Progress In Electromagnetics Research B, Vol. 16, 291–309, 2009

FOCUSING OF DARK HOLLOW GAUSSIAN ELECTROMAGNETIC BEAMS IN A PLASMA WITH RELATIVISTIC-PONDEROMOTIVE REGIME

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Abstract—This paper presents a theoretical model for the propagation of a hollow Gaussian electromagnetic beam [HGB], propagating in a plasma with dominant relativistic-ponderomotive nonlinearity. A paraxial like approach has been invoked to understand the nature of propagation; in this approach all the relevant parameters correspond to a narrow range around the irradiance maximum of the HGB. The critical curves for the propagation of various order HGBs have been discussed, and the dependence of the beam width parameter on distance of propagation has been evaluated for three typical cases *viz.* of steady divergence, oscillatory divergence and self focusing of the HGB.

1. INTRODUCTION

Among many of the nonlinear processes [1–15] in the laser-plasma interaction the phenomenon of self focusing [1–6] is of significant interest on account of the fact that the non-linear effects are highly sensitive to the irradiance distribution along the wavefront of the beam, which is significantly affected by self focusing. Further, the introduction of ultra high power laser [16] has led to many theoretical and experimental studies in the inertial confinement fusion (ICF) [17–20], charged particle acceleration [21–23] and ionospheric modification [24–27]. In many of such studies the ponderomotive as well as the relativistic nonlinearities have to be considered. Pukhov and Meyer-ter-Vehn [28] in their investigation proposed a three dimensional simulation model for short laser pulse propagation in a plasma with dominant relativistic nonlinearity and found that the incident laser beam creates a single propagation channel with considerably enhanced

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irradiance on the axis; however this work does not take cognizance of the role of the ponderomotive forces in the self focusing process.

Most of these investigations [29–32] are characterized by considering only the relativistic nonlinearity, while different kinds of nonlinearities are in fact operative, depending on the time scale of the pulse *viz.* (i) $\tau_{pe} \approx \tau$ and (ii) $\tau \approx \tau_{pi}$ where τ is the pulse duration, τ_{pi} is the ion plasma period and τ_{pe} is the electron plasma period. Case (i) corresponds to dominant relativistic nonlinearity while case (ii) refers to the situation when the ponderomotive [33] and relativistic [34] nonlinearities are simultaneously operative. In such a case the nonlinearity in the dielectric function occurs is caused by the electron mass variation due to large laser irradiance and the change in electron density as a consequence of the ponderomotive force. Very few studies on self focusing [35, 36] and cross focusing [37] of the laser beams have been made, incorporating the combined effect of relativistic and ponderomotive nonlinearities. Further the effect of an ultra intense laser pulse on the propagation of an electron plasma wave has been analyzed by Kumar et al. [38] in the relativistic-ponderomotive regime.

Most of the analyses on self focusing of the laser beams are devoted to the Gaussian beams [3–5, 33–43]; nevertheless a few studies have also been published on the self focusing of super Gaussian beams [44–46], self trapping of degenerate modes of laser beams [47], and self trapping of Bessel beams [48] by considering the nature of different irradiance distributions of the beams. Apart from these investigations, recently the optical beams with central shadow, usually known as dark hollow beams (DHB) have received much attention from the physics community because of their wide and attractive applications in the field of modern optics, atomic optics and plasmas [49–52]; numerous experimental techniques [53–55] have also been developed for the production of the DHBs. Further for the explanation of the dynamics and other propagation characteristics, several theoretical models for DHBs [the beam with zero central intensity] like the TEM_{01} mode doughnut beam, some higher order Bessel beams, superposition of off-axis Gaussian beams and dark-hollow Gaussian beams etc. have been introduced [56–59]. In relatively recent studies the propagation of DHBs in paraxial optical systems [60] and turbulent atmospheres [61] has been discussed in detail.

