Progress In Electromagnetics Research B, Vol. 18, 311–325, 2009

PHOTONIC BANDGAPS IN QUASIPERIODIC MULTI-LAYER STRUCTURES USING FOURIER TRANSFORM OF THE REFRACTIVE INDEX PROFILE

S. Golmohammadi

Nanophotonics Group
Department of Engineering-Emerging Technologies
University of Tabriz
Tabriz 5166614761, Iran

Y. Rouhani

Department of Engineering
Islamic Azad University
Tabriz, Iran

K. Abbasian and A. Rostami †

Nanophotonics Group
Department of Engineering-Emerging Technologies
University of Tabriz
Tabriz 5166614761, Iran

Abstract—In this paper, photonic bandgaps (PBGs) of the quasiperiodic structures is calculated using the Fourier transform of the refractive index profile. Comparing the reflectivity and Fourier spectrum of multilayer structure refractive index, we find that a peak in the Fourier spectrum is equivalent to a sinusoidal term in the refractive index. The wavelength of the peak location in the Fourier spectrum is half the wavelength where a PBG is located. Using Fourier transform analysis of the refractive index of any multilayer structure, we can determine the location of the PBGs of that structure. Peaks in the Fourier spectrum can be used to design reflective band optical filters in optical communication systems. The filtering wavelengths are twice the peaks in the Fourier spectrum.

Corresponding author: S. Golmohammadi (Sgolmohammadi@tabrizu.ac.ir).
† Also with Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz 5166614761, Iran.

1. INTRODUCTION

Today, next generation services have a great impact on design and topology of the communication networks. The rapidly growing demand for larger bandwidth motivates high bit rate communication networks toward dense wavelength division multiplexing (DWDM) technology as a solution. Using DWDM, the bandwidth of the optical fiber is used by a number of the wavelengths separated according to the ITU-T. Design and fabrication of the narrow and multiband filters for DWDM applications is a challenge for optical device engineers and designers.

Optical multilayer structures, such as fiber Bragg gratings [1, 2], multilayer thin-film filters [3], have been used as wavelength selective optical filters. The essential property of dielectric multilayer structure is the existence of photonic band gap that forbids propagation of a certain frequency range of light, where the structure acts as a filter in that frequency [4]. This property enables one to control light with amazing facility and produce effects that are impossible with conventional optics. Existence of a PBG in the spectra of multilayer structure is equivalent to a band of reflective filter. This filter can be used in optical communication systems. In order to use in DWDM systems, the filter should have several bands according to the wavelengths defined in the standard [5].

Multiband filter can be constructed by cascading single band filters, but there are some problems such as increased insertion loss and decreased reliability of the system. Narrow and multiband filters based on multilayer structures — as an alternative for cascaded single-band filters in DWDM systems — have some difficulty in manufacturing because of high number of layers with low refractive index difference between them [6]. In addition, quasiperiodic structures can be used as multiband filters in DWDM systems [7]. Importance of study of Photonic bandgaps in spectra of these structures is related to application of these structures as DWDM filters.

Quasiperiodic structures were discovered in 1984 by Sechtmann. These are non-periodic structures that are constructed following a defined recursive generation rule [8]. Light localization within quasiperiodic structures using Fibonacci sequence as the generation rule of the structure has been shown by Kohmoto et al. [9]. Then Sibilia et al. demonstrated self-similar patterns in transmission spectrum of quasiperiodic structures [10]. Existence of bandgaps in spectral response of these structures was demonstrated experimentally by Gellermann et al. [11]. Macia studied these structures numerically by means of Transfer Matrix Method [12]. Omnidirectional bandgaps of these structures are reported by Lusk et al. [13]. Perfect transmission in symmetric quasiperiodic multilayer structures based on Fibonacci sequence was observed by Peng et al. [14]. Fast Fourier Transform has already been used for spectral analysis of some periodic waveguide and grating structures [15, 16], but, so far, very little effort is reported on applying this transform to study of spectral contents of quasiperiodic multilayer structures [17].

In this paper, we will use Fourier transform of the quasiperiodic structure refractive index to find PBGs of the Fibonacci based quasiperiodic multilayer structures. We will compare spectral content of the Fourier transform of the refractive index profile with reflectance spectrum of the structure which is calculated by TMM. We will demonstrate that we can determine the PBGs of any multilayer structure only using the Fourier transform of the refractive index profile. Our case study is focused on the Fourier transform of generalized Fibonacci structures $G(j, n)$.

This paper is organized as following. In Section 2, Fibonacci and generalized Fibonacci sequence has been introduced, and basic idea behind realization of these sequences in photonic field is discussed. In Section 3, we apply Fourier transform to generalized Fibonacci based multilayer structures, and a recurrent relation for the Fourier terms has been obtained. In Section 4, reflectance of the Generalized Fibonacci structure using transfer matrix method (TMM) and spectral content using Fourier transform has been obtained and compared with each other. Finally, in Section 5, some conclusions of the paper are presented.

