m+1}\right) \tag{16}$$

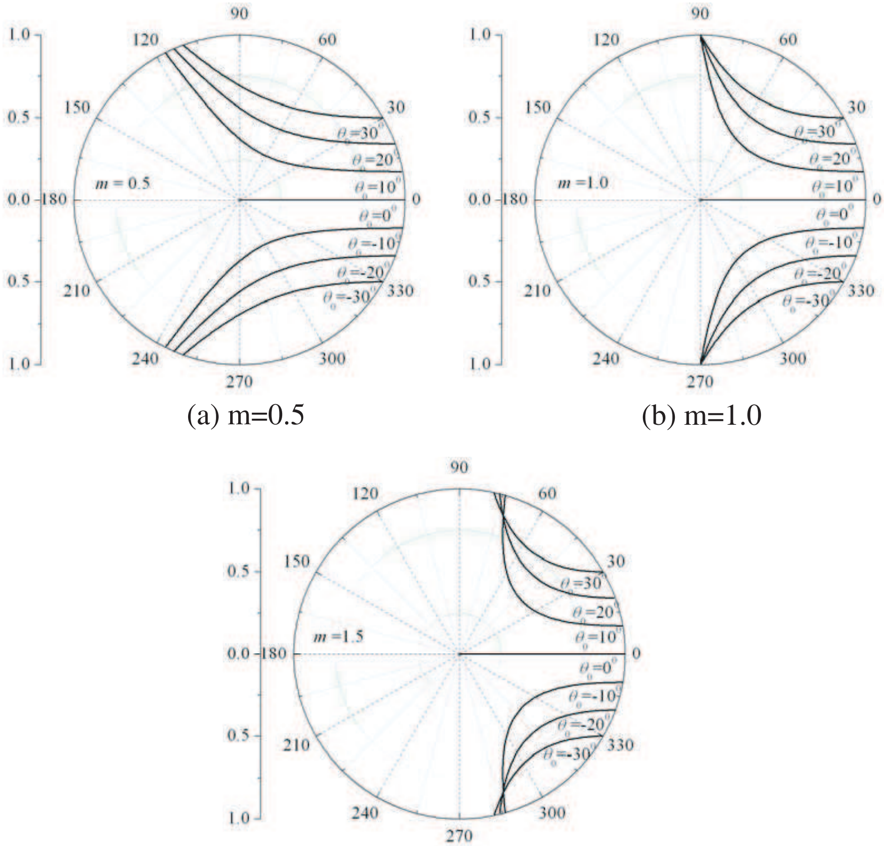
where θ_{10} is EMW incidence angle, θ_0, and θ_{20} is EMW emergence angle.

It is seen from Figure 1 that the angle between initial reflecting wave and incident wave is $2\theta_0$.

It is seen from Figure 2 that when EMW incidence angle θ_0 is $0°$, plasma refraction deviation angle is $0°$ too, and incidence EMW reflects completely along the original path. When EMW incidence angle θ_0 is 90°, plasma deviation angle is 180°, that is, EMW rays propagate straightly along the tangent of plasma cylinder.

Combining formula (16) and formula (14), we can obtain the relations between refraction deviation angle and incidence angle, and the relations between refraction angle and plasma numerical density are shown in Figures 3 and 4 respectively.

It is seen from Figures 3 and 4 that when EMW incidence angle is bigger than critical angle $\theta_{\min}$ but smaller than 90°, the bigger incidence angle is, the smaller refraction angle is, and the bigger m value is, the smaller refraction angle is. However, according to formula



(a) m=0.5 (b) m=1.0

(c) m=1.5

Figure 2. EMW tracks at different m values with different incidence angle.

(14), when incidence angle is constant, r_d will be longer as m value becomes larger. According to formula (15), when m value is constant, r_d will be longer as incidence angle gets larger. Therefore, as for refraction stealth, refraction angle affected by EMW incidence angle and plasma density distribution should be considered comprehensively.

