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SYNTHESIS OF MONOPULSE SUB-ARRAYED LINEAR AND PLANAR ARRAY ANTENNAS WITH OPTIMIZED SIDELOBES

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Abstract—In this paper, three approaches for the synthesis of the optimal compromise between sum and difference patterns for sub-arrayed linear and planar arrays are presented. The synthesis problem is formulated as the definition of the sub-array configuration and the corresponding sub-array weights to minimize the maximum level of the sidelobes of the compromise difference pattern. In the first approach, the definition of the unknowns is carried out simultaneously according to a global optimization schema. Differently, the other two approaches are based on a hybrid optimization procedures, exploiting the convexity of the problem with respect to the sub-array weights. In the numerical validation, representative results are shown to assess the effectiveness of the proposed approaches. Comparisons with previously published results are reported and discussed, as well.

1. INTRODUCTION

Monopulse tracking radars [1] are based on the simultaneous comparison of *sum* and *difference* signals to compute the angle-error and to steer the antenna patterns in the direction of the target (i.e., the boresight direction). Besides classical solutions where multi-feeder reflectors are considered, the two (sum and difference) or three (sum and double-difference) patterns, needed to determine the angular location of the target along a singular angular coordinate or both in azimuth and elevation, can be synthesized through linear or planar array antennas, respectively. Recent studies are mainly devoted to array solutions because of the larger number of degrees of freedom. As

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a matter of fact, such a solution allows one to control the illumination of the array directly on the aperture by modifying the excitations of the radiating elements. Moreover, the synthesized patterns are electronically steerable. This enables the fast change of the beam direction and it avoids the inertia problems due to the use of mechanical positioning systems. On the contrary, the drawbacks of the array implementation lay in the circuit complexity and the arising costs. Nevertheless, the elements of the aperture can be grouped into sub-arrays in order to simplify the antenna design and obtain cheaper tradeoff despite some reductions of the antenna performances [2, 3].

In antenna systems applied for real world applications [4], different strategies for implementing monopulse radars have been adopted. A well known technique considers the partition of the array aperture into two halves (linear array) o four quadrants (planar arrays). The outputs of the elements belonging to the same half/quadrant are combined and continuously compared with the output/s of the other half/quadrants to determine the error signal. Such a signal is used to steer the sum and difference beams and thus to track the moving target.

In such a framework, recent papers have dealt with the optimal compromise problem between sum and difference patterns, starting from an optimum sum pattern generated by a complete and dedicated feed network. The elements of the array are then grouped into sub-arrays with a proper weighting to obtain a "sub-optimal" difference pattern. Either the optimization of some specific pattern features (e.g., the directivity [5–7], the normalized difference slope [8], the sidelobe level (SLL) [9, 10]) or the fitting with an optimal pattern in the Dolph-Chebyshev sense [11, 12] have been considered. Among them, the SLL minimization of the compromise difference pattern has received particular attention. To deal with such a synthesis problem, different optimization strategies based on global optimization approaches [13, 14] as well as two-step hybrid techniques [9–11, 15] have been proposed. However, an effective and flexible procedure able to deal with both the synthesis of linear and planar structures has been previously proposed only in [9, 12, 16]. Such an event is mainly due to the exponential growth of the dimension of the solution space with the increase of the number of array elements.

The approach proposed in [12] and then extended in [16], named Contiguous Partition Method (CPM), takes advantage from the knowledge of the relationship between the independent distributions of the optimal sum and difference [17] coefficients to reduce the dimension of the solution space. Accordingly, the synthesis of large planar arrays is enabled and the converge of the synthesis procedure speeded up. Essentially based on an excitation matching procedure, the sub-array

configuration is first obtained by minimizing the distance between the reference/optimal and synthesized (sub-arrayed) difference coefficients. Accordingly, the sub-array gains are directly computed as a function of the optimal sum and difference excitations exploiting the guidelines of [20]. Nevertheless, the CPM procedure does not allow to control the level of the sidelobes. To overcome this drawback, preliminary results obtained by means of an iterative version of the CPM (the I-CPM) have been shown in [18, 19]. There, the optimal pattern to match is iteratively changed until the SLL of the compromise solution satisfied the user-defined constraints.

In this paper, three new approaches aimed at the minimization of the SLL of the compromise difference pattern are presented. In the first, the simultaneous optimization of the problem unknowns is dealt with likewise [12], but in this case the so-called *solution tree* (i.e., the representation of all the admissible sub-array configuration [12]) is explored looking the solution with minimum SLL. This strategy will be referred in the following as Modified CPM (M-CPM). The other two approaches consider the hybridization of the I-CPM (HI-CPM) and of the M-CPM (HM-CPM) with a Convex Programming (CP) procedure [10] to directly introduce SLL constraints in the optimization procedure.

The paper is organized as follows. In Section 2, the synthesis problem is mathematically formulated. The innovative CPM-based procedure aimed at the optimization of the SLL is pointed out in Section 3, where the one-step (Section 3.1) as well as the hybrid two-step (Section 3.2) are presented. A set of selected results concerning the synthesis of linear as well as planar arrays is reported in Section 4 to assess the effectiveness of the proposed methods. Comparison with previously published results are also reported where available. Finally, some conclusions are drawn (Section 5).

2. MATHEMATICAL FORMULATION

Let us consider either a linear or planar array with elements uniformly spaced in the xy-plane (Fig. 1). The array factor is

$$f(u,v) = \sum_{n=1}^{N} c_n e^{jk(ux_n+vy_n)} \tag{1}$$

where c_n, $n = 1, \ldots, N$, is the set of real excitations, $u = \sin\theta\cos\phi$ and $v = \sin\theta\sin\phi$, where the values (θ, ϕ), $\theta \in \left[0, \frac{\pi}{2}\right]$ and $\phi \in [0, 2\pi]$, indicate the angular direction, and $k = \frac{2\pi}{\lambda}$ is the wavenumber of the background medium. Moreover, (x_n, y_n) is the position of the n-th array element.

