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MICROWAVE FILTERING IN WAVEGUIDES LOADED WITH ARTIFICIAL SINGLE OR DOUBLE NEGATIVE MATERIALS REALIZED WITH DIELECTRIC SPHERICAL PARTICLES IN RESONANCE

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Abstract—The potential to implement microwave filters with special properties, by loading a waveguide with artificial Single Negative (SNG) or Double negative (DNG) materials was investigated. The SNG or DNG medium was structured with dielectric spherical particles of high permittivity embedded in a dielectric material of much smaller permittivity. Numerical analysis of the frequency response of the waveguide loaded with slabs of this type of composite dielectrics reveals that filtering performance, with attributes like very sharp attenuation at the bounds of frequency pass or stop band can be obtained. The central frequency as well as the bandwidth of the filtering can be controlled via the size and the dielectric constants of the particles, the dielectric constant of the hosting material and the size of the slab.

1. INTRODUCTION

Unconventional media of single and especially both negative relative permittivity, ε_r, and permeability, μ_r, have attracted the interest of theoretical investigation in virtue of their special properties. A medium with one of ε_r or μ_r, negative, can not support propagation of electromagnetic waves as they decay exponentially. However when both ε_r and μ_r are negative, waves can still propagate since the product $\varepsilon_r\mu_r$ remains positive. In this case, the phase of the wave moves in a direction opposite to that of the energy flow, and this means that the phase and group velocity have opposite signs. This type of materials is termed metamaterials, double negative (DNG), left

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handed (LH) or backward materials. The peculiar electromagnetic wave propagation inside them was first theoretically investigated by Veselago in 1967 [1] and was followed by works presenting theoretical and experimental verification of the phenomenon as well as models for its study, in various applications [2–9]. From the application side of view the question is how media with this performance could be constructed. Interesting artificial LH-structures have been proposed. Periodic arrays of interspaced conducting non-magnetic split ring resonators and thin wires [10–12] as well planar periodic microstrip arrangements [13–17] were proved theoretically, numerically and experimentally to support left handed waves. An alternative way to realize an artificial SNG or DNG medium is to compose a structure by creating a lattice of double positive (DPS) dielectric spherical particles with high dielectric constant, inside a DPS dielectric of much lower dielectric constant [18–22]. This hybrid medium exhibits a frequency depended performance and has the advantage of simple fabrication and mainly easy integration in a microwave device like a waveguide.

One of the applications of metamaterials, useful in practice, is the composition of structures that operate as electromagnetic wave filters of frequency [23, 24]. In the work at hand the study is focused to obtain microwave filters with specific operational properties, such as small frequency bandwidth, very rapid decrease of the signal at the boundaries of the frequency band, by the exploiting the special attributes of SNG and DNG dielectric slabs which load a waveguide. It is considered that the slabs are of type of the aforementioned materials of frequency dependent operation. Various methods can be used for the theoretical analysis of the propagation of electromagnetic waves in LH-materials as the geometrical analyses of plane waves [25, 26]. In the present work, the fields excited in the waveguide are approached by modal analysis [27–29] because no plane waves exist, and moreover higher order modes are excited, due to the reflection of waves at the metamaterial slabs. The calculations were made via the solution of a linear system of equations produced via the application of the boundary conditions on the interfaces of the arrangement. The permittivity and permeability of the slabs are functions of frequency and are taken into account via the dispersion functions as they are produced by the theoretical analysis of the hybrid particle-dielectric host scheme.

2. THEORY AND FORMULATION

2.1. The SNG and DNG Performance of the Lattice of Spherical Particles inside a Dielectric Host Medium

A medium composed of dielectric spherical particles periodically placed in a dielectric material, being the host material, exhibits electric and magnetic performance according to the distribution and the polarizability of the particles. A wave propagating through the array of these scatterers sees this non-uniform environment as an effective medium. The key of the propagation is the size, dielectric constant of the spheres and distance between them, as well as the dielectric constant of the host medium. When the size of the spherical scatterers is small compared to the wavelength in the surrounding dielectric, but not small compared to that in the material of the scatterers, the effective parameters of the composite medium become frequency dependent. Models that describe this performance and give the mathematical expression for the effective constant values have been introduced [18–22]. In the work of Lewin [30] a material composed of spherical dielectric particles arranged in a cubic lattice was considered. The solution of the boundary value problem for scattering by a sphere was used to obtain the description of the scattering of the global medium in terms of effective constants μ_{eff} and ε_{eff} assuming that the volume of the hybrid material contains a large number of cubic cells. In accordance to this formulation the relative effective permittivity and permeability of the non uniform medium is