2.3. EMW Reflectivity in Non-uniform Un-magnetized Plasma with Cylinder Envelopes

As mentioned above, plasma density changes gradually along R direction, and thus plasma refraction index function is as formula (8). Supposed non-uniform un-magnetized plasma envelopes are divided into n layers, and the refraction angle of EMW in i-th layer of plasma envelopes is θ_{ti}. When EMW reflects from the interface of air-to-plasma, the reflection coefficient is Γ_0, and the reflection coefficient about i-th to $(i+1)$-th plasma layers is Γ_i, and the coefficient of n-th plasma layer to inner conductor is Γ_n. Then, we get every kind of reflection coefficient formulae as follows [15]:

$$\Gamma_0 = 0 \tag{17a}$$

$$\Gamma_i = \frac{Z_{p,i+1}\cos\theta_{t,i+1} - Z_{p,i}\cos\theta_{t,i}}{Z_{p,i}\cos\theta_{t,i} + Z_{p,i+1}\cos\theta_{t,i+1}} \tag{17b}$$

$$\Gamma_n = -1 \tag{17c}$$

where $i = 1, 2, \ldots, n$; $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 377\Omega$ is wave impedance in vacuum; $Z_{pi} = \sqrt{\mu_0\mu_d/\varepsilon_{pi}\varepsilon_0}$ is the wave impedance in i-th layer plasma envelopes.

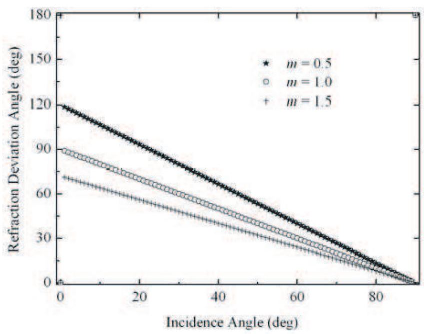


Figure 3. Relations between deviation angle and incidence angle.

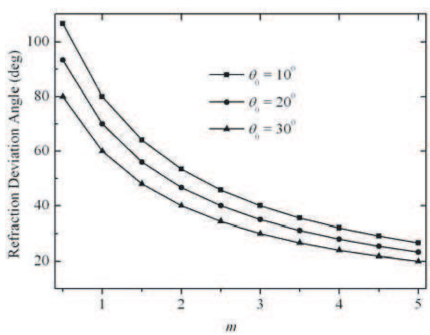


Figure 4. Deviation angle varying vs. that of m values.

By using Snell's law again, it yields

$$1 \cdot \sin\theta_0 = \sqrt{\varepsilon_{p1}} \cdot \sin\theta_{t1} = \sqrt{\varepsilon_{p,i}} \cdot \sin\theta_{t,i} = \sqrt{\varepsilon_{p,i+1}} \cdot \sin\theta_{t,i+1} \qquad (18)$$

Substituting formula (18) into formula (17) and arranging, we obtain the coefficient expressed by relative permittivity as

$$\begin{aligned}\Gamma_i &= \frac{\sqrt{\varepsilon_{p,i}}\cos\theta_{t,i+1} - \sqrt{\varepsilon_{p,i+1}}\cos\theta_{t,i}}{\sqrt{\varepsilon_{p,i+1}}\cos\theta_{t,i} + \sqrt{\varepsilon_{p,i}}\cos\theta_{t,i+1}} \\ &= \frac{\sqrt{\varepsilon_{p,i}}\sqrt{1-\sin^2\theta_0/\varepsilon_{p,i+1}} - \sqrt{\varepsilon_{p,i+1}}\sqrt{1-\sin^2\theta_0/\varepsilon_{p,i}}}{\sqrt{\varepsilon_{p,i+1}}\sqrt{1-\sin^2\theta_0/\varepsilon_{p,i}} + \varepsilon_{p,i}\sqrt{1-\sin^2\theta_0/\varepsilon_{p,i+1}}} \qquad (19)\end{aligned}$$