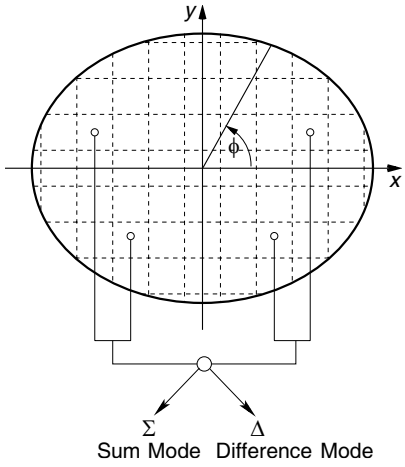

Figure 1. Planar array geometry.

To obtain sum and difference patterns, the distribution of the coefficients is supposed to be symmetric with respect to the physic center of the aperture. In particular and concerning the linear case, the two halves of the array are summed in phase and phase reversal, respectively. Differently, the aperture is supposed to be divided into four symmetric quadrants in the case of a planar array. Accordingly, the sum signal is obtained by adding in phase all the output of the four quadrants, while the difference modes, namely the *azimuth difference mode* (*H-mode*) and the *elevation difference mode* (*E-mode*), are given with pair of quadrants added in phase reversal.

The excitations of the "sub-optimal" difference pattern $c_n = d_n$, $n = 1, \ldots, N$, as obtained through the sub-arrayed feed network are

$$d_n = \begin{cases} \sum_{q=1}^{Q} s_n \delta_{a_n q} w_q & -\pi/2 < \phi \leq \pi/2 \\ \sum_{q=1}^{Q} (-1)\, s_n \delta_{a_n q} w_q & \pi/2 < \phi \leq 3\pi/2 \end{cases} \tag{2}$$

where $\underline{S} = \{s_n;\ n = 1, \ldots, N\}$ is a set of fixed excitations affording an optimal sum pattern [17], $\underline{W} = \{w_q;\ q = 1, \ldots, Q\}$ are the (unknown) sub-array weights, $\underline{A} = \{a_n;\ n = 1, \ldots, N\}$ is a integer vector where the element $a_n \in [0, Q]$ indicating the sub-array membership (when $a_n = 0$ it follows that $d_n = s_n$) and $\delta_{a_n q}$ is the Kronecker delta ($\delta_{a_n q} = 1$ if $a_n = q$ and $\delta_{a_n q} = 0$ otherwise). Since monopulse planar arrays require the generation of two spatially-orthogonal difference patterns [4], the coefficients of the first difference mode are given as in (2), while the second difference mode is obtained by adding the two pairs of quadrants shifted by $\pi/2$ in the ϕ-direction with respect to (2).

Hence, the problem at hand is formulated as follows : "*optimizing the sub-array configuration* $\underline{A}^{opt}$ *and the corresponding set of weights* $\underline{W}^{opt}$ *to obtain a compromise difference pattern with minimum sidelobe level for a given main lobes beamwidth.*"

3. SIDELOBE LEVEL OPTIMIZATION APPROACHES

In this section, three new approaches for the solution of the optimal compromise between sum and difference patterns are described, where the SLL optimization of the difference beams is dealt with. In particular, the simultaneous optimization of both the sub-array aggregation and the sub-array gains is firstly considered according to the $M\text{-}CPM$ (Section 3.1) and the main differences with respect to the $I\text{-}CPM$ [18] are pointed out. Then, their hybridized two-step versions, namely the $HI\text{-}CPM$ and the $HM\text{-}CPM$ are presented in Section 3.2, as well.

3.1. Simultaneous Definition of the Unknowns

As far as the simultaneous synthesis of the problem unknowns is concerned, the *Iterative Contiguous Partition Method* ($I\text{-}CPM$) has been successfully applied. Its procedure and some preliminary results have been already published in [18, 19], where linear and planar array synthesis problems have been dealt with, respectively. In particular, the $I\text{-}CPM$ is based on the following concept: by successively changing the reference/optimal target to approximate, at each step the CPM [12] is applied until the requirements on the SLL for the synthesized difference pattern are satisfied. It is worth to notice that in the $I\text{-}CPM$ [19], whose workflow is schematically outlined in Fig. 2, the optimization of the SLL is obtained as a by-product. As a matter of fact, the *bare* version of the CPM [12] concerns the definition of the "best compromise" difference pattern close as much as possible to the optimal one through an excitation matching procedure. Nevertheless, enforcing the CPM to iteratively approximate an optimal difference pattern with a reference SLL lower and lower, it allows to reduce the SLL of the synthesized pattern and therefore to satisfy user-defined constraints.

The strategy proposed in this work, namely the *Modified Contiguous Partition Method* ($M\text{-}CPM$), tries to to explore the *solution tree* [12], directly looking for the solution with minimum SLL, unlike the one guaranteeing the best least-square pattern matching. The solution with the lowest SLL is searched by means of the *border element method* (BEM) described in [12]. Towards this aim, the