$$\varepsilon_{eff} = \varepsilon_h \left(1 + \frac{3\nu_e}{(F(\theta) + 2b_e)/(F(\theta) - 2b_e) - \nu_e}\right) \tag{1}$$

$$\mu_{eff} = \mu_h \left(1 + \frac{3\nu_m}{(F(\theta) + 2b_m)/(F(\theta) - 2b_m) - \nu_m}\right) \tag{2}$$

where

$$F(\theta) = \frac{2(\sin\theta - \theta\cos\theta)}{(\theta^2 - 1)\sin\theta + \theta\cos\theta}, \quad \theta = k_o\alpha\sqrt{\varepsilon_p\mu_p},$$
$$k_o = 2\pi/\lambda_o, \quad b_e = \varepsilon_h/\varepsilon_p, \quad b_m = \mu_h/\mu_p$$

and α is the particle radius, ν_e, ν_m are the ratios of the volume of the spherical inclusions to the volume of the cell, ε_p, μ_p are the constants of the particles and ε_h, μ_h the constants of the host medium.

An electromagnetic wave propagating in the aforementioned hybrid structure excites certain modes in the particles. These modes are not strong eigenmodes of a spherical dielectric resonator, but they can be specified as H or E mode at the frequencies which are close to the spherical cavity eigenfrequencies [22]. The E_{111} mode, excited in a particle of specific radius, can give rise to a magnetic dipole moment and lead the entire medium to perform with a negative permittivity around the eigenfrequency of the E_{111} mode. In a particle with smaller radius, the H_{111} mode would be excited, offering an electric dipole moment which makes the medium perform with a negative permeability. The electric or magnetic dipole moments of the particles are calculated by integration, over their volume, of the electric and magnetic field of the modes. The frequency of resonance in both cases is controlled by the dielectric constant and/or the size of the particle. The size of the cell, which depends on the distance between the particles, affects a little the value of frequency of resonance. It mainly affects the frequency bandwidth in which the hybrid material has negative effective permittivity or/and permeability. As the distance between the particles reduces the bandwidth increases. If the cubic cell of the lattice is loaded with two types of spheres with different sizes and the radii of the spheres are properly selected in order the resonance frequencies of E_{111} and H_{111} modes to coincide, the bispherical hybrid medium can have a DNG performance.

A microwave waveguide would potentially exhibit an interesting frequency operation when loaded with components of either SNG or DNG composite materials. This potential is investigated in the work at hand.

2.2. Frequency Response of the SNG and DNG Slab Loaded Waveguide

Assume a rectangular waveguide loaded by dielectric slabs as shown in (Fig. 1(a)). The internal of the waveguide is divided in five regions $(1, 2$ and $\mathrm{I}, \mathrm{II}, \mathrm{III})$. In regions 1 (input) and 2 (output), air exists. Areas I and III are filled with DPS dielectrics with equal permittivity and permeability, whereas region II is filled with an SNG or a DNG material.

The first step of the modal analysis [27–29] is the determination of the propagation constants in the areas $\mathrm{I}, \mathrm{II}, \mathrm{III}$. Considering TE_{10} mode excitation the electric and magnetic field components are given next

$$\text{Region I}: E_y^{\mathrm{I}} = \sin\left(h_{\mathrm{I}} x\right) e^{jk_1 z} \tag{3a}$$

$$\text{Region II}: E_y^{\mathrm{II}} = \left[A \cos\left(h_{\mathrm{II}} x\right) + B \sin\left(h_{\mathrm{II}} x\right)\right] e^{jk_1 z} \tag{3b}$$

$$\text{Region III}: E_y^{\mathrm{III}} = A_{\mathrm{III}} \sin\left(h_{\mathrm{III}}(a - x)\right) e^{jk_1 z} \tag{3c}$$

