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TEMPORAL 1-SOLITON SOLUTION OF THE COMPLEX GINZBURG-LANDAU EQUATION WITH POWER LAW NONLINEARITY

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Abstract—This paper obtains the exact 1-soliton solution of the complex Ginzburg- Landau equation with power law nonlinearity that governs the propagation of solitons through nonlinear optical fibers. The technique that is used to carry out the integration of this equation is He's semi-inverse variational principle.

1. INTRODUCTION

The study of the propagation of solitons through optical fibers has been going on for the past few decades [1–25]. The nonlinear Schrödinger's equation (NLSE) is the main equation that studies this process [1]. There are various variants of this equation that also models this physical phenomena depending on the nonlinear perturbation effects. One such equation is the Radhakrishnan, Kundu, Lakshmanan (RKL) equation [22]. In this paper, the complex Ginzburg-Landau (CGL) equation, with power law nonlinearity, will be studied, which models optical solitons with a few perturbation effects.

In the presence of perturbation effects, the adiabatic parameter dynamics of optical solitons can be obtained also by using various mathematical techniques. They are Variational Principle [1], Soliton Perturbation Theory [9], Collective Variables Method [16] and others. But as a matter of fact, none of these methods can integrate the CGL equation. There has been a newly developed method called the He's Variational Principle (HVP) [10] that can carry out the integration of CGL equation. In this paper, HVP will be employed to carry out

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the integration of the CGL equation with power law nonlinearity. The closed form 1-soliton solution will also enable to obtain the parameter restrictions for the solitons to exist.

2. MATHEMATICAL ANALYSIS

The dimensionless form of the CGL equation, with power law nonlinearity, which will be studied in this paper, is given by [17]

$$\begin{aligned} & iq_t + aq_{xx} + b\left(|q|^{2m}\right)q \\ = & \ \alpha\frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + i\gamma q \end{aligned} \quad (1)$$

where x represents the non-dimensional distance along the fiber while; t represents time in dimensionless form; a, b, α, β and γ are real valued constants. The coefficients of a and b are due to dispersion and power law nonlinearity where the parameter m dictates the power law nonlinearity. The terms due to α, β and γ are from the perturbation effects [10, 17].

Equation (1) is a nonlinear partial differential equation that is not integrable by the classical method of Inverse Scattering Transform since (1) will fail the Painleve test of integrability [1]. However, the HVP will be available for integrating (1), and therefore this technique will be used in this paper to carry out the integration of (1).

The starting point is the hypothesis [10]:

$$q(x,t) = g(x - vt)e^{i(-\kappa x + \omega t + \theta)} \quad (2)$$

where the function g represents the pulse shape, and v is the velocity of the soliton. From the phase component, κ is the soliton frequency; ω is the soliton wave number, while θ is the phase constant. Substituting this hypothesis into (1) and decomposing into real and imaginary parts respectively yield

$$-\omega g + a\left(g'' - \kappa^2 g\right) + bg^{2m+1} = \frac{\alpha}{g}\left\{\left(g'\right)^2 + \kappa^2 g^2\right\} + \beta g'' \quad (3)$$

and

$$g'\left(v + 2a\kappa\right) = -\gamma g \quad (4)$$

where $g' = dg/ds$ and $g'' = d^2g/ds^2$ with

$$s = x - vt \quad (5)$$

Now (4) gives the velocity of the soliton as

$$v = -2a\kappa - \frac{\ln g}{t} \quad (6)$$

Multiplying (3) by g' and integrating, yields

$$-\left\{\omega+\kappa^2(a+\alpha)+\frac{\alpha\gamma^2}{(v+2a\kappa)^2}\right\}\frac{g^2}{2}+(\alpha-\beta)\frac{g'^2}{2}+\frac{bg^{2m+2}}{2m+2}=K \quad (7)$$

where K is the constant of integration. At this point, the quantity J is defined as

$$J=\int_{-\infty}^{\infty}Kds = \int_{-\infty}^{\infty}\left[-\left\{\omega+\kappa^2(a+\alpha)+\frac{\alpha\gamma^2}{(v+2a\kappa)^2}\right\}\frac{g^2}{2}\right.$$
$$\left.+(\alpha-\beta)\frac{g'^2}{2}+\frac{bg^{2m+2}}{2m+2}\right]ds \quad (8)$$