A look at the available literature reveals the fact that the interaction of DHBs with nonlinear media and plasmas has not been investigated significantly; as an exception the propagation of doughnut (TEM_{01}) beam in a plasma for regions around the axis and the maximum of irradiance, in the geometrical optics approximation [62, 63], has been investigated to some extent. Further a modified theory [64] for propagation of TEM_{01} mode of the beam, considering diffraction and the saturating nature of the non-linearity, has been developed. In a recent investigation Sodha et al. [65, 66] have presented a modified paraxial-like approach, similar to the one given by Akhmanov et al. [3] and developed by Sodha et al. [4, 5], to analyze the propagation characteristics of a hollow Gaussian beam in the vicinity of its irradiance maximum in the plasma by taking note of the saturating character of the nonlinearities. However, all the three basic nonlinearities of the plasma (i.e., ponderomotive, collisional and relativistic) have been analyzed separately to a significant extent but their combined effect has not been discussed in the context of the HGB. The present paper explores the propagation of a dark cylindrical hollow Gaussian electromagnetic beam (with a zero axial irradiance) in a plasma characterized by taking into account the relativistic-ponderomotive nonlinearity. It should however be realized that some interesting effects [67, 68] predicted by detailed numerical simulation like breaking up into a number of beams can not be recovered in the cylindrical geometry; hence the theory has some limitations, particularly for beams with powers above the critical value. However since cylindrical beams are commonly used, a theory for cylindrical beams (even approximate) is in order.

The present work is based on the modified approach followed by Sodha et al. [65, 66] and represents the extension of the theory to plasmas in which the relativistic and ponderomotive nonlinearities are operating simultaneously. Thus this investigation is inclusive of the following considerations:

(i) The diffraction term derived in the present analysis is appropriate for the vicinity of the maximum of the irradiance of the hollow Gaussian beam, occurring away from the central axis ($r = 0$).

(ii) All the relevant parameters have been expanded in terms of the radial distance from the maximum of the irradiance of the hollow Gaussian beam, which lies away from the axis $r = 0$.

(iii) The plasma is electrically neutral everywhere.

(iv) The pulse duration of the laser τ has been chosen so that both the nonlinearities *viz.* relativistic and ponderomotive are simultaneously operative.

This paper investigates some interesting aspects associated with the propagation of the various order HGBs in a paraxial like approximation and the results are appreciated through the critical curves and the dependence of the beam width parameter on various factors. The results have been discussed in Section 4 and a short summary of the investigation in Section 5 concludes the paper.

2. FOCUSING OF HOLLOW GAUSSIAN BEAM (HGB)

2.1. Propagation

Consider the propagation of a linearly polarized hollow Gaussian beamwith its electric vector polarized along the y-axis, propagating in a homogeneous plasma along the z-axis. In the steady state the electric field vector $\mathbf{E}$ for such a beam may be expressed in a cylindrical coordinate system with azimuthal symmetry as

$$\mathbf{E} = \hat{j} E_0(r, z) \exp(i\omega t), \tag{1}$$

where

$$(E_0)_{z=0} = E_{00} \left(\frac{r^2}{2r_0^2}\right)^n \exp\left(-\frac{r^2}{2r_0^2}\right), \tag{2}$$

E_0 refers to the complex amplitude of the hollow Gaussian beam of initial beam width r_0, E_{00} is a real constant characterizing the amplitude of the HGB, n is the order of the HGB and a positive integer, characterizing the shape of the HGB and position of its irradiance maximum, ω is the wave frequency, $\hat{j}$ is the unit vector along the y axis and E_{00} denotes the electric field maximum at $r = r_{\max} = r_0\sqrt{2n}$, corresponding to $z = 0$. For $n = 0$, Eq. (2) represents a fundamental Gaussian beam of width r_0; however, the interest of the present investigation lies in higher order HGBs [i.e., $n > 0$].

The electric field vector $\mathbf{E}$ satisfies the wave equation (stationary frame),

$$\nabla^2\mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \frac{\varepsilon(r, z)}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0, \tag{3}$$

where ε is the effective dielectric function of the plasma and c is the speed of light in free space.

For transverse beams, the second term of left hand side of Eq. (3) is zero. One can thus write the wave equation for the electromagnetic beam, as

$$\nabla^2\mathbf{E}_0 + (\omega^2/c^2)\varepsilon(r, z)\mathbf{E}_0 = 0. \tag{4}$$

Following Akhmanov et al. [3] and Sodha et al. [4, 5] the solution of Eq. (4) can be chosen as

$$\mathbf{E}_0(r, z) = jA(r, z) \exp\left(-i \int k(z)dz\right), \tag{5}$$

where $A(r, z)$ is a complex parameter, $k(z) = \frac{\omega}{c}\sqrt{\varepsilon_0(z)}$ and $\varepsilon_0(z)$ is the dielectric function, corresponding to the maximum electric field on the wavefront of the HGB [see Eq. (11)].