where $h_{\mathrm{I}} = \left(k_{\mathrm{I}}^2 - k_1^2\right)^{1/2}$, $h_{\mathrm{II}} = \left(k_{\mathrm{II}}^2 - k_1^2\right)^{1/2}$, $h_{\mathrm{III}} = h_{\mathrm{I}}$, $k_{\mathrm{I}} = k_{\mathrm{III}} = \omega\sqrt{\mu_{\mathrm{I}}\varepsilon_{\mathrm{I}}}$, $k_{\mathrm{II}} = \omega\sqrt{\mu_{\mathrm{II}}(f)\varepsilon_{\mathrm{II}}(f)}$ and ε_{I}, $\varepsilon_{\mathrm{II}}(f)$, μ_{I}, $\mu_{\mathrm{II}}(f)$ are the permittivity and the permeability of the loading materials and k_1 is the unknown transmission constant of the loaded non-uniform area $(\mathrm{I}, \mathrm{II}, \mathrm{III})$. The components of the magnetic field in all regions can be computed using the above expressions and Maxwell equations. Then applying the boundary conditions on the interfaces $(\mathrm{I}, \mathrm{II})$ and $(\mathrm{II}, \mathrm{III})$ the eigenvalue equation is derived and is decoupled in two equations as follows

$$\frac{h_{\mathrm{II}} \sin(h_{\mathrm{I}} d)}{h_{\mathrm{I}} \cos(h_{\mathrm{I}} d)(\mu_{\mathrm{II}}/\mu_{\mathrm{I}})} + \frac{\sin(h_{\mathrm{II}} t/2)}{\cos(h_{\mathrm{II}} t/2)} = 0 \tag{4a}$$

$$\frac{-h_{\mathrm{II}} \sin(h_{\mathrm{I}} d)}{h_{\mathrm{I}} \cos(h_{\mathrm{I}} d)(\mu_{\mathrm{II}}/\mu_{\mathrm{I}})} + \frac{\cos(h_{\mathrm{II}} t/2)}{\sin(h_{\mathrm{II}} t/2)} = 0 \tag{4b}$$

The above equations, being solved for k_1, provide the propagation constants of the excited wave modes.

In the input and output regions the fields are expanded in series of TE_{mn} modes, as they are described in a waveguide entirely filled by a dielectric. In the input region, incident and reflected fields exist. Considering that the incident field is transmitted in the direction of

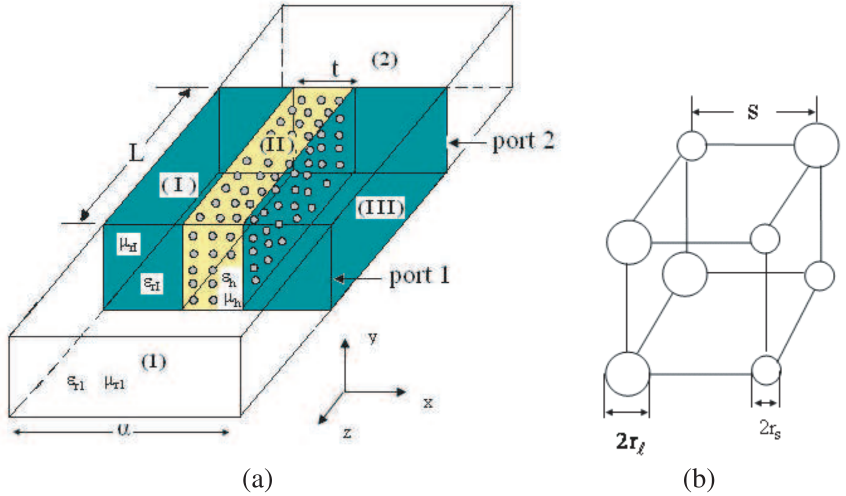


Figure 1. (a) The geometry of the waveguide loaded with SNG or DNG material (area II). (b) A cubic cell of the lattice of the spherical particles.

