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SURFACE WAVE MODES IN CHIRAL NEGATIVE REFRACTION GROUNDED SLAB WAVEGUIDES

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Abstract—The surface wave modes in the chiral negative refraction grounded slab waveguides in which the slab or cladding consists of chiral negative refraction metamaterial are investigated. The dispersion relations, electromagnetic fields, energy flow distribution and the total power of surface wave modes are presented. Some novel features have been found. The energy flow of surface wave mode is in opposite directions in the core and cladding. The total power is negative (corresponds to backward wave) in the grounded chiral negative refraction metamaterial slab waveguides.

1. INTRODUCTION

Grounded slab waveguides have important applications in the area of microwave and millimeter wave and optical wave. Recently, the negative refractive index material (NIM, also called left-handed material (LHM) or double negative (DNG) metamaterial) [1–9] has attracted much attention because NIM has many novel features and potential applications such as "perfect lens" [10], and novel devices [11, 12]. Guided and surface wave modes in the grounded DNG slab waveguides have been investigated intensively [13–18]. Grounded DNG slab waveguides have extraordinary dispersion characteristics. There exists a kind of unique electromagnetic waves termed as "surface waves" whose electromagnetic fields exponentially decay on both sides of the interfaces and cannot exist in conventional grounded waveguides. The grounded DNG slab has a potential as enhanced substrates for planar antennas and arrays with reduced edge-diffraction and mutual coupling between elements [14], or can be used as a bandpass filter [15].

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On the other hand, chiral medium grounded slab waveguides [19–23] and other chirowaveguides [24, 25] have been studied intensively last decades. They also have novel dispersion characteristics. More recently, the chiral medium has been suggested to achieve negative refractive index [26–29]. It has been shown that the chirality parameter can be greater than refraction index at least near the resonant frequency, thus one eigen-wave in the chiral medium becomes a backward wave, making negative refraction in the chiral medium possible [27, 28]. A slab of such negative refraction chiral medium can be used as a "perfect lens", which provides subwavelength resolution for circularly polarized waves [30, 31]. The effective negative indexes of refraction in chiral metamaterials have been experimentally realized at microwave frequencies [32–34] and THz frequencies [35] in 2009. The conditions for the existence of surface polariton modes on the surface of chiral medium or in the chiral slab [36], the novel features of guided modes in chiral negative refraction slab, grounded slab and parallel-plate waveguides [37], the single-mode backward wave in a planar dielectric waveguide with a strong chiral core [38] have been investigated. Chiral nihility metamaterial is a special case of chiral negative refraction medium, in which the permittivity and permeability are simultaneously zero [26, 39]. Waves in the parallel-plate waveguide containing two-layer chiral nihility metamaterials and one air layer [40] and in the chiral nihility metamaterial grounded slab [41–43] have been examined.

In our previous work, novel characteristics of guided modes and surface modes in asymmetric left-handed slab waveguides [44] and in planar chiral nihility metamaterial waveguides [45, 46] have been studied. In this paper, we investigate the features of surface wave modes in the chiral negative refraction grounded slab waveguides. Two cases are considered: the chiral negative refraction metamaterial grounded slab with achiral material cladding and the achiral material grounded slab with the chiral negative refraction metamaterial cladding. The formulas of electromagnetic fields and dispersion relations for surface wave mode are presented, and numerical results are given for typical parameters.

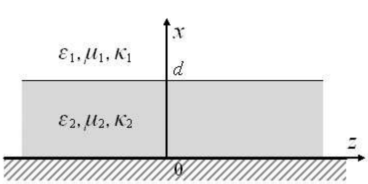

Figure 1. Geometry of the chiral negative refraction grounded slab waveguide.

2. FORMULATIONS

The chiral negative refraction grounded slab waveguide, in which the core or cladding materials are chiral, is shown in Fig. 1. The thickness of the slab is d. The constitutive relations in isotropic chiral media for a time-harmonic field with $\exp(j\omega t)$ are as follows [28]:

$$\mathbf{D} = \varepsilon_i \mathbf{E} - j\kappa_i \sqrt{\mu_0 \varepsilon_0}\, \mathbf{H}, \quad \mathbf{B} = \mu_i \mathbf{H} + j\kappa_i \sqrt{\mu_0 \varepsilon_0}\, \mathbf{E} \tag{1}$$

where $\mu_i, \varepsilon_i, \kappa_i$ $(i = 1, 2)$ are the permeability, permittivity and chirality parameter of chiral media, respectively. When $\kappa_i = 0$, the chiral medium becomes achiral material.