2. MULTILAYER STRUCTURES BASED ON FIBONACCI SEQUENCES AND GENERALIZED FIBONACCI SEQUENCES

By definition, the first two Fibonacci numbers are 0 and 1, and each remaining number is the sum of the previous two. Some sources omit the initial 0, instead beginning the sequence with two 1s. In mathematical terms, the sequence u_j of Fibonacci numbers is defined by the recurrence relation [18]:

$$u_j = u_{j-1} + u_{j-2}, \ j \geq 2 \tag{1}$$

The first seven terms of Fibonacci numbers are: **1, 1, 2, 3, 5, 8, and 13**.

For realization of this sequence in the field of photonics, two types of dielectric materials with different refractive indices are selected as building blocks (denoted by **A** and **B**), and summation in the number field is translated to concatenation of these building blocks. Applying the Fibonacci rule and starting by $S_0 = \mathbf{B}, S_1 = \mathbf{A}$, next three terms of the sequence will be $S_2 = \mathbf{AB}$, $S_3 = \mathbf{ABA}$, $S_4 = \mathbf{ABAAB}$, and general term of the multilayer structure based on this sequence will be in the following form:

$$S_j = S_{j-1}S_{j-2},\ j \geq 2 \tag{2}$$

Multilayer structures based on Fibonacci sequences are known as substitutional structures because some substitution rules can be found that by applying them to the first term next structures of the sequence can be constructed. Substitution rules of this structure can be easily found and are given by $\mathbf{B} \rightarrow \mathbf{A}$, $\mathbf{A} \rightarrow \mathbf{AB}$. There are two steps for generalizing of Fibonacci sequence. First step is to multiply previous terms by a positive integer as below:

$$u_j = nu_{j-1} + mu_{j-2},\ j \geq 2 \tag{3}$$

For realization of this generalization, multiplication by an integer is translated to repeat the building blocks. Second step in generalizing is using more than two terms for constructing general term of the sequence as shown below:

$$u_j = au_{j-1} + bu_{j-2} + cu_{j-3},\ j \geq 3 \tag{4}$$

Recently mentioned sequence by Eq. (4) is named Tribonacci [19]. In this generalization step, there is no limit on the number of used terms to construct the general term. In this paper, we will focus on the first step of generalization and use a special case by using $m = 1$. We also use $G(j, n)$ for notation of multilayer structures based on this generalized Fibonacci sequences which is given by:

$$G(j, n) = [G(j-1, n)]^n G(j-2, n),\ j \geq 2 \tag{5}$$

We refer j as generation number and n as structure order. It should be noted that in addition to recurrence relation (Eq. (5)), the first two terms of the sequence are required to describe the structure distinctively. In this paper, we will use $\mathbf{B}$ and $\mathbf{B}^{n-1}\mathbf{A}$ as the first two terms of the generalized Fibonacci sequence. By using the general term equation (Eq. (5)) three next terms of the sequence can be written as:

$$\begin{aligned} G(2, n) &= (\mathbf{B}^{n-1}\mathbf{A})^n\mathbf{B} \\ G(3, n) &= \left[(\mathbf{B}^{n-1}\mathbf{A})^n\mathbf{B}\right]^n \mathbf{B}^{n-1}\mathbf{A} \\ G(4, n) &= \left\{ \left[(\mathbf{B}^{n-1}\mathbf{A})^n\mathbf{B}\right]^n \mathbf{B}^{n-1}\mathbf{A} \right\}^n (\mathbf{B}^{n-1}\mathbf{A})^n\mathbf{B} \end{aligned} \tag{6}$$

Substitution rules of the sequence can be obtained easily:

$$\mathbf{B} \rightarrow \mathbf{B}^{n-1}\mathbf{A},\ \mathbf{A} \rightarrow \mathbf{B}^{n-1}\mathbf{AB} \tag{7}$$

3. FOURIER TRANSFORM OF REFRACTIVE INDEX OF THE ONE DIMENSIONAL MULTILAYER STRUCTURES BASED ON GENERALIZED FIBONACCI SEQUENCE

In this section, we use the Fourier transform to extract the spectral content of the refractive index profile. We start from first term of the generalized Fibonacci structure by using the Fourier transform formula:

$$F[f(z)] = F(k) = \int_{-\infty}^{+\infty} f(z)e^{-ikz}dz \tag{8}$$

To take Fourier transform of the $G(0,n) = \mathbf{B}$ by using Eq. (8), we have assumed horizontal axis as optical length equal to physical length multiplied by refractive index. Then we can write:

$$F[G(0,n,z)] = \int_0^D n_B e^{-ikz}dz = \frac{n_B}{-ik}\left(e^{-ikD} - 1\right), \tag{9}$$

where $k = 2\pi/\lambda$ is the wave vector, and λ is the wavelength in free space $D = n_B d_B = n_A d_A$, which is the optical length of both layers.