$-z$ axis, the field components are described by Eq. (5).

$$E_y^1 = \sin(\gamma_1 x)\, e^{ju_1 z} + \sum_{m=1}^{M} D_m \sin(\gamma_m x)\, e^{-ju_m z} \tag{5a}$$

$$H_x^1 = -\frac{j}{\omega\mu_0}\left[ju_1 \sin(\gamma_1 x)\, e^{ju_1 z} - j\sum_{m=1}^{M} D_m u_m \sin(\gamma_m x)\, e^{-ju_m z}\right] \tag{5b}$$

$$H_z^1 = \frac{j}{\omega\mu_0}\left[\gamma_1 \cos(\gamma_1 x)\, e^{ju_1 z} + \sum_{m=1}^{M} D_m \gamma_m \cos(\gamma_m x)\, e^{-ju_m z}\right] \tag{5c}$$

The far end of the waveguide is supposed to be matched, thereby in the output region only transmitted field exists:

$$E_y^2 = \sum_{m=1}^{M} E_m \sin(\gamma_m x)\, e^{ju_m z} \tag{6a}$$

$$H_x^2 = \frac{-j}{\omega\mu_0}\left[j\sum_{m=1}^{M} E_m u_m \sin(\gamma_m x)\, e^{ju_m z}\right] \tag{6b}$$

$$H_z^2 = \frac{j}{\omega\mu_0}\sum_{m=1}^{M} E_m \gamma_m \cos(\gamma_m x)\, e^{ju_m z} \tag{6c}$$

where $\gamma_1 = \frac{\pi}{a}$, $\gamma_m = \frac{m\pi}{\alpha}$, $u_m = \left(k_0^2 - \gamma_m^2\right)^{1/2}$ and $k_0 = \frac{2\pi}{\lambda}$.

In the interaction regions (I, II, III) incident and reflected waves are excited and the field is expanded in series of modes derived by Eqs. (4a), (4b). The field components must be expressed separately in each region in order the boundary conditions on the metallic waveguide walls to be satisfied.

Applying the continuity conditions on the interfaces of input-interaction and output-interaction regions, four complex linear equations are derived. Taking the inner product of each equation by $\sin(\gamma_m x)$, where $m = 1, 3, 5, \ldots$, we obtain a system of linear equations (7)–(10). The unknowns are the coefficients included in the terms of the summations by which the field is described.

$$\sum_{i=1}^{N} A_i \left(R_m^i + Q_m^i + I_1^i T_m^i + I_2^i W_m^i\right)$$
$$+\sum_{i=1}^{N} B_i \left(R_m^i + Q_m^i + I_1^i T_m^i + I_2^i W_m^i\right) = \begin{cases} \frac{a}{2} + D_1\frac{a}{2}, & m = 1 \\ D_m\frac{a}{2}, & m > 1 \end{cases} \tag{7}$$

$$\sum_{i=1}^{N} A_i \left[\frac{1}{\mu_{\mathrm{I}}}\left(jk_1^i\right)\left(R_m^i+Q_m^i\right)+\frac{1}{\mu_{\mathrm{II}}}\left(jk_1^i\right)\left(I_1^iT_m^i+I_2^iW_m^i\right)\right]$$
$$+\sum_{i=1}^{N} B_i \left[\frac{1}{\mu_{\mathrm{I}}}\left(-jk_1^i\right)\left(R_m^i+Q_m^i\right)+\frac{1}{\mu_{\mathrm{II}}}\left(-jk_1^i\right)\left(I_1^iT_m^i+I_2^iW_m^i\right)\right]$$
$$=\begin{cases} \frac{1}{\mu_0}\left(ju_1\frac{a}{2}-jD_1u_1\frac{a}{2}\right), & m=1 \\ -\frac{1}{\mu_0}jD_mu_m\frac{a}{2}, & m>1 \end{cases} \tag{8}$$

$$\sum_{i=1}^{N} A_i \left(R_m^i+Q_m^i+I_1^iT_m^i+I_2^iW_m^i\right)e^{-jk_1^iL}$$
$$+\sum_{i=1}^{N} B_i \left(R_m^i+Q_m^i+I_1^iT_m^i+I_2^iW_m^i\right)e^{jk_1^iL}=E_m\frac{a}{2}e^{-ju_mL} \tag{9}$$