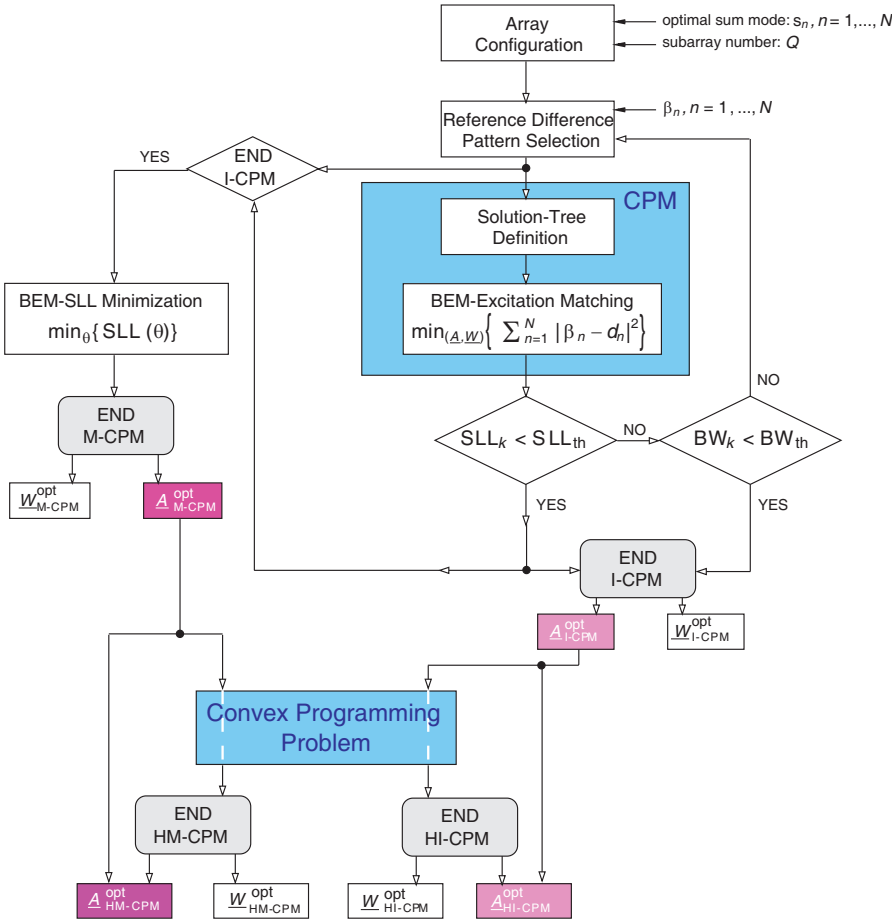

Figure 2. Pictorial representation of the CPM-based approaches.

following cost function is considered

$$\Psi^{M\text{-}CPM}\left(\underline{A},\underline{W}\right) = \min_{u,v}\left\{SLL\left(u,v\right)\right\} \tag{3}$$

for the linear and planar case, where $SLL\left(u,v\right)$ is the maximum level of the sidelobes outside the main lobe region. Let us we refer to this procedure as the.

It is worth noting that both the *I-CPM* and the *M-CPM* allow the simultaneous definition of all the problem unknowns in a reliable and efficient way since the are based on the *CPM*. As a matter of fact, whether on one hand the final sub-array aggregation is obtained through the *BEM*, which computational efficiency has been

pointed out in [2], on the other hand the definition of the sub-array weights does not increase the computational burden, since an analytical relationship [12] is considered:

$$w_q^{CPM} = \left[\frac{\sum_{n=1}^{N} \delta_{a_n q} (s_n \beta_n)}{\sum_{n=1}^{N} \delta_{a_n q} (s_n)^2} \right]; \; q = 1, \ldots, Q \tag{4}$$

where $\underline{B} = \{\beta_n; n = 1, \ldots, N\}$ is the set of optimal difference excitations [17].

3.2. Two-step Hybrid Approaches

Inspired by the investigations on the synthesis of difference patterns carried out in [21], it has been recently discussed in [10] how the definition of the sub-array weights can be formulated as the solution of a convex programming problem, once the clustering of the array elements is given. However, in [10] the solution of a the CP problem is required every time a new sub-array configuration is obtained by means of the an approach based on Simulated Annealing (SA). Therefore, the SA-CP approach turns out to be affected by an unavoidably and high computational cost.

In order to cope with this drawback, in the following two new hybrid (two-step) approaches are proposed, where the solution of the CP problem is required only once during the whole synthesis process. The flowchart of both the approaches is schematically depicted in Fig. 2. More specifically, at the first step the sub-array configurations are computed according to the principles of either the M-CPM or the I-CPM [18]. Successively, the sub-array weights, $\underline{W}^{opt} = \left\{ w_q^{(opt)}; q = 1, \ldots, Q \right\}$, of the compromise feed network are computed so that the SLL of the afforded pattern is below a pre-fixed threshold. The following cost function

$$\Psi^{CP} (\underline{W}) = \left. \frac{\partial \mathrm{Re} \{ f(u,v) \}}{\partial u \partial v} \right|_{\substack{u = u_0 \\ v = v_0}} \tag{5}$$

is minimized subject to $\left. \frac{\partial \mathrm{Im}\{f(u,v)\}}{\partial u \partial v} \right|_{\substack{u = u_0 \\ v = v_0}} = 0$, to $f(u_0, v_0) = 0$ and a function descriptive of an upper mask $UB(u,v)$ on the synthesized difference pattern. Moreover, Re and Im denotes the real and imaginary part, respectively and (u_0, v_0) is the boresight direction. Towards this end, a standard CP procedure is used, whose initial guess solution is given by $\underline{W}^{(0)}$ as computed through Eq. (4).

4. NUMERICAL SIMULATIONS AND RESULTS

In order to show the effectiveness and the versatility of the proposed approaches, different synthesis problems concerning linear (small and large) as well as planar monopulse array antennas are shown in this section. In order to better point out the advantages and limitations of the simultaneous/global optimization and of the hybrid procedures, the numerical analysis has been subdivided in two parts. The first one (Section 4.1) concerns with the syntheses of small linear arrays, where the total number of unknowns is small ($N \leq 20$) and both global and hybrid approaches reach the final solution in a limited amount of time (i.e., in the order of one minute or less). The capability to deal with large linear arrays and planar apertures, characterized by a large number of radiating elements, is then considered in Section 4.2. Comparisons with benchmarks already reported in the literature are considered where available.

4.1. Small Linear Arrays Synthesis

In the first test, let us consider a linear array of $N = 20$ elements equally spaced of $\lambda/2$. The sum excitations are chosen to afford a Villeneuve pattern with $SLL = -25\,\mathrm{dB}$ and $\overline{n} = 4$ [22]. The number of sub-arrays has been set equal to $Q = 5$. In this case the results obtained by means of the proposed approaches are compared with the pattern synthesized by means of the *constrained Excitation Matching Method* (EMM) of [11], where the final pattern was characterized by $SLL = -23.4\,\mathrm{dB}$.