Now, the 1-soliton solution ansatze, given by [10]

$$g(s)=\frac{A}{\cosh^{\frac{1}{m}}(Bs)} \quad (9)$$

is substituted into (8). Here, in (9), the parameters A and B represent the amplitude and inverse width of the soliton respectively. He's semi-inverse variational principle states that the parameters A and B are determined from the solution of the equations [10]

$$\frac{\partial J}{\partial A}=0 \quad (10)$$

and

$$\frac{\partial J}{\partial B}=0. \quad (11)$$

Thus, by virtue of the soliton hypothesis given by (9), Equation (8) reduces to

$$J=\left[-\frac{A^2}{2B}\left\{\omega+\kappa^2(a+\alpha)+\frac{\alpha\gamma^2}{(v+2a\kappa)^2}\right\}\right.$$
$$\left.+\frac{m(\alpha-\beta)A^2B}{2(m+2)}+\frac{bA^{2m+2}}{(m+1)(m+2)B}\right]\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{m}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{2}\right)} \quad (12)$$

From (12), Equations (10) and (11) respectively reduce to

$$\frac{m(\alpha-\beta)B^2}{m+2}+\frac{2bA^{2m}}{m+2}=\omega+\kappa^2(a+\alpha)+\frac{\alpha\gamma^2}{(v+2a\kappa)^2} \quad (13)$$

and

$$\frac{m(\alpha-\beta)B^2}{m+2}-\frac{2bA^{2m}}{(m+1)(m+2)}=-\left\{\omega+\kappa^2(a+\alpha)+\frac{\alpha\gamma^2}{(v+2a\kappa)^2}\right\} \quad (14)$$

After solving (13) and (14) yields

$$A = \left[\frac{m+1}{b(v+2a\kappa)^2} \left\{ \alpha\gamma^2 + \omega(v+2a\kappa)^2 + \kappa^2(a+\alpha)(v+2a\kappa)^2 \right\} \right]^{\frac{1}{2m}} \tag{15}$$

and

$$B = \frac{1}{v+2a\kappa} \left[\frac{\alpha\gamma^2 + \omega(v+2a\kappa)^2 + \kappa^2(a+\alpha)(v+2a\kappa)^2}{\beta - \alpha} \right]^{\frac{1}{2}} \tag{16}$$

Also from (13) and (14), one can obtain the relation between the soliton amplitude A and inverse width B as

$$B = A^m \sqrt{\frac{b}{(m+1)(\beta-\alpha)}} \tag{17}$$

Now, since A and B are constants, Equation (17) also implies that

$$\frac{b}{\beta - \alpha} = \text{constant} > 0 \tag{18}$$

Also, (17) yields the domain restriction as

$$b(\beta - \alpha) > 0 \tag{19}$$

Finally, Equations (15) and (16) respectively imply

$$b \left\{ \alpha\gamma^2 + \omega(v+2a\kappa)^2 + \kappa^2(a+\alpha)(v+2a\kappa)^2 \right\} > 0 \tag{20}$$

and

$$(\beta - \alpha) \left\{ \alpha\gamma^2 + \omega(v+2a\kappa)^2 + \kappa^2(a+\alpha)(v+2a\kappa)^2 \right\} > 0 \tag{21}$$

Thus, finally the 1-soliton solution to (1) is given by

$$q(x,t) = \frac{A}{\cosh^{\frac{1}{m}} [B(x - vt)]} e^{i(-\kappa x + \omega t + \theta)} \tag{22}$$

where the amplitude and width are given by (15) and (16), and the velocity of the soliton is given by (6) while the constraint relation between the soliton parameters are given by (18)–(21).

3. CONCLUSION

This paper obtains the 1-soliton solution of the CGL equation with power law nonlinearity. the HVP is used to carry out the integration. In the process of determining the solution, several constraint relations between the soliton parameters are obtained. In future, this technique can be applied to CGL equation with perturbation terms.

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