In the chiral media, there exist two eigen-waves, whose effective refractive indexes are $n_{i\pm} = n_i \pm \kappa_i$, where $n_i = \sqrt{\mu_i \varepsilon_i / \mu_0 \varepsilon_0}$; the subscription $+$ and $-$ correspond to right-handed and left-handed circularly polarized eigen-waves, denoted as RCP and LCP. If the chirality parameter is very large $(\kappa_i > n_i)$, then the effective refractive index n_{i-} for the LCP eigen-wave is negative $(n_{i-} < 0)$. We call this type of strong chiral medium as chiral negative refraction medium.

In our previous paper [46], the formulas of dispersion equation, electromagnetic fields of surface wave mode in the planar chiral nihility metamaterial waveguide are presented. Here,we present the formulas of surface wave modes in the chiral negative refraction grounded slab waveguides. The derived procedure is similar to that in [46] and omitted here. It is noted that in the planar chiral nihility metamaterial waveguide, the electromagnetic fields and dispersion equations can be expressed as even or odd mode according to symmetry; however, in the chiral negative refraction grounded slab waveguides, they cannot.

2.1. The Chiral Negative Refraction Metamaterial Grounded Slab with Achiral Material Cladding

The electromagnetic fields for surface wave mode are:

$$\begin{pmatrix} E_{z2} \\ H_{z2} \\ E_{y2} \\ H_{y2} \\ E_{x2} \\ H_{x2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ j/\eta_2 & j/\eta_2 & -j/\eta_2 & -j/\eta_2 \\ k_{2+}/\gamma_{2+} & -k_{2+}/\gamma_{2+} & -k_{2-}/\gamma_{2-} & k_{2-}/\gamma_{2-} \\ jk_{2+}/(\eta_2\gamma_{2+}) & -jk_{2+}/(\eta_2\gamma_{2+}) & jk_{2-}/(\eta_2\gamma_{2-}) & -jk_{2-}/(\eta_2\gamma_{2-}) \\ j\beta/\gamma_{2+} & -j\beta/\gamma_{2+} & j\beta/\gamma_{2-} & -j\beta/\gamma_{2-} \\ -\beta/(\eta_2\gamma_{2+}) & \beta/(\eta_2\gamma_{2+}) & \beta/(\eta_2\gamma_{2-}) & -\beta/(\eta_2\gamma_{2-}) \end{pmatrix}$$

$$\begin{pmatrix} A_+ \exp(\gamma_{2+} x) \\ B_+ \exp(-\gamma_{2+} x) \\ A_- \exp(\gamma_{2-} x) \\ B_- \exp(-\gamma_{2-} x) \end{pmatrix} \qquad (0 \le x \le d) \tag{2a}$$

$$\begin{pmatrix} E_{z1} \\ H_{z1} \\ E_{y1} \\ H_{y1} \\ E_{x1} \\ H_{x1} \end{pmatrix} = \begin{pmatrix} C\exp[-\gamma_1(x-d)] \\ D\exp[-\gamma_1(x-d)] \\ (j\eta_1 k_1/\gamma_1)D\exp[-\gamma_1(x-d)] \\ -[jk_1/(\eta_1\gamma_1)]C\exp[-\gamma_1(x-d)] \\ -(j\beta/\gamma_1)C\exp[-\gamma_1(x-d)] \\ -(j\beta/\gamma_1)D\exp[-\gamma_1(x-d)] \end{pmatrix} \quad (x \geq d) \tag{2b}$$

where $\exp[j(\omega t - \beta z)]$ is omitted for simplicity; β is the longitudinal propagation constant in the waveguide which can be determined by dispersion relation; A_+, B_+, A_-, B_-, C, D are constants whose relationships can be determined by boundary condition; $\gamma_{2\pm} = (\beta^2 - k_{2\pm}^2)^{1/2}$, $k_{2\pm} = k_0(n_2 \pm \kappa_2)$, $\gamma_1 = (\beta^2 - k_1^2)^{1/2}$, $k_1 = n_1 k_0$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $n_i = \sqrt{\mu_i\varepsilon_i/\mu_0\varepsilon_0}$, $\eta_i = \sqrt{\mu_i/\varepsilon_i}$ $(i = 1, 2)$.