As far as the proposed approaches are concerned, the optimal difference excitation set considered in the M-CPM is chosen to correspond to the one used at the last step of the I-CPM. Moreover,

Table 1. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 5$) — Sub-array configurations and weights.

$N = 20$	$\underline{A}^{ICPM}$, $\underline{A}^{H\text{-}ICPM}$	3 4 5 5 5 4 3 3 2 1 1 2 3 3 4 5 5 5 4 3				
	$\underline{A}^{MCPM}$, $\underline{A}^{H\text{-}MCPM}$	3 4 5 5 5 4 4 3 2 1 1 2 3 4 4 5 5 5 4 3				
$Q = 5$	$\underline{W}^{ICPM}$	0.1738	0.5083	0.9561	1.3299	1.4775
	$\underline{W}^{MCPM}$	0.1738	0.5083	0.8358	1.2042	1.4775
	$\underline{W}^{H\text{-}ICPM}$	0.2896	0.7476	1.4378	2.1858	2.3207
	$\underline{W}^{H\text{-}MCPM}$	0.3423	0.7816	1.6012	2.1233	2.7166

since the constrained EMM [11] is also an excitation matching procedure, we force the $I\text{-}CPM$ to avoid a reference target with SLL lower than that considered in [11] (i.e., a modified Zolotarev difference pattern with $SLL = -25\,\text{dB}$, $\overline{n} = 4$ and $\epsilon = 3$ [23]).

The sub-array configurations $\underline{A}^{opt}_{I\text{-}CPM}$, $\underline{A}^{opt}_{M\text{-}CPM}$ as well as the corresponding sub-array gains $\underline{W}^{opt}_{I\text{-}CPM}$, $\underline{W}^{opt}_{M\text{-}CPM}$ obtained at the final iterations by the two global optimization techniques are summarized in Tab. 1. The corresponding patterns are shown in Fig. 3. As expected, improvements in term of SLL minimization are given by the $M\text{-}CPM$

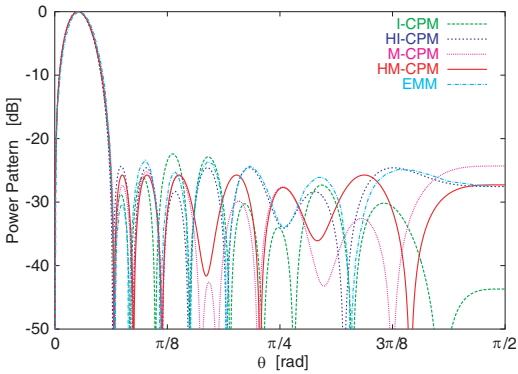

Figure 3. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 5$) — Relative power patterns obtained by means of the proposed approaches and the EMM [11].

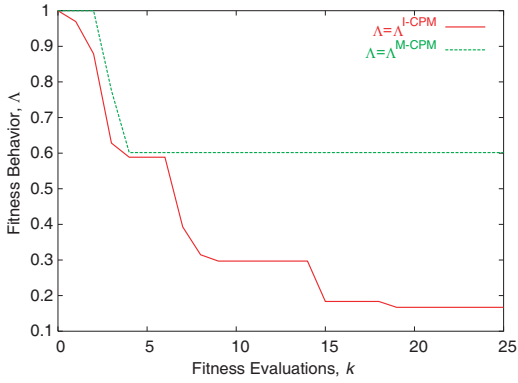

Figure 4. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 5$) — Behavior of the cost function of the $I\text{-}CPM$ and $M\text{-}CPM$ versus the iteration index k.

with a SLL lowered of almost 2 dB (i.e., $SLL_{I\text{-}CPM} = -22.4$ dB vs. $SLL_{M\text{-}CPM} = -24.3$ dB). In this experiment, only the $M\text{-}CPM$ outperforms the EMM in terms of SLL minimization. As far as the computational burden is concerned, thanks to the computational efficiency of the BEM, *Manica 2008b* and by virtue of the fact that the sub-array weights are computed analytically, the required CPU time is equal to $T_{I\text{-}CPM} = 0.05$ sec and $T_{M\text{-}CPM} = 0.24$ sec, while $k_{I\text{-}CPM} = 19$ and $k_{M\text{-}CPM} = 4$ is the total number of cost function evaluations.

In order to complete the analysis, Fig. 4 reports the values of the cost function of the $I\text{-}CPM$ as well as that of the $M\text{-}CPM$. Since two incommensurable quantities are minimized, in order to make the comparison meaningful the following relationship has been considered for the plots of the fitness

$$\Lambda = 1 - \frac{|\xi_k - \xi^{\max}|}{|\xi^{\max}|} \ , \ k = 1, \ldots, K \tag{6}$$

where ξ_k assumes either the value $\Psi_k^{M\text{-}CPM}$ (3) or $\Psi_k^{I\text{-}CPM}$ [18], according to the use of the $M\text{-}CPM$ or $I\text{-}CPM$, respectively. Moreover, $\xi^{\max} = \max_{i=1,\ldots,K}\{\xi_i\}$ is the maximum fitness value obtained throughout the whole optimization process.

As a second step, the final aggregations obtained by means of the bare approaches (Tab. 1) are considered as fixed clustering in the $H\text{-}ICPM$ and $H\text{-}MCPM$, i.e., $\underline{A}_{H\text{-}ICPM}^{opt} = \underline{A}_{ICPM}^{opt}$ and $\underline{A}_{H\text{-}MCPM}^{opt} = \underline{A}_{MCPM}^{opt}$, respectively. Then, the sub-array weights are determined through the subroutine FMINCON [24], where the mask $UB(\theta)$ has been set to have $BW = BW_{EMM}$ and uniform level of sidelobes. Accordingly, starting from a guess solution equal to $\underline{W}_{H\text{-}ICPM}^{(0)} = \underline{W}_{ICPM}^{opt}$ and $\underline{W}_{H\text{-}MCPM}^{(0)} = \underline{W}_{MCPM}^{opt}$, the weights of the sub-arrays are computed by the two hybrid approaches and the corresponding results are reported in Tab. 1. Also in this case, the synthesized patterns are shown in Fig. 3. It is worth noting that both the solutions achieved by the hybrid approaches have a SLL below the one obtained with the EMM [11], i.e., $SLL_{HI\text{-}CPM} = -24.4$ dB, $SLL_{HM\text{-}CPM} = -25.8$ dB vs. $SLL_{EMM} = -23.4$ dB. Moreover, the hybrid versions are more effective in term of SLL minimization than the respective bare procedures, with an improvement of 2 dB and 1 dB for the $HI\text{-}CPM$ and $HM\text{-}CPM$, respectively. As a matter of fact, notwithstanding the CP problem is aimed at the maximization of the difference slope, the same hybrid approaches can be used for the optimization of the SLL, as pointed out in [10].