Substituting for $\mathbf{E}_0(r,z)$ from Eq. (5) in Eq. (4) and neglecting the term $(\partial^2 A/\partial z^2)$ (assuming $A(r,z)$ to be a slowly varying function of z), one obtains

$$2ik\frac{\partial A}{\partial z}+iA\frac{\partial k}{\partial z}=\left(\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}\right)+\frac{\omega^2}{c^2}(\varepsilon-\varepsilon_0). \tag{6}$$

The complex amplitude $A(r,z)$ may be expressed as,

$$A(r,z)=A_0(r,z)\exp[-ik(z)S(r,z)], \tag{7}$$

where $S(r,z)$ is termed the eikonal associated with the hollow Gaussian beam; both A_0 and S are real parameters.

Substitution for $A(r,z)$ from Eq. (7) in Eq. (6) and the separation of the real and imaginary parts, yields

$$\frac{2S}{k}\frac{\partial k}{\partial z}+2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)+\frac{\omega^2}{c^2}(\varepsilon-\varepsilon_0) \tag{8a}$$

and

$$\frac{\partial A_0^2}{\partial z}+A_0^2\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)+\frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r}+\frac{A_0^2}{k}\frac{\partial k}{\partial z}=0. \tag{8b}$$

To proceed further one can adopt an approach, analogous to the paraxial approximation. Thus one may start by expressing Eqs. (8a) and (8b) in terms of variables η and z, where

$$\eta=\left[(r/r_0f)-\sqrt{2n}\right], \tag{9}$$

$r_0f(z)$ is the width of the beam and $r=r_0f\sqrt{2n}$ is the position of the maximum irradiance for the propagating beam; it is shown later that in the paraxial like approximation, i.e., when $\eta\ll\sqrt{2n}$, Eqs. (8a) and (8b) lead to the maintenance of the HGB character during propagation. Since the irradiance of the beam is a function of r and z only, expansions of expressions for relevant parameters made along r, near the irradiance maximum *viz.* $r=r_0f(z)\sqrt{2n}$, are certainly justified in the paraxial like approximation; for $n=0$ (Gaussian beam), the expansion is made (like wise) around $r=0$ (as usual). Like the paraxial theory, the present analysis is strictly applicable when $\eta\ll\sqrt{2n}$.

The transformation [Eq. (9)] leads to,

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial z}-\frac{\left(\sqrt{2n}+\eta\right)}{f}\frac{df}{dz}\frac{\partial}{\partial\eta} \tag{9a}$$

and

$$\frac{\partial}{\partial r}=\frac{1}{r_0f}\frac{\partial}{\partial\eta} \tag{9b}$$

Thus with the help of Eqs. (9a) and (9b), the set of focusing equations [i.e., Eqs. (8a) and (8b)] may in terms of variables (η, z) be expressed as,

$$\frac{2S}{k}\frac{\partial k}{\partial z}+2\left(\frac{\partial S}{\partial z}-\frac{\left(\sqrt{2n}+\eta\right)}{f}\frac{df}{dz}\frac{\partial S}{\partial \eta}\right)+\frac{1}{r_0^2 f^2}\left(\frac{\partial S}{\partial \eta}\right)^2$$
$$=\frac{1}{k^2 A_0 r_0^2 f^2}\left(\frac{\partial^2 A_0}{\partial \eta^2}+\frac{1}{\left(\sqrt{2n}+\eta\right)}\frac{\partial A_0}{\partial \eta}\right)+\frac{\omega^2}{c^2}(\varepsilon-\varepsilon_0) \quad (10a)$$

and

$$\left(\frac{\partial A_0^2}{\partial z}-\frac{\left(\sqrt{2n}+\eta\right)}{f}\frac{df}{dz}\frac{\partial A_0^2}{\partial \eta}\right)+\frac{A_0^2}{r_0^2 f^2}\left(\frac{\partial^2 S}{\partial \eta^2}+\frac{1}{\left(\sqrt{2n}+\eta\right)}\frac{\partial S}{\partial \eta}\right)$$
$$+\frac{1}{r_0^2 f^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S}{\partial \eta}+\frac{A_0^2}{k}\frac{\partial k}{\partial z}=0. \quad (10b)$$

In the paraxial like approximation the relevant parameters (i.e., the dielectric function $\varepsilon(r, z)$, eikonal and irradiance) may be expanded around the maximum of the HGB, i.e., around $\eta = 0$. Thus one can express the dielectric function $\varepsilon(\eta,\, z)$ around the maximum ($\eta = 0$) of the HGB as

$$\varepsilon(\eta,\, z)=\varepsilon_0(z)-\eta^2\varepsilon_2(z), \quad (11)$$

where $\varepsilon_0(z)$ and $\varepsilon_2(z)$ are the coefficients associated with η^0 and η^2 in the expansion of $\varepsilon(\eta,\, z)$ around $\eta = 0$. The expressions for these coefficients have been derived later.