The dispersion relation for surface wave mode can be obtained by boundary condition:

$$\begin{aligned} &2\frac{k_{2+}}{\gamma_{2+}}\frac{k_{2-}}{\gamma_{2-}}\frac{k_1}{\gamma_1}\left[\frac{\eta_1^2}{\eta_2^2} - 1 + \left(\frac{\eta_1^2}{\eta_2^2} + 1\right)\cosh v_+ \cosh v_-\right] \\ &+ \frac{k_1}{\gamma_1}\left(\frac{k_{2+}^2}{\gamma_{2+}^2} + \frac{k_{2-}^2}{\gamma_{2-}^2}\right)\left(\frac{\eta_1^2}{\eta_2^2} + 1\right)\sinh v_+ \sinh v_- \\ &+ 2\frac{\eta_1}{\eta_2}\frac{k_{2+}}{\gamma_{2+}}\left(\frac{k_{2-}^2}{\gamma_{2-}^2} + \frac{k_1^2}{\gamma_1^2}\right)\cosh v_+ \sinh v_- \\ &+ 2\frac{\eta_1}{\eta_2}\frac{k_{2-}}{\gamma_{2-}}\left(\frac{k_{2+}^2}{\gamma_{2+}^2} + \frac{k_1^2}{\gamma_1^2}\right)\sinh v_+ \cosh v_- = 0 \end{aligned} \tag{3}$$

where $v_\pm = \gamma_{2\pm} d$.

The energy flow for surface wave mode along the z-axis in the waveguide is defined by

$$S_z = \frac{1}{2}\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot \hat{\mathbf{z}} = \frac{1}{2}\mathrm{Re}(E_x H_y^* - E_y H_x^*) \tag{4}$$

The energy flow in the slab can be expressed as:

$$S_{z2} = S_{z2+} + S_{z2-} \tag{5}$$

where $S_{z2\pm}$ correspond to RCP and LCP waves, respectively:

$$\begin{aligned} S_{z2\pm} = \; & \frac{\beta k_{2\pm}}{\eta_2 \gamma_{2\pm}^2}\mathrm{Re}[|A_\pm|^2 \exp(2\gamma_{2\pm}x) \\ & + |B_\pm|^2 \exp(-2\gamma_{2\pm}x) - (A_\pm B_\pm^* + A_\pm^* B_\pm)] \end{aligned} \tag{6}$$

The energy flow in the cladding is

$$S_{z1} = \frac{\beta k_1}{2\eta_1\gamma_1^2}(|C|^2 + \eta_1^2|D|^2)\exp[-2\gamma_1(x-d)] \tag{7}$$

The normalized total power can be expressed as

$$P = \frac{P_1 + P_2}{|P_1| + |P_2|} \tag{8}$$

where $P_1 = \int_d^\infty S_{z1}dx$ and $P_2 = \int_0^d S_{z2}dx$ are the power in the cladding and slab, respectively.

2.2. The Achiral Material Grounded Slab with Chiral Negative Refraction Metamaterial Cladding

The electromagnetic fields for surface wave mode are:

$$\begin{pmatrix} E_{z2} \\ H_{z2} \\ E_{y2} \\ H_{y2} \\ E_{x2} \\ H_{x2} \end{pmatrix} = \begin{pmatrix} A\sinh(\gamma_2 x) \\ B\cosh(\gamma_2 x) \\ -(j\eta_2 k_2/\gamma_2)B\sinh(\gamma_2 x) \\ [jk_2/(\eta_2\gamma_2)]\,A\cosh(\gamma_2 x) \\ (j\beta/\gamma_2)A\cosh(\gamma_2 x) \\ (j\beta/\gamma_2)B\sinh(\gamma_2 x) \end{pmatrix} \qquad (0 \le x \le d) \tag{9a}$$