For the next term $G(1,n) = \mathbf{B}^{n-1}\mathbf{A}$, the Fourier transform of the refractive index profile is:

$$F[G(1,n,z)] = \int_0^{(n-1)D} n_B e^{-ikz}dz + \int_{(n-1)D}^{nD} n_A e^{-ikz}dz \tag{10}$$

$$F[G(1,n,z)] = \frac{n_B}{-ik}(e^{-ik(n-1)D} - 1) + \frac{n_A}{-ik}e^{-ik(n-1)D}(e^{-ikD} - 1) \tag{11}$$

Now, we can proceed to the next term $G(2,n) = (\mathbf{B}^{n-1}\mathbf{A})^n\mathbf{B}$; its refractive index profile is demonstrated in Fig. 1.

As can be seen in Fig. 1, this structure is mainly constructed from shifted versions of the previous term $G(1,n,z)$, except the last layer which is constructed from shifted $G(0,n,z)$ structure.

$$\begin{aligned} G(2,n,z) \;=\; & G(1,n,z)G(1,n,z)_{nD}G(1,n,z)_{2nD}\ldots \\ & G(1,n,z)_{(n-1)nD}G(0,n,z)_{n^2D} \end{aligned} \tag{12}$$

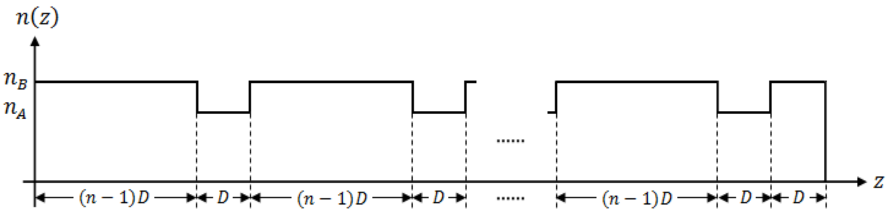

Figure 1. Refractive index Profile of the $G(2,n,z)$.

In Eq. (12), the indices $nD, 2nD, \ldots, (n-1)D$ and n^2D are the amount of refractive index profile of the $G(1,n,z)$ in the z axis. Using the shift theorem of Fourier transform we have:

$$\begin{aligned} F[G(1,n,z)_{nD}] &= e^{-iknD}F[G(1,n,z)] \\ F[G(1,n,z)_{2nD}] &= e^{-ik2nD}F[G(1,n,z)] \\ F[G(1,n,z)_{(n-1)nD}] &= e^{-ik(n-1)nD}F[G(1,n,z)], \\ &\vdots \end{aligned} \tag{13}$$

Finally, by adding set of equations denoted by (13) we can write:

$$\begin{aligned} F[G(2,n,z)] &= \left[1+e^{-iknD}+\cdots+e^{-ik(n-1)nD}\right]F[G(1,n,z)] \\ &\quad +F\left[G(0,n,z)_{n^2D}\right] \end{aligned} \tag{14}$$

$$F[G(2,n,z)] = \frac{e^{-ikn^2D}-1}{e^{-iknD}-1}F[G(1,n,z)] + e^{-ikn^2D}F[G(0,n,z)] \tag{15}$$

Equation (15) is a recurrence relation for Fourier transform of $G(2,n,z)$ in terms of Fourier transforms of $G(1,n,z)$ and $G(0,n,z)$. Similarly, we can write for the refractive index of the next term of the generalized Fibonacci sequence:

$$\begin{aligned} G(3,n,z) &= G(2,n,z)G(2,n,z)_{(n^2+1)D}\cdots \\ &\quad G(2,n,z)_{(n-1)(n^2+1)D}G(1,n,z)_{n(n^2+1)D} \end{aligned} \tag{16}$$

Applying the Fourier transform to $G(3,n,z)$ we have:

$$F[G(3,n,z)] = \frac{e^{-ikn(n^2+1)D}-1}{e^{-ik(n^2+1)D}-1}F[G(2,n)] + e^{-ikn(n^2+1)D}F[G(1,n)] \tag{17}$$

Equation (17) is a similar relation for $F[G(3,n,z)]$ as a recurrence function of $F[G(2,n,z)]$ and $F[G(1,n,z)]$. We can continue this operation to the next terms of $G(j,n,z)$, and recurrence relations are obtained as follows:

$$\begin{aligned} F[G(4,n,z)] &= \left(\frac{e^{-ikn(n^3+2n)D}-1}{e^{-ik(n^3+2n)D}-1}\right)F[G(3,n,z)] \\ &\quad +e^{-ikn(n^3+2n)D}F[G(2,n,z)] \end{aligned} \tag{18}$$

$$\begin{aligned} F[G(5,n,z)] &= \left(\frac{e^{-ikn(n^4+3n^2+1)D}-1}{e^{-ik(n^4+3n^2+1)D}-1}\right)F[G(4,n,z)] \\ &\quad +e^{-ikn(n^4+3n^2+1)D}F[G(3,n,z)] \end{aligned} \tag{19}$$

By comparing the obtained results for $F[G(3,n,z)]$, $F[G(4,n,z)]$ and $F[G(5,n,z)]$, we can suggest a general recurrence relation for the Fourier transform of the refractive index of multilayer structures based on generalized Fibonacci sequence:

$$F[G(j,n,z)] = \frac{[P(j,n)]^n - 1}{[P(j,n)] - 1} F[G(j-1,n,z)] + [P(j,n)]^n F[G(j-2,n,z)], \; j \geq 2 \tag{20}$$