$$\left[\sum_{i=1}^{N} A_i \left[\frac{1}{\mu_{\mathrm{I}}}\left(jk_1^i\right)\left(R_m^i+Q_m^i\right)+\frac{1}{\mu_{\mathrm{II}}}\left(jk_1^i\right)\left(I_1^iT_m^i+I_2^iW_m^i\right)\right]e^{-jk_1^iL}\right]$$
$$+\left[\sum_{i=1}^{N} B_i \left[\frac{1}{\mu_{\mathrm{I}}}\left(-jk_1^i\right)\left(R_m^i+Q_m^i\right)+\frac{1}{\mu_{\mathrm{II}}}\left(-jk_1^i\right)\left(I_1^iT_m^i+I_2^iW_m^i\right)\right]e^{jk_1^iL}\right]$$
$$=\frac{1}{\mu_0}E_m\left(ju_m\right)\frac{a}{2}e^{-ju_mL} \tag{10}$$

where, I_1^i, I_2^i, T_m^i, W_m^i, Q_m^i, R_m^i are quantities derived via the procedure of boundary condition application, the inner production by $\sin(\gamma_m x)$, the integration over the interface areas etc. The unknowns for which the system has to be solved are the coefficients A_i, B_i of the field in the interaction region and the coefficients D_m, E_m of the field in the input and output regions respectively. The coefficient D_1 stands for the scattering coefficient S_{11} at the port 1, of the slab loaded region, and E_1 is the transfer coefficient S_{12}, between port 1 and port 2 constant.

3. RESULTS

The frequency response of a waveguide loaded with SNG and DNG dielectric components was studied as a function of the geometry and the electromagnetic constants of the configuration. The selected waveguide is the WR-90 of size $2.826\,\mathrm{cm}\times 1.016\,\mathrm{cm}$, and the numerical calculations were made with the theoretical analysis of the previous section.

3.1. Loads with $\varepsilon_{eff} < 0$ and $\mu_{eff} > 0$

Figure 2 depicts the variation of the relative effective permittivity and the relative effective permeability of the hybrid medium composed of dielectric with relative permittivity $\varepsilon_h = 10$ and relative permeability $\mu_h = 1$ that hosts a lattice of spheres (Fig. 1(b)). All the dielectric spheres have equal radii $r_\ell = r_s = 0.7\,\text{mm}$, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_p = 1$. Ferroelectric single crystals or ceramic materials are proposed as media with dielectric constant of this order of value. It has also to be to pointed out that for frequencies of resonance between 8 GHz and 12 GHz, the radius of the spheres is expected to be in the order of mm. In the literature it is referred that around 60 GHz it is some tens of μm [21] and at frequencies in the order of THz the radius is tens of nm [19]. The size of the cell of the lattice is $s = 1.5\,\text{mm}$ and the distance between the particles is 0.1 mm. This small value was chosen in order the frequency range, in which $e_{eff} < 0$, to be wide as possible. It is shown that $\mu_{eff} > 0$ $(0.004 < \mu_{eff} < 0.6)$ in the entire frequency range whereas $\varepsilon_{eff} < 0$ in a frequency band $Bf_{eff} = 0.25\,\text{GHz}$ which extends from $10.55\,\text{GHz} < f < 10.8\,\text{GHz}$. It is the frequency area in which the E_{111} mode of resonance is developed in the spheres. This type of composite medium was used to load the central area of the WR-90 (Fig. 1(a)). The frequency performance of the waveguide shows that the global propagation constant k_1 of the loaded area (I, II, III) does not exist for frequencies extending in a range $Bf_w \leq 0.25\,\text{GHz}$. So, in this frequency range the structure can perform as a perfect band stop filter. The value of Bf_w depends on the width t of the loaded area and the dielectric constants $\varepsilon_{r\text{I}} = \varepsilon_{r\text{III}}$ (Fig. 3). The maximum value of Bf_w is equal to the Bf_{eff} in which $\varepsilon_{eff} < 0$ and it is observed for small values of t and $\varepsilon_{r\text{I}}$. As the $\varepsilon_{r\text{I}}$ gets larger, Bf_w reduces and tends to zero. The greater being the value of t, the larger has to be the value of $\varepsilon_{r\text{I}}$ in order Bf_w to be equal to zero. This behavior admits a reasonable physical interpretation. The central area of width t does not permit the propagation of the wave as $\varepsilon_{eff} < 0$, whereas the adjacent areas with dielectric constants $\varepsilon_{r\text{I}}$ and $\varepsilon_{r\text{III}}$ and width $(\alpha - 2t)/2$ offer a passage to the wave. The smaller their width is, the larger has to be the value of their permittivity in order to concentrate the field lines of the wave inside their volume and, in this way, to permit the pass of the wave from region 1 to region 2 (Fig. 1(a)).