$$\begin{pmatrix} E_{z1} \\ H_{z1} \\ E_{y1} \\ H_{y1} \\ E_{x1} \\ H_{x1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ j/\eta_1 & -j/\eta_1 \\ -k_{1+}/\gamma_{1+} & k_{1-}/\gamma_{1-} \\ -jk_{1+}/(\eta_1\gamma_{1+}) & -jk_{1-}/(\eta_1\gamma_{1-}) \\ -j\beta/\gamma_{1+} & -j\beta/\gamma_{1-} \\ \beta/(\eta_1\gamma_{1+}) & -\beta/(\eta_1\gamma_{1-}) \end{pmatrix} \begin{pmatrix} C_+\exp\left[-\gamma_{1+}(x-d)\right] \\ C_-\exp\left[-\gamma_{1-}(x-d)\right] \end{pmatrix}$$
$$(x \ge d) \tag{9b}$$

where A, B, C_+, C_- are constants whose relationships can be determined by boundary condition;

$$\gamma_2 = (\beta^2 - k_2^2)^{1/2}, k_2 = n_2 k_0, \gamma_{1\pm} = (\beta^2 - k_{1\pm}^2)^{1/2}, k_{1\pm} = k_0(n_1 \pm \kappa_1).$$

The dispersion relation for surface wave mode can be obtained by boundary condition:

$$\frac{k_2}{\gamma_2}\left(\frac{k_{1+}}{\gamma_{1+}} + \frac{k_{1-}}{\gamma_{1-}}\right)\left[\frac{\eta_1^2}{\eta_2^2} - 1 + (\frac{\eta_1^2}{\eta_2^2} + 1)\cosh 2v\right]$$
$$+2\frac{\eta_1}{\eta_2}\left[\frac{k_{1+}k_{1-}}{\gamma_{1+}\gamma_{1-}} + \left(\frac{k_2}{\gamma_2}\right)^2\right]\sinh 2v = 0 \tag{10}$$

where $v = \gamma_2 d$.

The energy flow in the slab is

$$S_{z2} = \frac{\beta k_2}{2\eta_1 \gamma_2^2}\left[|A|^2 \cosh^2(\gamma_2 x) + |B|^2 \sinh^2(\gamma_2 x)\right] \tag{11}$$

The energy flow in the cladding can be expressed as:

$$S_{z1} = S_{z1+} + S_{z1-} \tag{12}$$

where

$$S_{z1\pm} = \frac{\beta k_{1\pm}}{\eta_1 \gamma_{1\pm}^2} |C_\pm|^2 \exp[-2\gamma_{1\pm}(x-d)] \tag{13}$$

3. NUMERICAL RESULTS

3.1. The Chiral Negative Refraction Metamaterial Grounded Slab Case

Since chiral negative refraction metamaterial parameters occur near resonances, and they are frequency dependent, here we will confine ourselves to the discussion of the geometric dispersion, i.e., effective refractive index versus reduced slab thickness (or normalized frequency). In the following calculations, we assume $\mu_2 = \mu_1 = \mu_0$, and $\eta_2 = \eta_0/n_2$, $\eta_1 = \eta_0/n_1$, where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance in vacuum.

It is found that the dispersion Equation (3) has solution only if $k_{2+} > k_1 > |k_{2-}|$. Thus, surface wave modes cannot exist in the

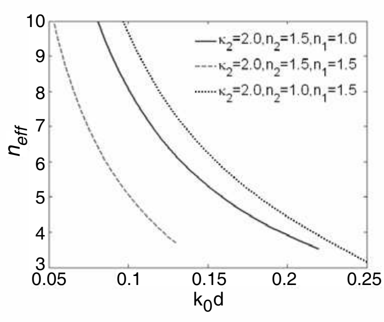

Figure 2. Dispersion curves of surface wave modes in the chiral negative refraction grounded slab waveguides.

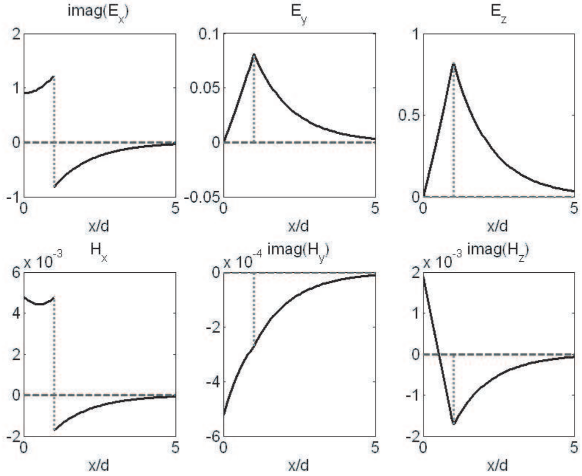

Figure 3. Amplitudes of electromagnetic field components at $k_0d = 0.10$ for surface wave mode with refractive index parameters $n_2 = 1.5, n_1 = 1.0$.