Where:

$$F[G(0,n,z)] = \frac{n_B}{-ik}\left(e^{-ikD} - 1\right) \tag{21}$$

$$F[G(1,n,z)] = \frac{n_B}{-ik}\left(e^{-ik(n-1)D} - 1\right) + \frac{n_A}{-ik} e^{-ik(n-1)D}\left(e^{-ikD} - 1\right) \tag{22}$$

and

$$P(0,n) = 1 \tag{23}$$

$$P(1,n) = e^{-ikD} \tag{24}$$

$$P(j,n) = [P(j-1,n)]^n P(j-2,n), \; j \geq 2 \tag{25}$$

$P(j,n)$ is an exponential function, and its exponent can be written as $-ikf(n,D)$, where $f(n,D)$ is the optical thickness of $G(j-1,n)$. For example, $P(j,n)$ for $j = 2, 3, 4, 5$ is given as below:

$$\begin{aligned} P(2,n) &= [P(1,n)]^n P(0,n) = e^{-iknD} \\ P(3,n) &= [P(2,n)]^n P(1,n) = e^{-ikn^2D} e^{-ikD} = e^{-ik(n^2+1)D} \\ P(4,n) &= [P(3,n)]^n P(2,n) = e^{-ikn(n^2+1)D} e^{-iknD} = e^{-ik(n^3+2n)D} \\ P(5,n) &= [P(4,n)]^n P(3,n) = e^{-ik(n^3+2n)D} e^{-ikn(n^2+1)D} \\ &= e^{-ik(n^4+3n^2+1)D} \end{aligned} \tag{26}$$

Eq. (20) demonstrates a recurrence relation between Fourier transforms of the refractive index profiles of the generalized Fibonacci structure. Comparing Eq. (20) with Eq. (5), we realize that the recurrence relation between $G(j,n)$, $G(j-1,n)$ and $G(j-2,n)$ in the direct domain will appear in the Fourier domain between $F[G(j,n,z)]$, $F[G(j-1,n,z)]$ and $F[G(j-2,n,z)]$. Then, the Fourier transform of the refractive index of the generalized Fibonacci structures can be given in terms of refractive index Fourier transforms of the two previous structures according to Eq. (20). In order to construct the spectral content of any structure, we should plot $|F[G(j,n,z)]|$ which can be calculated using Eq. (20). From the investigation of $|F[G(j,n,z)]|$ plot, we are going to argue about PBGs of the whole structure. Here, we want to answer the question "What is the relationship between the Fourier spectrum and PBGs of a multilayer structure?”. We will compare the Fourier transform spectrum with the optical reflectivity of the generalized Fibonacci structure that is calculated using (TMM) [20] to answer this question.

In any multilayer structure the electric field of optical signals is composed of forward and backward components [21]. Using TMM, first matrices are derived in such a way that they relate the forward and backward components of the electric field across an interface and a given thickness, in order to properly represent wave propagation in each layer of the multilayer structure. Wave propagation through each layer is calculated by using the matrices that relate the field at one side to another side of that layer. In TMM the reflectivity from first layer is calculated by the field propagation from the end layer to the first layer using matrix relations. A PBG in the reflectivity occurs when the forward components of a propagating field at a given wavelength becomes low due to the instructive interference of partially Fresnel reflected fields at each interface.

4. SIMULATION RESULTS

In this section, we present the simulation results using two different methods, Fourier transform and TMM. We compare the spectral content of the Fourier transform of the refractive index profile and optical reflectivity spectrum of $G(j, n)$, in order to justify PBGs of this structure.

First, we discuss the optical length of the layers **A** and **B**. It is well known that for achieving maximum reflectivity in a multilayer structure we should set the optical length of the layers [20]:

$$D = \frac{m\lambda_c}{4}, \ (m = 1, 3, 5, \ldots) \tag{27}$$

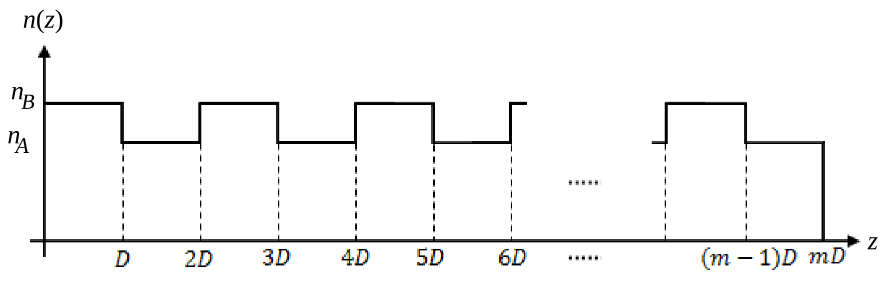

Figure 2. Refractive index profile of the one dimensional periodic structure.

where, λ_c is the center wavelength of the optical reflection spectrum. In order to compare the spectral contents of the Fourier transform and optical reflectivity spectrum of different structures, we start from a well known periodic structure, which is composed of two different layers denoted by **A** and **B** as shown in Fig. 2. In this figure, the period of optical length variations of layers **A** and **B** is $2D$. It is well-known that the Fourier transform of a square function with period D contains superposition of sinusoidal terms with periods $2D, 6D, \ldots, 2(2n+1)D$. The sinusoidal terms in the Fourier transform of the refractive index profile will appear as a peak in the spectrum of the Fourier transform. For the periodic structure of Fig. 2, the peaks will appear in $\lambda = 2D, 6D, \ldots$. The height of these peaks will be decreased with increasing the wavelength.