The operation of the loaded waveguide exhibits interesting properties near the Bf_w. Fig. 4(a) depicts the scattering coefficients S_{11} and S_{12} at the ports 1 and 2 (Fig. 1(a)) of the loaded area with $\varepsilon_{r\text{I}} = 10$, $t = 8\,\text{s}$ and $L = 4\,\text{s}$. Fig. 4(b) illustrates the values of the propagation constants in the entire frequency range $8.7\,\text{GHz} < f <$

12 GHz. In Fig. 4(a), we see a band-pass filtering performance with flat response in the pass band and very abrupt rejection of the signal at the bounds of this area. In other words the scheme performs as a perfect band pass microwave filter. Similar are the results shown in Fig. 5(a) received with $\varepsilon_{r\mathrm{I}} = 2.8$, $t = 4\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. In this case the loaded area leads the waveguide to a band stop filtering operation with sharp signal rejection at the frequency band bounds. The results of Fig. 6, received with $\varepsilon_{r\mathrm{I}} = 10$, $t = 12\,\mathrm{s}$ and $L = 4\,\mathrm{s}$, present successive frequency bands with stop and pass properties.

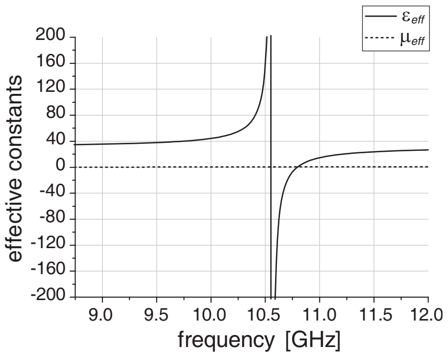


Figure 2. The relative effective permittivity and permeability of the dielectric structure with $\varepsilon_h = 10$ and $\mu_h = 1$ that hosts a lattice of spheres with equal radii 0.7 mm, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_p = 1$. The size of the cell of the lattice is $s = 1.5\,\mathrm{mm}$.

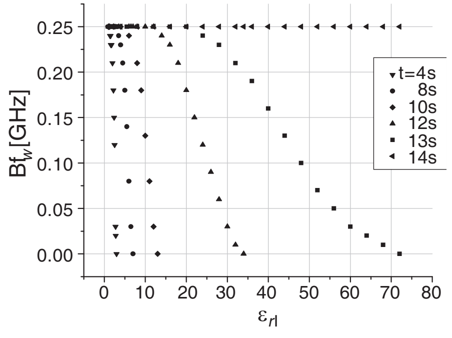


Figure 3. The frequency range of forbidden propagation of the waves versus the size of the loaded area and the dielectric constant of the neighbouring areas. The SNG load has effective parameters shown in Fig. 2.

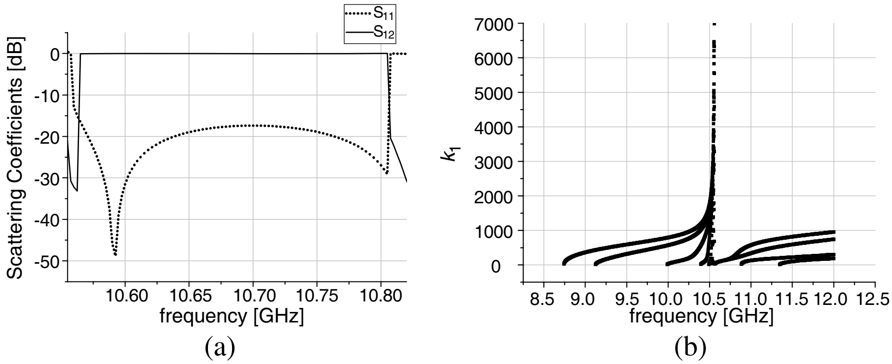


Figure 4. (a) The scattering coefficients of the wave at the ports 1 and 2 of the loaded area of the waveguide. The SNG load's performance is depicted in Fig. 2. Size of the load: $t = 8\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 10$. (b) The propagation constants inside the hybrid area $(\mathrm{I}, \mathrm{II}, \mathrm{III})$.