Figure 5 reports the values Ψ_k^{CP}, $k = 1, \ldots, K$ (k being the iteration index) as well as the maximum distance C_θ between the actual

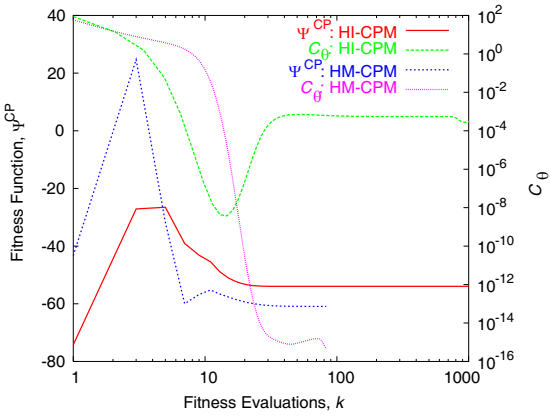

Figure 5. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 5$) — Behavior of the cost function and evolution of the distance from the constraints for the $HI\text{-}CPM$ and $HM\text{-}CPM$ versus the iteration index k.

pattern and the mask

$$C_k^{\theta} = \max_{\theta} \{f_k(\theta) - UB(\theta)\} \quad -\tfrac{\pi}{2} \leq \theta \leq \tfrac{\pi}{2} \tag{7}$$

where $f_k(\theta)$ is the array factor of the trail solution at the k-th iteration. As far as the costs of the subroutine FMINCON [24] are concerned, let us first point out that the number of function evaluations to reach the final solutions is equal to $k_{H\text{-}ICPM} = 1001$ and $k_{H\text{-}MCPM} = 83$. The overall CPU-time required to obtain $\underline{W}_{H\text{-}ICPM}^{opt}$ and $\underline{W}_{H\text{-}MCPM}^{opt}$ amounts to $T_{H\text{-}ICPM} = 61.22\,\text{sec}$ and $T_{H\text{-}MCPM} = 9.66\,\text{sec}$, with a non-negligible cost saving of almost six times for the $HM\text{-}CPM$ against the $HI\text{-}CPM$.

As a second experiment, let us consider one of the benchmark of [10], previously proposed in [14]. The number of sub-array was set to $Q = 6$ and the sum excitations fixed to those of a Dolph-Chebyshev pattern with $SLL = -20\,\text{dB}$ [25], while the difference excitations are those of a Zolotarev pattern with $SLL = -31\,\text{dB}$ [26]. Similarly to the previous case, the synthesis problems consists in defining the sub-array clustering and weights in order to obtain a compromise difference beam with the lowest SLL, once the pattern beamwidth has been fixed to that obtained by *Differential Evolution* (DE) optimization in [14].

The sub-array configuration achieved in [10] in the case of SLL optimization was $\underline{A}_{SA\text{-}CP}^{opt} = [1\,5\,2\,3\,3\,4\,2\,5\,6\,1\,1\,6\,5\,2\,4\,3\,3\,2\,5\,1]$ with a maximum $SLL = -30\,\text{dB}$. For the sake of comparison, the result achieved by the $SA\text{-}CP$ in the case of maximization of the slope (where a value $SLL = -29.50\,\text{dB}$ was reached) has been reported in

Fig. 6 as well as the one obtained with the *DE*-based approach [14], together with those synthesized through the proposed approaches. Concerning the two global *CPM*-based approaches, the *I-CPM* and the *M-CPM* achieve two different sub-array configurations, namely $\underline{A}^{opt}_{I\text{-}CPM} = [2\,4\,5\,6\,6\,6\,5\,4\,3\,1\,1\,3\,4\,5\,6\,6\,6\,5\,4\,2]$ and $\underline{A}^{opt}_{M\text{-}CPM} = [1\,3\,4\,5\,6\,6\,4\,3\,2\,1\,1\,2\,3\,4\,6\,6\,5\,4\,3\,1]$, among the 126 solutions defined in the solution tree [12]. The corresponding sub-array weights turns out being $\underline{W}^{opt}_{I\text{-}CPM} = \{0.1641, 0.2422, 0.4652, 0.6917, 0.8776, 0.9991\}$ and $\underline{W}^{opt}_{M\text{-}CPM} = \{0.2081, 0.4652, 0.6917, 0.8776, 0.9840, 1.0044\}$.

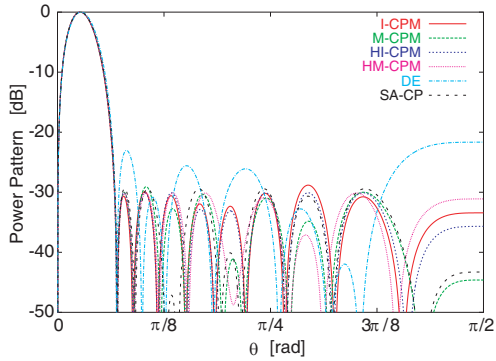

Figure 6. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 6$) — Relative power patterns obtained by means of the proposed approaches, the *SA-CP* [10] and the *DE* [14].