chiral nihility metamaterial (in which $|k_{2+}| = |k_{2-}|$) grounded slab waveguide. We choose $\kappa_2 = 2.0$ to solve the dispersion Equation (3). Fig. 2 shows the dispersion curves of surface wave modes in the chiral negative refraction metamaterial grounded slab waveguides for different parameters, where $n_{eff} = \beta/k_0$ is the effective refractive index, k_0d is the normalized frequency. Solid, dashed and dotted curves correspond to refractive index parameters $n_2 = 1.5, n_1 = 1.0$; $n_2 = 1.5, n_1 = 1.5$; and $n_2 = 1.0, n_1 = 1.5$; respectively. We choose these parameter values in order to ensure that $n_{2-} < 0$ and $k_{2+} > k_1 > |k_{2-}|$. The curves of effective refractive index versus normalized frequency decrease monotonically. When n_{eff} is less than the value of refractive index of RCP n_{2+} in the slab, the surface wave mode becomes guided mode.

Figures 3 and 4 show the amplitudes of electromagnetic field components and the energy flow at normalized frequency $k_0d = 0.10$ for the surface wave mode with refractive index parameters $n_2 = 1.5, n_1 = 1.0$. All electric field components decay exponentially from interface between slab and cladding. S_{z2+} is positive, and S_{z2-} is negative in the slab. However, the absolute S_{z2-} is larger than $S_{z2+,}$, thus S_z is negative in the slab. It indicates that the energy flow of surface wave mode is in opposite directions in the slab and cladding. The normalized total power P versus normalized frequency k_0d is plotted

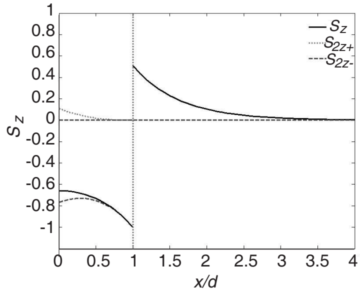

Figure 4. Energy flow at $k_0d = 0.10$ for surface wave mode with refractive index parameters $n_2 = 1.5, n_1 = 1.0$.

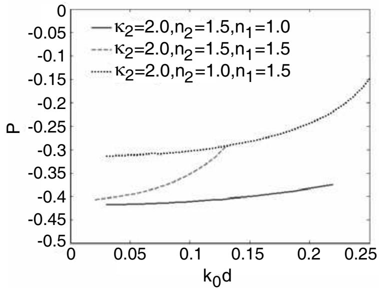

Figure 5. Normalized total power P versus normalized frequency k_0d for surface wave modes in the chiral negative refraction metamaterial grounded slab waveguides.

in Fig. 5 for surface wave modes (correspond to dispersion curves for different parameters in Fig. 2). P is always negative, and the absolute value of P increases as k_0d increases. It implies that the absolute power in the slab is always larger than that in the cladding, and the total power flows opposite to the wave vector propagation direction, thus it is a backward wave.

3.2. The Chiral Negative Refraction Metamaterial Cladding Case

It is found that the dispersion Equation (10) has solution only if $|k_{1\pm}| > k_2$ and $n_1 > n_2$. Thus surface wave modes cannot exist in the chiral nihility metamaterial cladding waveguide, too. We choose $\kappa_1 = 3.2, n_2 = 1.5, n_1 = 1.6$ and $\kappa_1 = 3.8, n_2 = 1.5, n_1 = 2.0$ to solve the dispersion Equation (10). These parameter values satisfy the conditions $n_{1-} < 0$, $|k_{1\pm}| > k_2$ and $n_1 > n_2$.

Figure 6 displays the dispersion curves of surface wave modes in the grounded slab waveguides with chiral negative refraction metamaterial cladding. Solid and dashed curves correspond to parameters $\kappa_1 = 3.2, n_2 = 1.5, n_1 = 1.6$ and $\kappa_1 = 3.8, n_2 = 1.5, n_1 = 2.0$, respectively. The characteristic of dispersion curves is also anomalous and similar to Fig. 2. However, when n_{eff} is less than the value of refractive index of RCP n_{1+} in the cladding, the surface wave mode becomes guided mode.