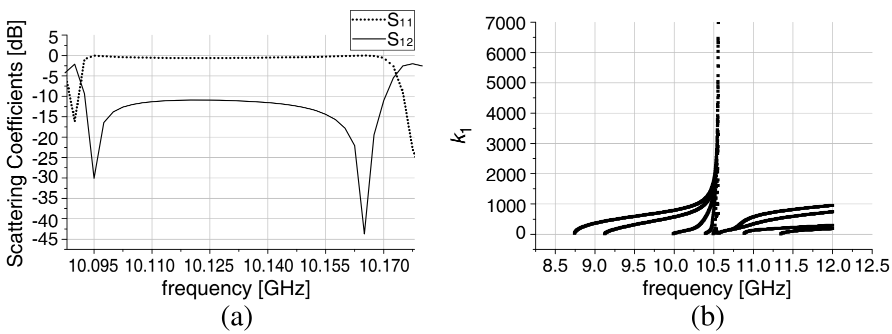


Figure 5. (a) Scattering coefficients of the wave at the ports 1 and 2 of the loaded area of the waveguide. The SNG load has performance shown in Fig. 2. Size of the load: $t = 4\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 2.8$. (b) The propagation constants inside the hybrid area $(\mathrm{I}, \mathrm{II}, \mathrm{III})$.

3.2. Loads with $\varepsilon_{eff} > 0$ and $\mu_{eff} < 0$

In this case the area of size $(t \times L)$, in the waveguide, is filled with a composite medium of a dielectric material with $\varepsilon_h = 10$ and $\mu_h = 1$ which hosts a lattice of cubic cells with spherical particles of radius $r_\ell = r_s = 0.477\,\mathrm{mm}$, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_p = 1$. The size of cell is $s = 1.054\,\mathrm{mm}$. In this case also the distance between the spheres is very small.

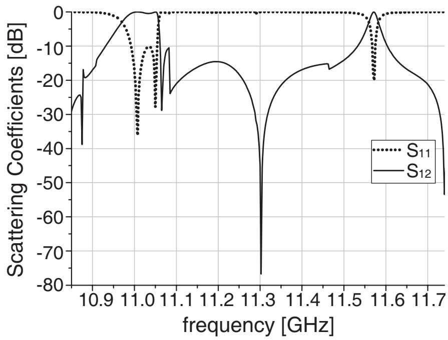


Figure 6. Scattering coefficients of the wave at the ports 1 and 2 of the loaded area of the waveguide. The performance of the SNG load is that of Fig. 2. Size of the load: $t = 12\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 10$.

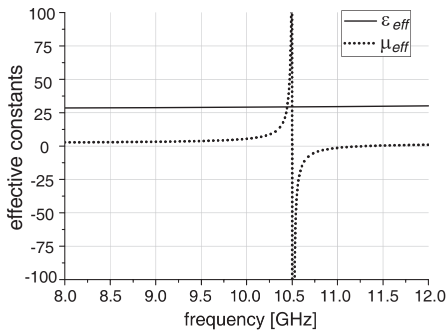


Figure 7. The relative effective permittivity and permeability of the dielectric structure with $\varepsilon_h = 10$ and $\mu_h = 1$ that hosts a lattice of spheres with equal radii 0.477 mm, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_p = 1$. The size of the cell of the lattice is $s = 1.054\,\mathrm{mm}$.