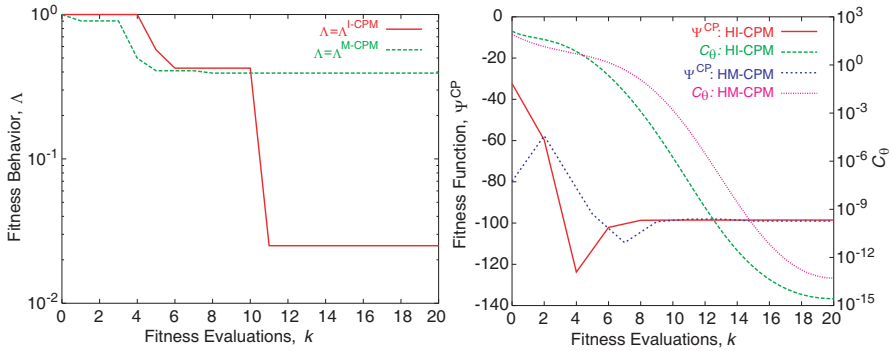

Figure 7. Small linear array ($N = 20$, $d = \frac{\lambda}{2}$, $Q = 6$) — Behavior of the cost function of the (a) *I-CPM* and *M-CPM* and of the (b) *HI-CPM* and *HM-CPM* versus the iteration index k.

Moreover, $T_{I\text{-}CPM} = 0.001\,\text{sec}$, $T_{M\text{-}CPM} = 0.267\,\text{sec}$ and $k_{I\text{-}CPM} = 12$, $k_{M\text{-}CPM} = 10$. Also the solutions achieved by the hybrid versions are shown in Fig. 6. In these cases, $k_{HI\text{-}CPM} = 15$ and $k_{HM\text{-}CPM} = 16$ function evaluations were needed with a required CPU time of $T_{HI\text{-}CPM} = 2.703\,\text{sec}$ and $T_{HM\text{-}CPM} = 2.719\,\text{sec}$. The corresponding sub-array weights are $\underline{W}^{opt}_{HI\text{-}CPM} = \{0.6676, 0.9174, 1.7668, 2.6966, 3.4241, 3.8810\}$ and $\underline{W}^{opt}_{HM\text{-}CPM} = \{0.8019, 1.8409, 2.6401, 3.5552, 3.7342, 3.9490\}$. It is interesting to note how all the solutions defined by means of the proposed approaches outperform that of [14], whereas only the solutions obtained by means of hybrid approaches $HI\text{-}CPM$ and $HM\text{-}CPM$ are able to enhance the performances of [10]. As a matter of fact $SLL_{I\text{-}CPM} = -28.81\,\text{dB}$, $SLL_{M\text{-}CPM} = -29.12\,\text{dB}$, $SLL_{HI\text{-}CPM} = -30.09\,\text{dB}$ and $SLL_{MI\text{-}CPM} = -30.13\,\text{dB}$. In order to complete the analysis, the behavior of the objective functions for the global optimization procedures as well as their hybrid versions are reported in Figs. 7(a) and 7(b), respectively.

4.2. Large Linear Arrays and Planar Apertures

This section is aimed at analyzing the performances of the proposed approaches when dealing with the synthesis of array with a large number of elements. In the first example a linear aperture of length 100λ is considered, whit $N = 200$ elements equi-spaced of $\frac{\lambda}{2}$. The sum excitations are fixed to afford a Dolph-Chebyshev pattern [25] with $SLL = -25\,\text{dB}$. The number of available sub-array is $Q = 6$.

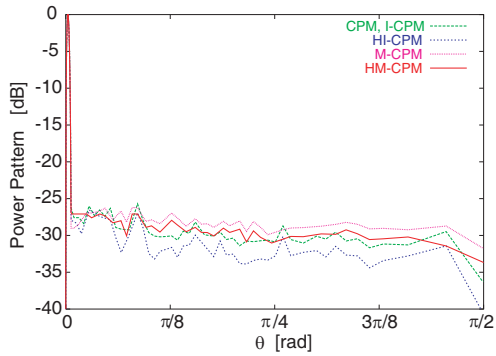

Figure 8. Large linear array ($N = 200$, $d = \frac{\lambda}{2}$, $Q = 6$) — Relative power patterns obtained by means of the proposed approaches and the CPM [12].

This synthesis problem was previously dealt with in [12]. Since a well known trade-off exists between pattern beamwidth and SLL, the I-CPM is not allowed to use reference targets whose SLL is below the one taken into account in [12] (i.e., a Zolotarev difference pattern [26] $SLL = -30\,\mathrm{dB}$). Fig. 8 shows the compromise difference patterns synthesized by means of the proposed procedures. As expected, the solution obtained with the I-CPM is the same obtained with the CPM [12]. The behavior of the fitness values for the global and hybrid approaches are shown in Figs. 9(a) and 9(b), respectively.

Although all the solutions show a good behavior in term of sidelobes rejection, the HM-CPM outperformed the other approaches with $SLL_{HM\text{-}CPM} = -27.1\,\mathrm{dB}$, while $SLL_{I\text{-}CPM} = -25.2\,\mathrm{dB}$, $SLL_{M\text{-}CPM} = -26.2\,\mathrm{dB}$ and $SLL_{HI\text{-}CPM} = -26.5\,\mathrm{dB}$. The sub-array configurations as well as the corresponding sub-array weights are given in Tab. 2.

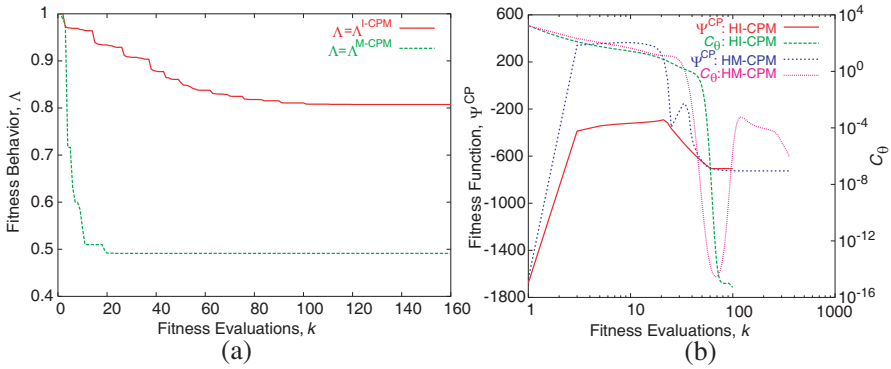

Figure 9. Large linear array ($N = 200$, $d = \frac{\lambda}{2}$, $Q = 6$) — Behavior of the cost function of the (a) I-CPM and M-CPM and of the (b) HI-CPM and HM-CPM versus the iteration index k.