Substitution for $\varepsilon(\eta,\, z)$ from Eq. (11) in Eqs. (10a) and (10b) leads to

$$\frac{2S}{k}\frac{\partial k}{\partial z}+2\left(\frac{\partial S}{\partial z}-\frac{\left(\sqrt{2n}+\eta\right)}{f}\frac{df}{dz}\frac{\partial S}{\partial \eta}\right)+\frac{1}{r_0^2 f^2}\left(\frac{\partial S}{\partial \eta}\right)^2$$
$$=\frac{1}{k^2 A_0 r_0^2 f^2}\left(\frac{\partial^2 A_0}{\partial \eta^2}+\frac{1}{\left(\sqrt{2n}+\eta\right)}\frac{\partial A_0}{\partial \eta}\right)-\eta^2\frac{\omega^2}{k^2c^2}\varepsilon_2 \quad (12a)$$

and

$$\left(\frac{\partial A_0^2}{\partial z}-\frac{\left(\sqrt{2n}+\eta\right)}{f}\frac{df}{dz}\frac{\partial A_0^2}{\partial \eta}\right)+\frac{A_0^2}{r_0^2 f^2}\left(\frac{\partial^2 S}{\partial \eta^2}+\frac{1}{\left(\sqrt{2n}+\eta\right)}\frac{\partial S}{\partial \eta}\right)$$
$$+\frac{1}{r_0^2 f^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S}{\partial \eta}+\frac{A_0^2}{k}\frac{\partial k}{\partial z}=0. \quad (12b)$$

In the paraxial like approximation, one can express the solution of Eq. (12b) as

$$A_0^2 = \frac{E_0^2}{2^{2n} f^2}\left(\sqrt{2n}+\eta\right)^{4n}\exp[-\left(\sqrt{2n}+\eta\right)^2], \tag{13a}$$

where

$$S(\eta,\, z) = \frac{\left(\sqrt{2n}+\eta\right)^2}{2}\beta(z)+\varphi(z); \tag{13b}$$

one has

$$\frac{\partial A_0^2}{\partial z} = -\frac{2}{f}\frac{df}{dz}A_0^2, \tag{13c}$$

$$\frac{\partial A_0^2}{\partial \eta} = \left[\frac{4n}{\left(\sqrt{2n}+\eta\right)} - 2\left(\sqrt{2n}+\eta\right)\right]A_0^2, \tag{13d}$$

$$\frac{\partial S}{\partial z} = \frac{\left(\sqrt{2n}+\eta\right)^2}{2}\frac{d\beta}{dz}+\frac{d\varphi}{dz}, \tag{13e}$$

$$\frac{\partial S}{\partial \eta} = \left(\sqrt{2n}+\eta\right)\beta, \tag{13f}$$

$$\beta(z) = r_0^2 f\frac{df}{dz}, \tag{13g}$$

$$E_0^2 = E_{00}^2\left(\frac{k(0)}{k(z)}\right) = E_{00}^2\left(\frac{\varepsilon_0(0)}{\varepsilon_0(z)}\right)^{1/2}, \tag{13h}$$

$\varphi(z)$ is a function of z, and $f(z)$ is the beam width parameter for the HGB.

Most of the power of the beam is concentrated in the region around $\eta = 0$. There is certainly some power of the beam beyond this limitation, which is accounted for in an approximate manner by Eq. (13a), which in common with the variational and moment approaches, assures that the nature of r dependence of irradiance does not change with propagation. Eq. (13a) also ensures conservation of power as the beam propagates.