All electric field components of the surface wave mode decay exponentially from interface between slab and cladding too and are similar to Fig. 3, except that E_y and H_y change sign. Fig. 7 shows the

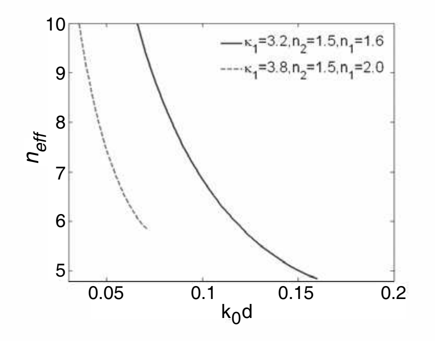

Figure 6. Dispersion curves of surface wave modes in the grounded slab with the chiral negative refraction cladding waveguides.

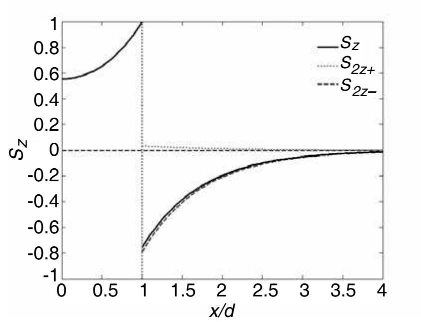

Figure 7. Energy flow at $k_0d = 0.10$ for the surface wave mode with chirality and refractive index parameters $\kappa_1 = 3.2, n_2 = 1.5, n_1 = 1.6$.

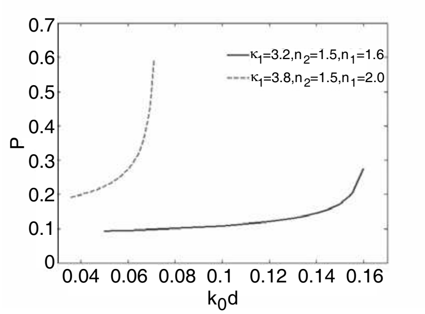

Figure 8. Normalized total power P versus normalized frequency k_0d for surface wave modes in the grounded slab with the chiral negative refraction cladding waveguides.

energy flow at normalized frequency $k_0d = 0.10$ for the surface wave mode with chirality and refractive index parameters $\kappa_1 = 3.2, n_2 = 1.5, n_1 = 1.6$. S_z is positive in the slab and negative in the cladding. The normalized total power P versus normalized frequency k_0d is plotted in Fig. 8 for surface wave modes (correspond to curves for different parameters in Fig. 6). P is always positive. It indicates that the absolute power in the cladding is always smaller than that in the slab, and the total power flows to the wave vector propagation direction, thus it is a forward wave.

4. CONCLUSION

The surface wave modes, whose electromagnetic fields exponentially decay on both sides of the interface between chiral media and achiral materials, in the chiral negative refraction grounded slab waveguides which consist of a chiral slab with an achiral cladding, and a chiral cladding with an achiral slab have been investigated. The dispersion relations, electromagnetic fields, and energy flow of surface wave modes are presented in explicit forms. The dispersion curves, electromagnetic fields distribution, energy flow and the power of surface wave modes in the chiral negative refraction grounded slab waveguides are given for typical parameters. Some novel features such as anomalous dispersion curves, the power flows opposite to the wave vector propagation direction in the chiral negative refraction metamaterial grounded slab waveguide have been found. It must be noted that surface wave modes cannot exist in the chiral nihility metamaterial grounded slab waveguide, which is different from the result in [46] in which surface wave modes can exist in the planar chiral nihility metamaterial waveguide.

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REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Sov. Phys. Usp.*, Vol. 10, No. 4, 509–514, 1968.
2. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, Vol. 84, No. 18, 8184–4187, 2000.
3. Shelby, R. A., D. R. Smith, and S. Shultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, 77–79, 2001.
4. Chen, H., B. I. Wu, and J. A. Kong, "Review of electromagnetic theory in left-handed materials," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 15, 2137–2151, 2006.