Table 2. Large linear array ($N = 200$, $d = \frac{\lambda}{2}$, $Q = 6$) — Sub-array configurations and weights.

$M = 100$	$a_n^{I\text{-}CPM}$, $n = 1, ..., M$	111111111111122222223 333333344444444555555556666666666666666666666666666555555554444443331				
	$a_n^{M\text{-}CPM}$, $n = 1, ..., M$	111111111222222233333333 333333344444444455555555555666666666666666666655555555544444444443332				
$Q = 6$	$\underline{W}^{I\text{-}CPM}$	0.8206	1.4472	2.0200	2.5000	2.9000
	$\underline{W}^{M\text{-}CPM}$	0.3739	1.0060	1.8017	2.5520	3.0300
	$\underline{W}^{HI\text{-}CPM}$	0.2132	0.7236	0.9411	1.0909	1.2754
	$\underline{W}^{HM\text{-}CPM}$	0.1134	0.3327	0.6773	1.1001	1.1871

Table 3. Large linear array ($N = 200$, $d = \frac{\lambda}{2}$, $Q = 6$) — Fitness evaluations and CPU time.

Approach	k	T [sec]
I-CPM	128	15.6
HI-CPM	383	4105.17
M-CPM	24	519.98
HM-CPM	95	957.51

Concerning the computational costs, the number of cost function evaluation and the required CPU time for each approach are reported in Tab. 3. It is worth noting that in this case the computational burden of the CP problem is non-negligible (i.e., $T_{HI\text{-}CPM} = 4105.12$ and $T_{HM\text{-}CPM} = 957.51$ sec). Such a drawback is principally due to the computation of C_θ, where the pattern has to be sampled densely in order to obtain satisfactory results. Likewise, the computation of the power pattern is necessary also in the M-CPM to evaluate the SLL for each trial solution. Therefore, the I-CPM [18] turns out to be in this case the most efficient strategy.

In the last example, in order to fully exploit the capabilities of the CPM-based approaches, let us consider a planar array with circular boundary $r = 4.85\lambda$ and $N = 300$ elements equally-spaced of $d = \frac{\lambda}{2}$ along the two coordinates. The sum mode is set to a circular Taylor pattern [27] with $SLL = -35$ dB and $\overline{n} = 6$. Moreover, $Q = 3$ sub-arrays have been considered. The synthesis problem has been originally dealt with in [9] by means of a SA-based algorithm and then considered as benchmark in [16, 19]. There, the *sidelobe ratio* (SLR) defined as

$$SLR\left(\phi\right) = \frac{SLL\left(\phi\right)}{max_\theta\left[f\left(\theta,\phi\right)\right]},\ \ 0 \leq \theta < \frac{\pi}{2} \tag{8}$$

was optimized. Unlike [19], in this case we are aimed at synthesizing a compromise difference pattern with a SLL low as much as possible. As far as the I-CPM is concerned, the reference excitations (at the last iteration) was set in [19] to those a Bayliss pattern [28] with $SLL = -35$ dB and $\overline{n} = 6$. In this case, the SLL was equal to the one obtained with the SA-based approach (i.e., $SLL_{SA} = SLL_{I\text{-}CPM} - 19$ dB). Although an improvement of the performances was expected by using its hybrid version, in this case the achieved compromise configuration affords a pattern with $SLL_{HI\text{-}CPM} = -18.9$ dB, worse than the one obtained with the I-CPM. On the contrary, the M-CPM synthesized a solution with $SLL_{M\text{-}CPM} = -24.45$ dB, almost than 5 dB below the solution of [9]. Moreover, an additional improvement of more than 2 dB was gained when using the HM-CPM (i.e., $SLL_{M\text{-}CPM} =$

$-26.55\,\mathrm{dB}$).

Figure 10 shows the 2D plots of the relative power patterns for all the compromise solutions. The corresponding sub-array configurations are shown in Fig. 11, while the sub-array weights for the four approaches are summarized in Tab. 4. Although the proposed approaches are aimed the optimization of the maximum SLL on the whole aperture, in this case both M-CPM and HM-CPM guaranteed that also the values of SLR were lower than that of [9] (Fig. 12).