When plasma collision is considered, there exists attenuation in the processing of EMW propagating trough plasma envelopes. Due to double-path attenuation, the reflection coefficient in i-th layer plasma envelopes should be modified, and it yields

$$\left|\tilde{\Gamma}_i\right|^2 = |\Gamma_i|^2 \prod_{q=1}^{i} \left(1 - |\Gamma_{q-1}|^2\right) \exp\left(\frac{-4k_{i,q}l}{\sqrt{1-\frac{\sin^2\theta_0}{\varepsilon_{p,q}}}}\right) \qquad (20)$$

where, as $p = q = 1$, Γ_{q-1} is Γ_0; as $l = i = n$, $\Gamma_q = -1$; $k_{i,q}$ is q-th layer plasma attenuation constant.

Therefore, the total power reflection coefficient of EMW incidence in cylinder plasma envelopes and going out air is

$$|\Gamma_{tol}|^2 = \sum_{j=1}^{n}\left(|\Gamma_j|^2 \prod_{q=1}^{j} (1 - |\Gamma_{q-1}|^2) \exp\left(\frac{-4k_{i,q}l}{\sqrt{1-\frac{\sin^2\theta_0}{\varepsilon_{p,q}}}}\right)\right) \qquad (21)$$

In addition, the reflection loss is

$$R_L(\text{dB}) = 10\log(|\Gamma_{tol}|^2) \qquad (22)$$

Supposed incidence EMW frequencies vary from 2 GHz to 14 GHz. When $m = 0.5$, and incidence angle is 30°, EMW reflectivity curves in the case of changing plasma collision are obtained as shown in Figure 5. When plasma collision frequency is 5 GHz, as m values change, EMW reflectivity curves are plotted as Figure 6.

It is seen from Figure 5 that, as plasma collision frequency enlarges, EMW reflectivity will decrease when EMW frequency above 5 GHz. When plasma collision frequency is constant, as EMW frequency grows, EMW reflectivity will increase.

It is seen from Figure 6 that, as m value become bigger, EMW reflectivity will increase first, then it decreases. As for three m values used above, when $m = 1.0$, the EMW reflectivity is minimal. Therefore, as for absorption and attenuation stealth, plasma collision frequency should be enlarged. At the same time, m value should be chosen properly.

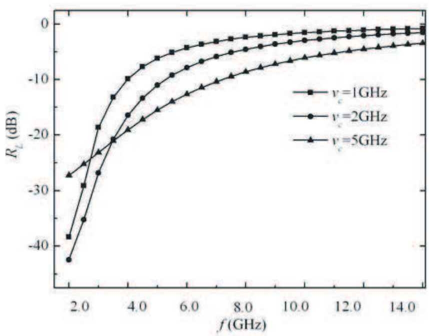


Figure 5. Reflectivity with different plasma collision frequency.

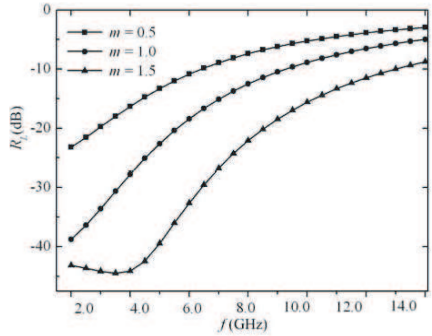


Figure 6. Reflectivity with different m values.

3. CONCLUSION

When EMW enters cylinder conductor covered with concentric cylinder plasma envelopes because of EMW refraction, reflection and absorption by plasma, in the case of EMW rays with larger incidence angle, EMW rays will deviate greatly from original direction, and the refraction angle is θ_d, so such EMW rays cannot reach inner conductor. EMW rays with smaller incidence angle may arrive at inner conductor after refraction and absorption by plasma envelopes. However, the back-wave deviates from incidence direction $2\theta_0$, with energy attenuates a lot, and the minimal reflectivity is below -40 dB. From above, it is concluded that cylinder plasma envelopes can be used to protect objects.

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