On substituting for A_0^2 and S and their derivatives from the set of Eqs. (13a) to (13h) in Eq. (12a) and equating the coefficients of η^0 and η^2 on both sides of the resulting equation, one obtains

$$\varepsilon_0 f\frac{d^2 f}{d\xi^2} = \left(\frac{4}{f^2}-\rho_0^2\varepsilon_2\right)-\frac{1}{2}f\frac{df}{d\xi}\frac{d\varepsilon_0}{d\xi} \tag{14a}$$

and

$$\frac{1}{\varepsilon_0}\left(\left(nf\frac{df}{d\xi}+\Phi\right)\frac{d\varepsilon_0}{d\xi}+\frac{2}{f^2}\right)+2nf\frac{d^2 f}{d\xi^2}+2\frac{d\Phi}{d\xi}=0, \tag{14b}$$

where $\xi = (c/r_0^2\omega)z$ is the dimensionless distance of propagation, $\rho_0 = (r_0\omega/c)$ is the dimensionless initial beam width, and $\Phi = (\omega/c)\varphi$ is the dimensionless function associated with the eikonal of the HGB.

It may be noted that Eq. (12a) has a term proportional to η on the left hand side but none on the right hand side, giving an absurd result that the double derivative of the beam width parameter $f(z)$ equals zero. This implies that ideally the eikonal should also have an odd power term in η, which introduces complications in mathematics. However, such an asymmetry will not significantly influence the focusing of the maximum irradiance of the HGB; hence its neglect is reasonable.

The dependence of the beam width parameter f on the dimensionless distance of propagation ξ can be obtained by the numerical integration of Eq. (14a) after putting suitable expressions for ε_0 and ε_2, and using the initial boundary conditions $f = 1$, $(df/d\xi) = 0$ at $\xi = 0$; Φ is obtained by simultaneous solution of Eqs. (14a) and (14b), taking the additional boundary condition $\Phi = 0$, at $\xi = 0$ into account.

2.2. Dielectric Function

Following Sodha et al. [5], the effective dielectric function of the plasma can be expressed as

$$\varepsilon(r, z) = 1 - \Omega^2(N_{0e}/N_0), \tag{15}$$

where $\Omega = (\omega_{pe}/\omega)$, $\omega_{pe} = (4\pi N_0 e^2/m)^{1/2}$ is the electron plasma frequency, N_0 is the undisturbed electron density of the plasma, N_{0e} is the electron density of the plasma in the presence of the electromagnetic field, m is the mass of the electron and e is the electronic charge.

Following the paraxial like approximation one can expand the dielectric function $\varepsilon(\eta, z)$ in axial and radial parts around the maximum of the ring ripple ($\eta = 0$). Thus one obtains from Eq. (11) and Eq. (15),

$$\varepsilon_0(z) = \varepsilon(\eta, z)_{\eta=0}, \tag{16}$$

$$\text{and} \quad \varepsilon_2(z) = -\left(\frac{\partial\varepsilon(\eta, z)}{\partial\eta^2}\right)_{\eta=0}. \tag{17}$$

2.3. Evaluation of the Effective Dielectric Function

The present study considers a plasma, characterized by simultaneously operative relativistic and ponderomotive nonlinearities, caused by the relativistic change in the mass of electron and the modification of the background electron density due to ponderomotive nonlinearity. The relativistic ponderomotive force on an electron in the presence of an intense electromagnetic beam may be represented as [35, 69] as,

$$F_p = -m_0 c^2 \nabla(\gamma - 1) \tag{18}$$

where γ is the relativistic factor given by

$$\gamma = [1 + (e^2/m_0^2 c^2 \omega^2) E E^*]^{1/2} = [1 + \alpha E E^*]^{1/2} \tag{19}$$

and $\alpha = (e^2/m_0^2 c^2 \omega^2)$.

Using the electron continuity equation and current density equation the second order correction in the electron density equation i.e., N_2, can with the help of Eq. (18) be written as [35],

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_{p0}^2}{\gamma}\right) N_2 = -\nabla(N_0 F_p / m_0 \gamma) + K,$$

where K is a corrective term, given by

$$K = \nabla \left(\frac{N_2}{\gamma}\frac{\partial \gamma}{\partial t} + \frac{\partial}{\partial t}\right) v.$$

In the steady state the expression reduces to

$$\begin{aligned} N_2 &= \frac{\gamma}{\omega_{p0}^2} \nabla(N_0 F_p / m_0 \gamma) \\ \text{or} \quad N_2 &= \frac{c^2 N_0}{\omega_{p0}^2}\left(\nabla^2 \gamma - \frac{(\nabla \gamma)^2}{\gamma}\right). \end{aligned}$$