The spectral content of a multilayer structure may contain several peaks, where each peak demonstrates a sinusoidal term in the Fourier transform of the refractive index profile of that structure. As an example the spectral content of the Fourier transform of a sample periodic structure is demonstrated in Fig. 3. From the Bragg condition, the PBG of this structure is located at $\lambda_B = 2(n_B d_B + n_A d_A) = 4D$, while as seen from Fig. 3, the peak with largest height is located at $\lambda = 2D$, in the Fourier spectrum. From the Bragg condition we know that the PBG of this structure is located at $\lambda_B = 4D = 1550\,\text{nm}$. From Fig. 5, we see a peak with large height is located at $\lambda = 2D = 750\,\text{nm}$. The peak at $\lambda = 750\,\text{nm}$ is due to the sinusoidal term in the Fourier transform of the refractive index profile of the periodic structure. It should be noted that the refractive index profile variation is a square wave, and its main sinusoidal term has the same frequency as that of square wave.

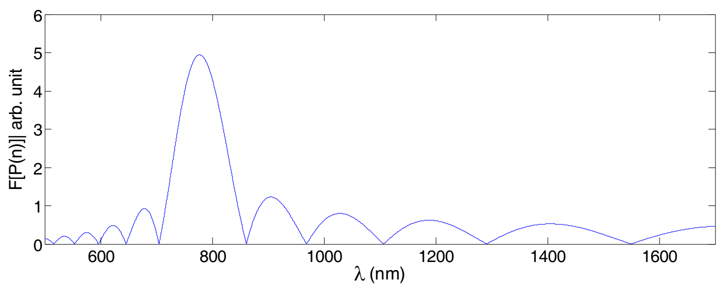

Figure 3. Spectral content of the Fourier transform of the refractive index profile of a sample periodic structure. The parameters are $D = 387.5\,\text{nm}$, $\lambda_C = 1550\,\text{nm}$, $n_A = 1.45$, $n_B = 1.65$, $d_A = 267.24\,\text{nm}$ and $d_B = 234.85\,\text{nm}$.

After a general study on Fourier transform of the periodic structure, we are going to investigate PBGs of the generalized Fibonacci structure using both discussed methods of Fourier transform and TMM. We calculate spectral response of this structure for various values of j and n. In all structures, we use $n_A = 1.45$, $n_B = 1.65$, $\lambda_C = 1550\,\text{nm}$, $D = \lambda_C/4 = 387.5\,\text{nm}$, $d_A = 267.24\,\text{nm}$ and $d_B = 234.85\,\text{nm}$. And refractive indices of media before and after of the structure are the same as material A.

In Figs. 4–9, the obtained reflectance of these structures using TMM and spectral content of the same structure using Fourier transform algorithm is depicted respectively. It should be discussed that average value of refractive index distribution of the structures has been canceled from refractive index profile to remove spectral content of the average value from the Fourier transform spectrum.

Figures 4(a) and 4(c) demonstrate the calculated reflectivity from TMM and Fourier spectrum of the $G(3,2)$. Figs. 4(b) and 4(d) demonstrate an enlarged section of Figs. 4(a) and 4(c) respectively. Figs. 4(b) and 4(d) show that the appeared peaks have similar contents, but the peak wavelengths in the Fourier spectrum are