5. Grzegorczyk, T. M. and J. A. Kong, "Review of left-handed metamaterials: Evolution from theoretical and numerical studies to potential applications," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 2053–2064, 2006.
6. Ding, W., L. Chen, and C.-H. Liang, "Numerical study of Goos-Hänchen shift on the surface of anisotropic left-handed materials," *Progress In Electromagnetics Research B*, Vol. 2, 151–164, 2008.
7. Tang, W. X., H. Zhao, X. Zhou, J. Y. Chin, and T. J. Cui, "Negative index material composed of meander line and SRRs," *Progress In Electromagnetics Research B*, Vol. 8, 103–114, 2008.
8. Wang, J., S. Qu, H. Ma, Y. Yang, X. Wu, Z. Xu, and M. Hao, "Wide-angle polarization-independent planar left-handed metamaterials based on dielectric resonators," *Progress In Electromagnetics Research B*, Vol. 12, 243–258, 2009.
9. Wang, J., S. Qu, H. Ma, J. Hu, Y. Yang, X. Wu, Z. Xu, and M. Hao, "A dielectric resonator-based route to left-handed metamaterials," *Progress In Electromagnetics Research B*, Vol. 13, 133–150, 2009.
10. Pendry, J. B., "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, 2000.
11. Li, Z. and T. Cui, "Novel waveguide directional couplers using left-handed materials," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1053–1062, 2007.
12. Hsu, H. T. and C. J. Wu, "Design rules for a Fabry-Perot narrow band transmission filter containing a metamaterial negative-index defect," *Progress In Electromagnetics Research Letters*, Vol. 9, 101–107, 2009.
13. Baccarelli, P., P. Burghignoli, G. Lovat, and S. Paulotto, "Surface wave suppression in a double-negative metamaterial grounded slab," *IEEE Antennas Wireless Propag. Lett.*, Vol. 2, 269–272, 2003.
14. Baccarelli, P., P. Burghignoli, F. Frezza, A. Galli, P. Lampariello, G. Lovat, and S. Paulotto, "Fundamental modal properties of surface waves on metamaterial grounded slabs," *IEEE Trans. Microwave Theory Tech.*, Vol. 53, No. 4, 1431–1442, 2005.
15. Bait-Suwailam, M. M. and Z. (D.) Chen, "Surface waves on a grounded double-negative (DNG) slab waveguide," *Microwave Opt. Technol. Lett.*, Vol. 44, No. 6, 494–498, 2005.
16. Mahmoud, S. F. and A. J. Viitanen, "Surface wave character on a slab of metamaterial with negative permittivity and permeability," *Progress In Electromagnetics Research*, PIER 51, 127–137, 2005.

17. Li, C., Q. Sui, and F. Li, "Complex guided wave solutions of grounded dielectric slab made of metamaterials," *Progress In Electromagnetics Research*, PIER 51, 187–195, 2005.
18. Shu, W. and J. M. Song, "Complete mode spectrum of a grounded dielectric slab with double negative metamaterials," *Progress In Electromagnetics Research*, PIER 65, 103–123, 2006.
19. Paiva, C. R. and A. M. Barbosa, "A method for the analysis of biisotropic planar waveguides — Application to a grounded chiroslabguide," *Electromagnetics*, Vol. 11, No. 1, 209–221, 1991.
20. Mariotte, F., P. Pelet, and N. Engheta, "A review of recent study of guided waves in chiral media," *Progress In Electromagnetics Research*, PIER 09, 311–350, 1994.
21. Cory, H., "Chiral devices — An overview of canonical problems," *Journal of Electromagnetic Waves and Applications*, Vol. 9, Nos. 5–6, 805–829, 1995.
22. Zhou, H. and K. Zhang, "Analysis of a metal-coated planar dielectric waveguide with chiral cladding," *Int. J. Infrared Millim. Waves*, Vol. 20, No. 11, 1977–1987, 1999.
23. Zhou, H. and K. Zhang, "Mode characteristics in a metal-coated planar chiral dielectric waveguide," *Microwave Opt. Technol. Lett.*, Vol. 26, No. 2, 137–140, 2000.
24. Hussain A., M. Faryad, and Q. A. Naqvi, "Fractional curl operator and fractional chiro-waveguide," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1119–1129, 2007.
25. Nair, A. and P. K. Choudhury, "On the analysis of field patterns in chirofibers," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2277–2286, 2007.
26. Tretyakov, S., I. Nefedov, A. Sihvola, S. Maslovski, and C. Simovski, "Waves and energy in chiral nihility," *Journal of Electromagnetic Waves and Applications*, Vol. 17, 695–706, 2003.
27. Pendry, J. B., "A chiral route to negative refraction," *Science*, Vol. 306, 1353–1355, 2004.
28. Tretyakov, S., A. Sihvola, and L. Jylhä, "Backward-wave regime and negative refraction in chiral composites," *Photonics and Nanostructures*, Vol. 3, Nos. 2–3, 107–115, 2005.
29. Faryad, M. and Q. A. Naqvi, "Cylindrical reflector in chiral medium supporting simultaneously positive phase velocity and negative phase velocity," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 4, 563–572, 2008.
30. Monzon, C. and D. W. Forester, "Negative refraction and focusing of circularly polarized waves in optically active media," *Phys. Rev.*