Thus the total electron density may be represented by [35]

$$\begin{aligned} N_{0e} &= N_0 + N_2 = N_0 + (c^2 N_0/\omega_{p0}^2)\left(\nabla^2 \gamma - \frac{(\nabla \gamma)^2}{\gamma}\right) \\ \text{or,} \quad (N_{0e}/N_0) &= 1 + (c^2/\omega_{p0}^2)\left(\nabla^2 \gamma - \frac{(\nabla \gamma)^2}{\gamma}\right) \end{aligned} \tag{20}$$

The effective dielectric function in the case of relativistic ponderomotive nonlinearity may be given by

$$\varepsilon(r, z) = 1 - \Omega_0^2 (N_{0e}/\gamma N_0) \tag{21}$$

where $\Omega_0 = (\omega_{p0}/\omega)$, $\omega_{p0} = (4\pi N_0 e^2/m_0)^{1/2}$ and m_0 is the rest mass of the electron.

Using Eq. (20), Eq. (21) reduces to the form

$$\begin{aligned} \varepsilon(r, z) &= 1 - (\Omega_0^2/\gamma)\left[1 + (c^2/\omega_{p0}^2)\left(\nabla^2 \gamma - \frac{(\nabla \gamma)^2}{\gamma}\right)\right] \\ &= 1 - (\Omega_0^2/\gamma) - (c^2/\omega^2)\nabla\left(\frac{\nabla \gamma}{\gamma}\right). \end{aligned} \tag{22}$$

For further algebraic analysis, it is convenient to expand the solution for A_0^2 as a polynomial in η^2; thus

$$A_0^2 = g_0 + g_2\eta^2, \tag{23}$$

where

$$g_0 = \frac{E_0^2}{f^2} n^{2n} \exp[-2n], \tag{24a}$$

$$g_2 = -2g_0 = -\frac{2E_0^2}{f^2} n^{2n} \exp[-2n], \tag{24b}$$

Following the paraxial like approximation one can expand the dielectric function $\varepsilon(\eta, z)$ in axial and radial parts around the maximum of the HGB ($\eta = 0$). Thus from the set of Eqs. (11), (22) and (24), one obtains

$$\varepsilon_0(z) = 1 - \left(\frac{\Omega_0^2}{(1+g_0)^{1/2}}\right) + \frac{1}{\rho^2 f^2}\left[\frac{2g_0}{(1+g_0)}\right] \tag{25}$$

and

$$\varepsilon_2(z) = -\left(\frac{\Omega_0^2}{2(1+g_0)^{3/2}}\right) g_2 + \frac{1}{\rho^2 f^2}\left[\frac{(8g_0 - 2g_2)}{(1+g_0)} + \frac{(2g_0 g_2 - 2g_2^2)}{(1+g_0)^2}\right]. \tag{26}$$

Using Eqs. (24) one obtains

$$\varepsilon_2(z) = \left(\frac{\Omega_0^2}{(1+g_0)^{3/2}}\right) g_0 + \frac{1}{\rho^2 f^2}\frac{12g_0}{(1+g_0)^2} \tag{27}$$

2.4. Critical Condition for Focusing: Critical Curves

With initially ($\xi = 0$) plane wave front $[(df/d\xi) = 0]$ of the beam and $f = 1$ at $\xi = 0$, the condition $(d^2f/d\xi^2)_{\xi=0} = 0$ leads to $f(\xi) = 1$ or propagation of the HGB without convergence or divergence; this condition is known as the critical condition. Thus putting $(d^2f/d\xi^2)_{\xi=0} = 0$ in Eq. (14a) one obtains a relation between dimensionless initial width of the HGB $\rho_0[= r_0\omega/c]$ and αE_{00}^2, corresponding to the propagation of the HGB in the self trapped mode. Further For $(d^2f/d\xi^2) < 0$ the HGB displays oscillatory self focusing, while for $(d^2f/d\xi^2) > 0$ HGB undergoes either oscillatory or steady divergence.

The critical curve can thus be represented as,

$$\rho_0^2\varepsilon_2(0) = 4. \tag{28}$$

Using the appropriate expression for $\varepsilon_2(z)$ at $z = 0$ from Eq. (27) for relativistic-ponderomotive nonlinearity, Eq. (28) reduces to

$$g_0\rho_0^2 = \frac{(1+g_0)^{3/2}}{\Omega_0^2}\left[4 - \frac{12g_0}{(1+g_0)^2}\right] \tag{29}$$

On substitution for the coefficient g_0 from Eqs. (24), Eq. (29) represents the critical curve ρ_0^{-2} *vs.* αE_{00}^2 and separates the self focusing region from the rest. The critical curves, which exhibit a relationship between the initial dimensionless amplitude αE_{00}^2 and the width ρ_0, correspond to the propagation of the HGB without convergence or divergence. Points above the curve correspond to divergence (or dissipation) while points below the curve refers to self focusing of the HGB.