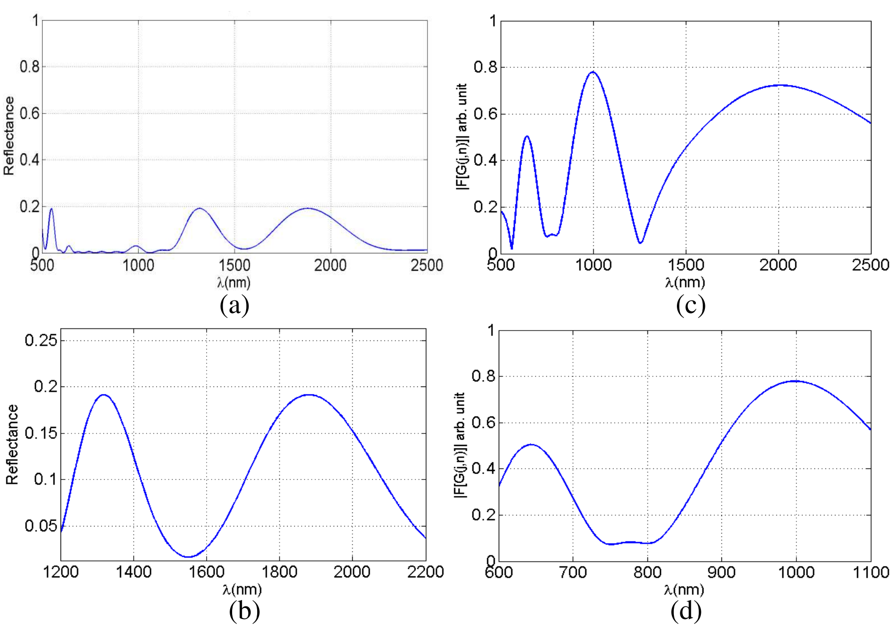

Figure 4. (a) Reflectance, (b) enlarged section of (a), (c) Fourier transform, and (d) enlarged section of (c) for $G(3,2)$.

half of the wavelengths where the peaks of the TMM spectrum are located. This pattern is repeated for the structures, $G(3,3)$, $G(3,5)$ and $G(3,15)$ where the reflectivity calculated using TMM and the spectrum calculated using Fourier transform for these structures are demonstrated in Figs. 5, 6 and 7 respectively. A peak in the reflectivity from TMM calculation at a given wavelength is equivalent to existence of a PBG at that wavelength. Also a peak at the Fourier spectrum of the refractive index of the structure is equivalent to existence of a sinusoidal term in the refractive index of the structure. The Bragg condition for a peak in the Fourier spectrum occurs at twice of that wavelength as we mentioned about periodic structure. A PBG occurs in the structure at wavelength twice while only a peak appears in the Fourier spectrum. This shows Fourier transform of the refractive index is applicable to analysis of reflectance and transmission profile of these structures, usually takes less processing time of the PC and is useful to reach reflectance profile of the large structures in very short time in comparison with TMM method. To have an insight into high order structures, spectral content and reflectance of $G(4,n)$ for $n = 11, 15$ are depicted in Figs. 8 and 9.

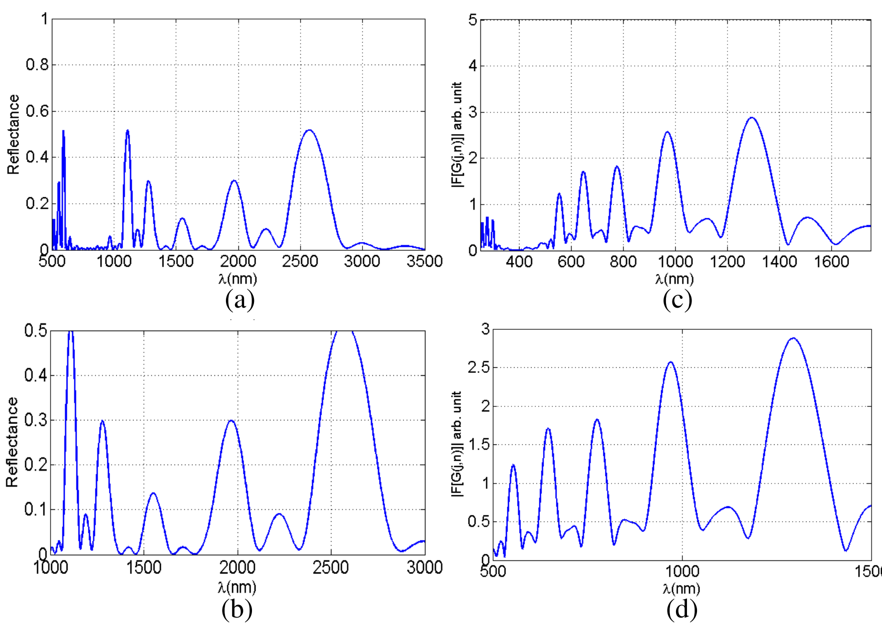

Figure 5. (a) Reflectance, (b) enlarged section of (a), (c) Fourier transform, and (d) enlarged section of (c) for $G(3,3)$.