Lett., Vol. 95, 123904, 2005.
31. Jin, Y. and S. He, "Focusing by a slab of chiral medium," *Optics Express*, Vol. 13, No. 13, 4974–4979, 2005.
32. Plum, E., J. Zhou, J. Dong, V. A. Fedotov, T. Koschny, C. M. Soukoulis, and N. I. Zheludev, "Metamaterial with negative index due to chirality," *Phys. Rev. B*, Vol. 79, 035407, 2009.
33. Zhou, J., J. Dong, B. Wang, T. Koschny, M. Kafesaki, and C. M. Soukoulis, "Negative refractive index due to chirality," *Phys. Rev. B*, Vol. 79, 121104 (R), 2009.
34. Wang, B., J. Zhou, T. Koschny, and C. M. Soukoulis, "Nonplanar chiral metamaterials with negative index," *Appl. Phys. Lett.*, Vol. 94, 151112, 2009.
35. Zhang, S., Y. Park, J. Li, X. Lu, W. Zhang, and X. Zhang, "Negative refractive index in chiral metamaterials," *Phys. Rev. Lett.*, Vol. 102, 023901, 2009.
36. Jin, Y., J. He, and S. He, "Surface polaritons and slow propagation related to chiral media supporting backward waves," *Phys. Lett. A*, Vol. 351, 354–358, 2006.
37. Dong, J. F., Z. J. Wang, L. L. Wang, and B. Liu, "Novel characteristics of guided modes in chiral negative refraction waveguides," *Proceedings of International Symposium on Biophotonics, Nanophotonics and Metamaterials, Metamaterials 2006*, 517–520, Oct. 2006.
38. Zhang, C. and T. J. Cui, "Chiral planar waveguide for guiding single-mode backward wave," *Opt. Commun.*, Vol. 280, No. 2, 359–363, 2007.
39. Naqvi, Q. A., "Fractional dual interface in chiral nihility medium," *Progress In Electromagnetics Research Letters*, Vol. 8, 135–142, 2009.
40. Cheng, Q. and C. Zhang, "Waves in planar waveguide containing chiral nihility metamaterial," *Opt. Commun.*, Vol. 276, No. 2, 317–321, 2007.
41. Naqvi, Q. A., "Planar slab of chiral nihility metamaterial backed by fractional dual/PEMC interface," *Progress In Electromagnetics Research*, PIER 85, 381–391, 2008.
42. Naqvi, Q. A., "Fractional dual solutions in grounded chiral nihility slab and their effect on outside field," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 5–6, 773–784, 2009.
43. Illahi, A. and Q. A. Naqvi, "Study of focusing of electromagnetic waves reflected by a PEMC backed chiral nihility reflector using Maslov's method," *Journal of Electromagnetic Waves and*

Applications, Vol. 23, No. 7, 863–873, 2009.

44. Wang, Z. J. and J. F. Dong, "Analysis of guided modes in asymmetric left-handed slab waveguides," *Progress In Electromagnetics Research*, PIER 62, 203–215, 2006.
45. Dong, J. and C. Xu, "Characteristics of guided modes in planar chiral nihility meta-material waveguides," *Progress In Electromagnetics Research B*, Vol. 14, 107–126, 2009.
46. Dong, J. F. and C. Xu, "Surface polaritons in planar chiral nihility meta-material waveguides," *Opt. Commun.*, 2009, doi:10.1016/j.optcom.2009.06.054.