3. COMPUTATIONAL SCHEME

To have a better understanding of the underlying physics and the numerical appreciation of the results, the critical curves and the dependence of the beam width parameter f (in the vicinity of the maximum of the irradiance of the HGB), on ξ for a chosen set of parameters and ponderomotive-relativistic nonlinearity, has been computed.

The critical curves for the propagation of the HGB in a plasma, between the irradiance αE_{00}^2 and the initial dimensionless width of the HGB ρ_0, have been plotted with the help of Eq. (29), by using Eqs. (24), corresponding to the applicable plasma nonlinearity and chosen sets of parameters n and Ω_0. Further the computations have also been made to investigate the variation of the dimensionless beam width parameter f, associated with the propagation of the HGB

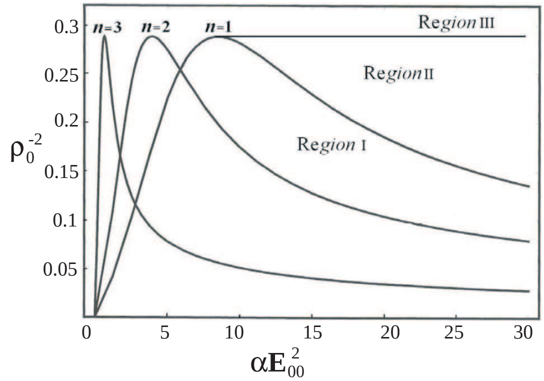

Figure 1. The dependence of the initial beam width ρ_0^{-2} with the initial irradiance αE_{00}^2, for the propagation of various orders HGBs with dominant ponderomotive-relativistic plasma nonlinearity, for the parameter $\Omega_0^2 = 0.8$; the orders of the HGB are indicated over the curves and the Regions I, II and III correspond to $n = 1$.

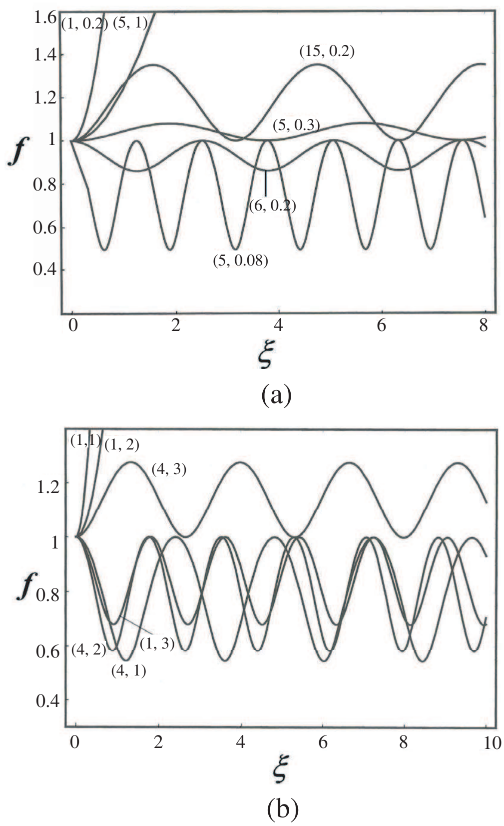

Figure 2. (a) The dependence of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, with dominant ponderomotive-relativistic nonlinearity for various order HGBs for the parameters $n = 2$ and $\Omega_0^2 = 0.8$; the curves refer to an arbitrarily chosen set of initial irradiance and initial beam width $(\alpha E_{00}^2, \rho_0^{-2})$ as indicated over the curve. (b) The dependence of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, with dominant ponderomotive-relativistic nonlinearity for various order HGBs for the parameters $\rho_0^{-2} = 2$ and $\Omega_0^2 = 0.8$; the curves refer to an arbitrarily chosen set of initial irradiance and initial beam width $(\alpha E_{00}^2, n)$ as indicated over the curves.

on the dimensionless distance of propagation ξ in a homogeneous plasmas. Starting with a combination of parameters αE_{00}^2, ρ_0 and Ω_0, one can obtain the solution for the beam width parameter f by numerical integration of Eq. (14a) using appropriate expressions for the parameters ε_0 and ε_2 from Eqs. (25) and (27); appropriate boundary conditions *viz.* $f = 1$, $df/d\xi = 0$ at $\xi = 0$ have been used.