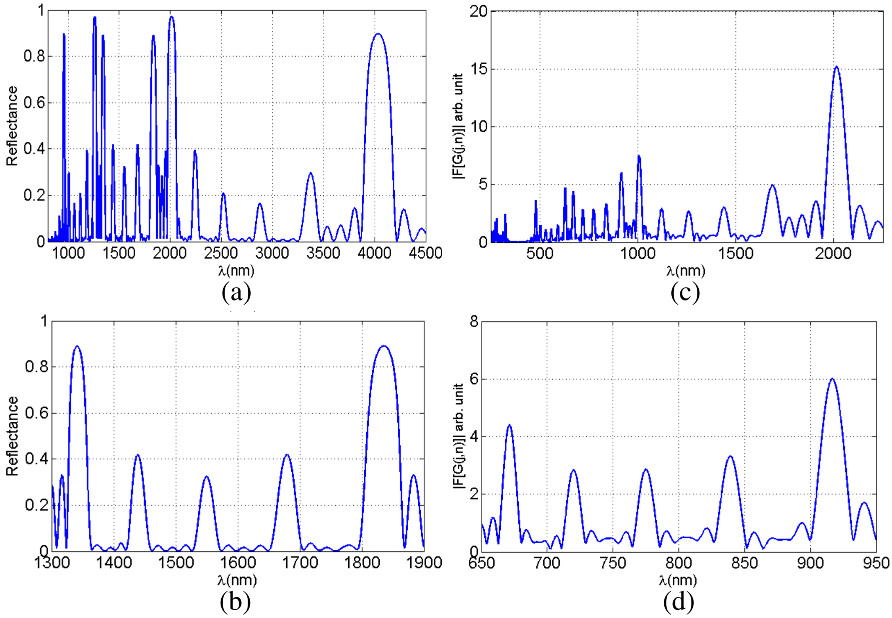

Figure 6. (a) Reflectance, (b) enlarged section of (a), (c) Fourier transform, and (d) enlarged section of (c) for $G(3,5)$.

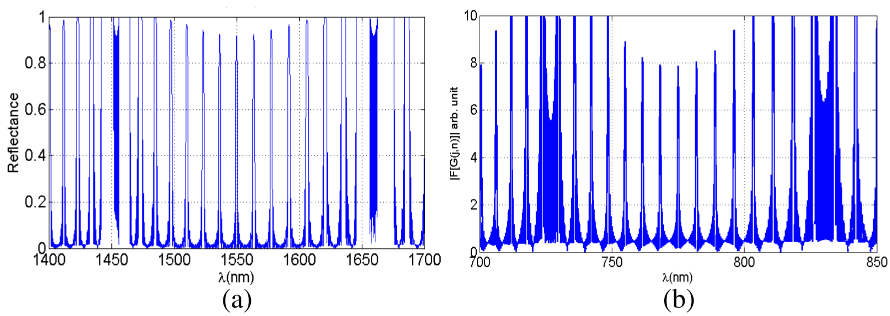

Figure 7. (a) Reflectance and (b) Fourier transform for $G(3,15)$.

Very sharp photonic bandgaps in spectra of these structures reveal the potential for applications in dense wavelength division multiplexing systems (DWDM) such as filtering, interleaving/de-interleaving devices, DWDM dispersion compensation and multi-wavelength narrow linewidth lasers.

We can calculate Fourier transform of the refractive index profile of any multilayer structure using tools such as fast Fourier transform

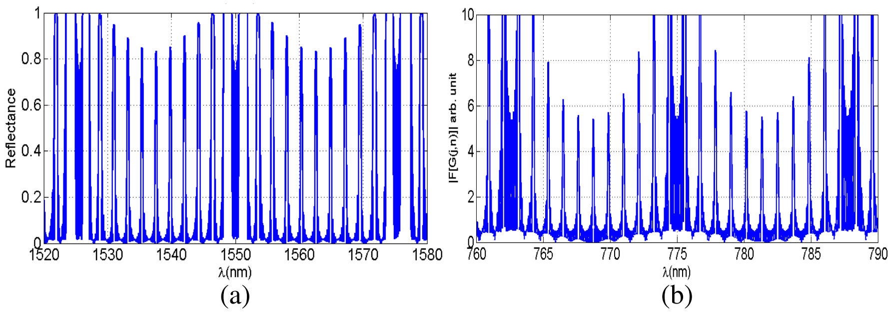

Figure 8. (a) Reflectance and (b) Fourier transform for $G(4, 11)$.

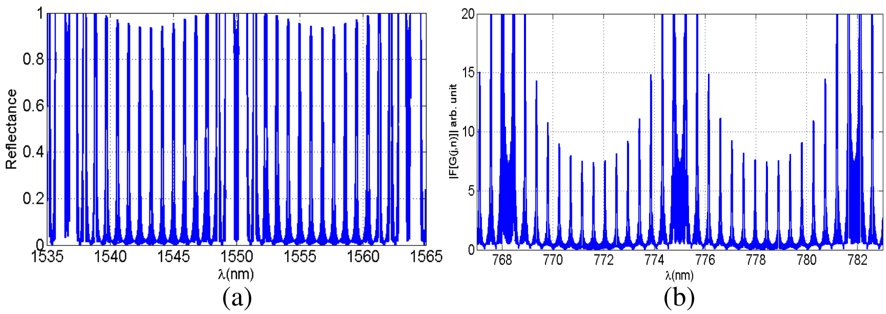

Figure 9. (a) Reflectance and (b) Fourier transform for $G(4, 15)$.

(FFT). From the calculated spectrum, one can determine the peaks of the spectrum and the PBGs of the structure, while the wavelength of one PBG is twice of the wavelength of that peak. Since each PBG is equivalent to a reflective band in optical communication systems, we can design these filters using the FFT spectrum of the refractive index profile. Central wavelengths of multilayer filters are strongly dependent on the structure parameters such as layer thicknesses, refractive indices and arrangement of layers [7]. Effects of physical and geometrical parameters on the central wavelength location, bandwidth and other properties of the multilayer filter can be studied using analysis of Fourier spectrum variations under variation of structure parameters [7].