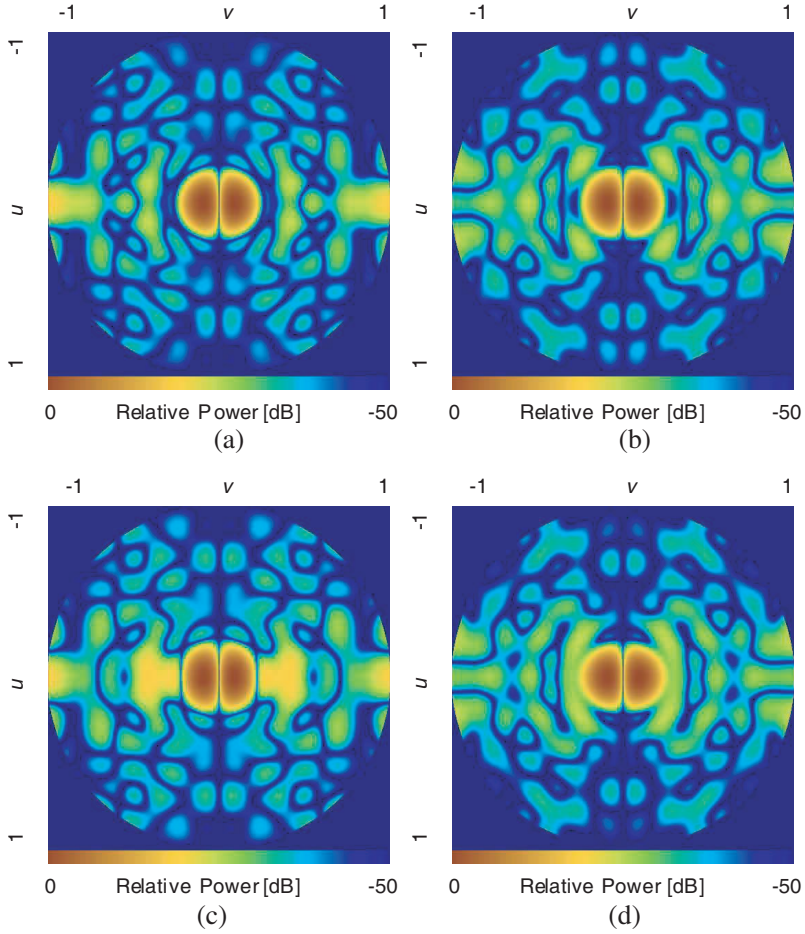

Figure 10. Planar array synthesis ($N = 300$, $d = \frac{\lambda}{2}$, $r = 4.85\lambda$, $Q = 3$) — Relative power patterns obtained by means of (a) the I-CPM, (b) the M-CPM, (c) the HI-CPM and (d) HM-CPM.

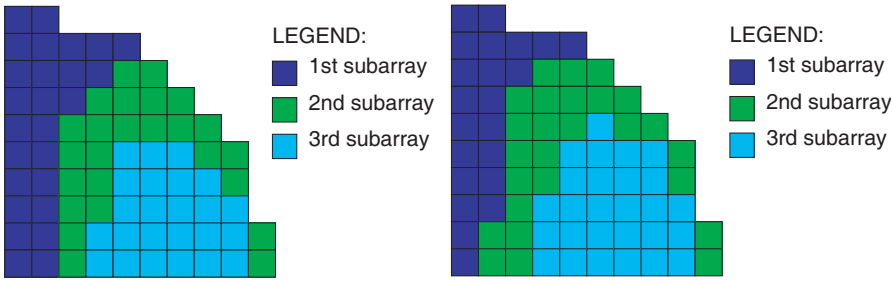

Figure 11. Planar array synthesis ($N = 300$, $d = \frac{\lambda}{2}$, $r = 4.85\lambda$, $Q = 3$) — Sub-array configurations obtained with (a) the *I-CPM* and (b) the *M-CPM*.

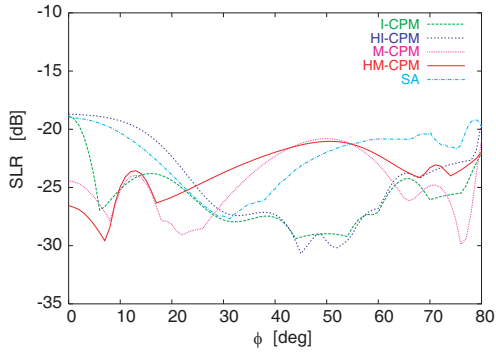

Figure 12. Planar array synthesis ($N = 300$, $d = \frac{\lambda}{2}$, $r = 4.85\lambda$, $Q = 3$) — Plots of the synthesized *SLR* values by means of the proposed approaches and the *SA* [9] in the range $\phi \in [0°, 80°]$.

Table 4. Planar array synthesis ($N = 300$, $d = \frac{\lambda}{2}$, $r = 4.85\lambda$, $Q = 3$) — Sub-array weights obtained by means of the proposed approaches and the *SA* [9]).

Approach	w_1^H	w_2^H	w_3^H
I-CPM	0.3499	0.9333	1.4170
M-CPM	0.2870	0.8120	1.3886
HI-CPM	0.3684	2.4088	4.0573
HM-CPM	0.3313	0.9719	1.4113
SA [9]	1.69	3.69	5.00

Concerning the computational costs, it turns out that $T_{HI\text{-}CPM} = 24186.6\,\text{sec}$ (almost seven hours) and $T_{HM\text{-}CPM} = 39036.8\,\text{sec}$ (more than ten hours). Moreover, $k_{HI\text{-}CPM} = 6621$ and $k_{HM\text{-}CPM} = 10001$. On the contrary, the computational cost reduces to $T_{M\text{-}CPM} = 537.9\,\text{sec}$, $T_{I\text{-}CPM} = 165.5\,\text{sec}$, and $k_{M\text{-}CPM} = 6$, $k_{I\text{-}CPM} = 81$ for the bare approaches.

5. CONCLUSIONS

In this paper, innovative approaches to the synthesis of the optimal compromise between sum and difference patterns for sub-arrayed monopulse array antennas have been presented. The synthesis of linear and planar array has been deal with, where the problem at hand has been formulated as the definition of the sub-array configuration and weights of these latter to minimize the SLL of the synthesized difference beam. The definition of the unknowns has been simultaneously carried out according to a global optimization schema, the $M\text{-}CPM$, and the results have been compared with the previously proposed $I\text{-}CPM$. Unlike the $I\text{-}CPM$, the compromise solution with minimum SLL has been directly looked for among the solutions belonging to the *solution tree*. In a different fashion, the $HI\text{-}CPM$ and the $HM\text{-}CPM$ have shown better performance in term of SLL minimization with respect to the corresponding one-step approaches. In these case, the convexity of the problem with respect to a part of the unknowns has been exploiting, where the synthesis problem has been reduced to solve a CP problem for a fixed clustering. The effectiveness of the proposed techniques in terms of SLL minimization has been assessed by showing some experiments concerned with small as well as large array synthesis problems, hardly to manage with stochastic optimization procedures for the arising computational burden. Moreover, by virtue of the fact that the solution of the CP problem is required only once, the hybrid CPM-based strategies seem to represent promising tools to be further analyzed and extended to other antenna geometries.

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