4. NUMERICAL RESULTS AND DISCUSSION

In the present analysis, we have investigated the propagation of HGB (of various orders) in a homogeneous plasma; the plasma is characterized by the dielectric function corresponding to the relativistic-ponderomotive nonlinearity. The theory is based on a paraxial like approach in which all the relevant parameters have been expanded around the axis of maximum irradiance of the HGB. The irradiance distribution profile of the HGB [Eq. (2)] shows that its maximum lies at $r = r_{\max} = r_0\sqrt{2n}$. This indicates that the radius of the bright ring (corresponding to irradiance maximum) increases with the increasing order; further the area of the dark region across the HGB also increases with increasing n. It is interesting to notice that the third term of the right hand side in the expression of dielectric function $\varepsilon(r, z)$ [from Eq. (22)] is independent of the background electron density and if one ignores this term the expression for the dielectric function gets reduced to the simpler form of relativistic nonlinearity. Thus the third term represents the combined effect of relativistic and ponderomotive forces and strongly depends on the width and irradiance of the electromagnetic beam.

In the context of the present study, the critical curves and the plot of the beam width parameter f as a function of the dimensionless distance of propagation ξ has been obtained for a chosen set of parameters αE_0^2, n and Ω_0 corresponding to relativistic-ponderomotive nonlinearity. The critical curve for the HGB characterizes the self focusing region in the $\rho_0^{-2} - \alpha E_{00}^2$ space. The points $(\rho, \alpha E_{00}^2)$ above the critical curve display self focusing, while points lying below the critical curve lead to oscillatory divergence or steady divergence.

Figure 1 illustrates the dependence of the initial irradiance on the dimensionless width ρ_0, for the self trapping mode of the propagation of various order HGBs corresponding to the relativistic-ponderomotive nonlinearity. The figure indicates that the $\rho_0^{-2} - \alpha E_{00}^2$ space can be divided in three regions *viz.* oscillatory focusing (Region I), oscillatory divergence (Region II) and steady divergence (Region III). The regions for self focusing and oscillatory divergence are separated by the critical curve while an approximate line curve divides the area above the critical curve into two regions namely oscillatory divergence and steady divergence. For the parameters corresponding to an initial point $(\rho_0^{-2},\ \alpha E_{00}^2)$ lying on the critical curve $(d^2f/d\xi^2)$ vanishes at $\xi = 0$; since for initially plane wavefront $df/d\xi = 0$, it continues to be zero and f remains constant throughout the propagation of the HGB; this is known as stationary spatial soliton propagation. For the initial points lying below the critical curves (Region I), $d^2f/d\xi^2 < 0$ and hence as the beam propagates the beam width parameter oscillates between the initial value unity and a minimum. Similarly for the initial point $(\rho_0^{-2},\ \alpha E_{00}^2)$ lying in (Region II), f oscillates between a maximum and the initial value unity. For the points lying in (Region III) the beam displays steady divergence. Further it is seen that the region for self focusing decreases with increasing order of the HGB.

The set of Fig. 2 describes the dependence of the beam width parameter f on the dimensionless distance of propagation ξ for a plasma with dominant relativistic-ponderomotive nonlinearity. Fig. 2(a) describes the propagation of the second order $(n = 2)$ HGB in the characteristic three regions namely of self focusing, oscillatory divergence and steady divergence; this behavior is characterized by saturating nature of the nonlinearity. Fig. 2(b) shows the curve for dependence of the beam width parameter f on ξ with varying order n and the initial irradiance αE_{00}^2; the relevant parameters are $\rho_0^{-2} = 0.13$ and $\Omega_0^2 = 0.8$. The curves are in conformance with the above discussed critical curves.

5. CONCLUSION

A paraxial like approach has been adopted to analyze the propagation of various order HGBs, in a homogeneous plasma, where both relativistic and ponderomotive nonlinearities are simultaneously operative. It is seen that the critical curves and self focusing show strong dependence on the order of the HGB; the propagation of the HGB follows the characteristic three regimes in the vicinity of the maximum irradiance.

ACKNOWLEDGMENT

The authors are grateful to Department of Science and Technology, Government of India for financial support and to Prof. M. S. Sodha and Prof. M. P. Verma for helpful discussions.

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