5. CONCLUSION

In this paper, photonic bandgaps (PBGs) of the quasiperiodic structure have been obtained using the Fourier transform of the refractive index profile. Fourier transform of refractive index profile for generalized

Fibonacci structure has been calculated analytically, and a recursive relation between Fourier transform of these structures with different generation numbers has been introduced. Comparing the reflectivity that has been calculated using TMM with Fourier spectrum that has been calculated using Fourier recursive relation for generalized Fibonacci structure, we have found that a peak in the Fourier spectrum is equivalent to a sinusoidal term in the refractive index. The wavelength of the peak location in the Fourier spectrum is half the wavelength where a PBG is located. So, by analyzing the Fourier transform of the refractive index of any multilayer structure we can determine the location of the PBGs of that structure. Since each PBG is equivalent to a stop band of a filter, we can design multiband filters using FFT spectrum of any given multilayer structure.

REFERENCES

1. Archambault, J. L., L. Reekie, and P. St. J. Russell, "High reflectivity and narrow bandwidth fiber gratings written by a single excimer pulse," *Electron. Lett.*, Vol. 26, 730–731, 1990.
2. Kashyap, R., J. R. Armitage, R. Wyatt, S. T. Davey, and D. L. Williams, "Allfiber narrowband reflection gratings at 1500 nm," *Electron. Lett.*, Vol. 30, 1977–1978, Nov. 1994.
3. Angus Macleod, H., *Thin Film Optical Filter*, 2nd edition, McGraw-Hill, New York, 1989.
4. Yablonovitch, E., "Inhibited spontaneous emission in solid state physics and electronics," *Phys. Rev. Lett.*, Vol. 58, 2059–2062, 1987.
5. ITU-T Recommendation G.698.1, 2005.
6. Packiaraj, D., K. J. Vinoy, and A. T. Kalghatgi, "Analysis and design of a compact multi-layer ultra wide band filter," *Progress In Electromagnetics Research C*, Vol. 7, 111–123, 2009.
7. Golmohammadi, S., M. K. Moravvej-Farshi, A. Rostami, and A. Zarifkar, "Narrowband DWDM filters based on Fibonacci-class quasiperiodic structures," *Opt. Express*, Vol. 15, 10520–10532, 2007.
8. Shechtman, D., I. Blech, D. Gratias, and J. W. Cahn, "Metallic phase with long-range orientational order and no translational symmetry," *Physical Review Letters*, Vol. 53, 1951, 1984.
9. Kohmoto, M., B. Sutherland, and K. Iguchi, "Localization inoptics: Quasiperiodic media," *Physical Review Letters*, Vol. 58, 2436, 1987.
10. Sibilia, C., P. Masciulli, and M. Bertolotti, "Optical properties of

quasiperiodic (self-similar) structures," *Pure Appl. Opt.*, Vol. 7, 383–391, 1998.
11. Gellermann, W., M. Kohmoto, B. Sutherland, and P. C. Taylor, "Localization of light waves in Fibonacci dielectric multilayers," *Physical Review Letters*, Vol. 72, 633, 1994.
12. Macia, E., "Optical engineering with Fibonacci dielectric multilayers," *Applied Physics Letters*, Vol. 73, 3330, 1998.
13. Lusk, D., I. Abdulhalim, and F. Placido, "Omnidirectional reflection from Fibonacci quasiperiodic one-dimensional photonic crystal," *Optics Communications*, Vol. 198, 273, 2001.
14. Peng, R. W., M. Mazzer, X. Q. Huang, F. Qiu, M. Wang, A. Hu, and S. S. Jian, "Symmetry-induced perfect transmission of light waves in quasiperiodic dielectric multilayers," *Applied Physics Letters*, Vol. 80, 3063, 2002.
15. Watanabe, K. and K. Kuto, "Numerical analysis of optical waveguides based on periodic Fourier transform," *Progress In Electromagnetics Research*, PIER 64, 1–21, 2006.
16. Khalaj-Amirhosseini, M., "Analysis of periodic and aperiodic coupled nonuniform transmission lines using the Fourier series expansion," *Progress In Electromagnetics Research*, PIER 65, 15–26, 2006.
17. Golmohammadi, S., M. K. Moravvej-Farshi, A. Rostami, and A. Zarifkar, "Spectral analysis of Fibonacci-class one-dimensional quasi-periodic structures," *Progress In Electromagnetics Research*, PIER 75, 69–84, 2007.
18. Vorob'ev, N. N., *Fibonacci Numbers*, a translation of *Chisla Fibonachchi* by H. Moss, Gostekhteoretizdat, Moscow-Leningrad, 1951.
19. Dumitriu, I., "On generalized Tribonacci sequences and additive partitions," *Disc. Math.*, Vol. 219, 65–83, 2000.
20. Yeh, P., *Optical Waves in Layered Media*, Wiley, New York, 1988.
21. Born, E. and M. Wolf, *Fundamentals of Optics*, 3rd edition, Cambridge University Press, 1999.