Figure 7 illustrates the variation of the relative effective permittivity and the relative effective permeability of the composite medium versus frequency. It is shown that $\varepsilon_{eff} > 0$ in the entire frequency range whereas $\mu_{eff} < 0$ in a frequency band $Bf_{eff} = 0.81\,\mathrm{GHz}$ which extends from 10.5 GHz to 11.31 GHz. It is the frequency area in which the H_{111} mode of resonance arises in the spheres. In this case the global propagation constant of the hybrid area (I, II, III) does not exist for frequencies in a range Bf_w. The value of Bf_w depends on the width t of the loaded area and the dielectric constants $\varepsilon_{r\mathrm{I}} = \varepsilon_{r\mathrm{III}}$ (Fig. 8). The maximum value of Bf_w is equal to the Bf_{eff} in which $\mu_{eff} < 0$ and it is observed for small values of t and $\varepsilon_{r\mathrm{I}}$. As the $\varepsilon_{r\mathrm{I}}$ gets larger Bf_w reduces and tends to zero. The greater is the value of t the larger has to be the value of $\varepsilon_{r\mathrm{I}}$ in order Bf_w to be equal to zero. This behavior is explained as in the case with the load of $\varepsilon_{eff} < 0$.

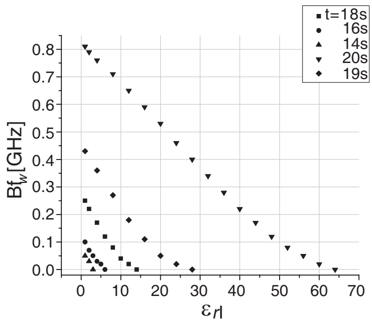


Figure 8. The frequency range of forbidden propagation of the waves versus the size of the loaded area and the dielectric constant of the neighbouring areas. The SNG load has effective parameters shown in Fig. 8.

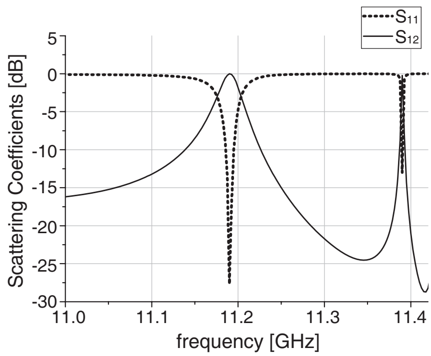


Figure 9. Scattering coefficients of the wave at the ports 1 and 2 of the loaded area. The performance of the SNG load is that of Fig. 7. Size of the load: $t = 18\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 10$.

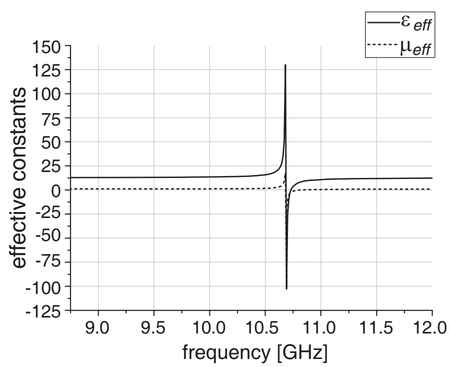


Figure 10. The relative effective permittivity and permeability of the dielectric structure with $\varepsilon_h = 10$ and $\mu_h = 1$ that hosts a biosphere lattice with radii $r_\ell = 0.7\,\mathrm{mm}$ and $r_s = 0.494\,\mathrm{mm}$, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_h = 1$. The size of cell is $s = 2.588\,\mathrm{mm}$.

Indicative results for scattering coefficients S_{11} and S_{12} at the ports 1 and 2 of the hybrid area are depicted in Fig. 9. The results were received with $\varepsilon_{r\mathrm{I}} = 10$, $t = 18\,\mathrm{s}$ and $L = 4\,\mathrm{s}$.

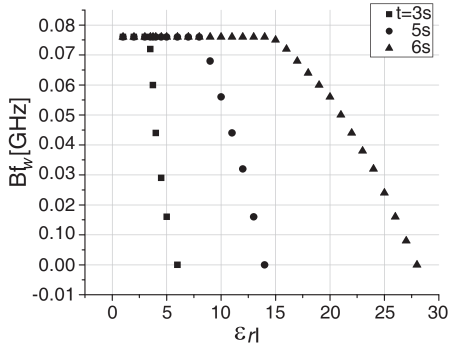


Figure 11. The frequency range of forbidden propagation of the waves versus the size of the loaded area and the dielectric constant of the neighbouring areas. The SNG load has effective parameters shown in Fig. 8.

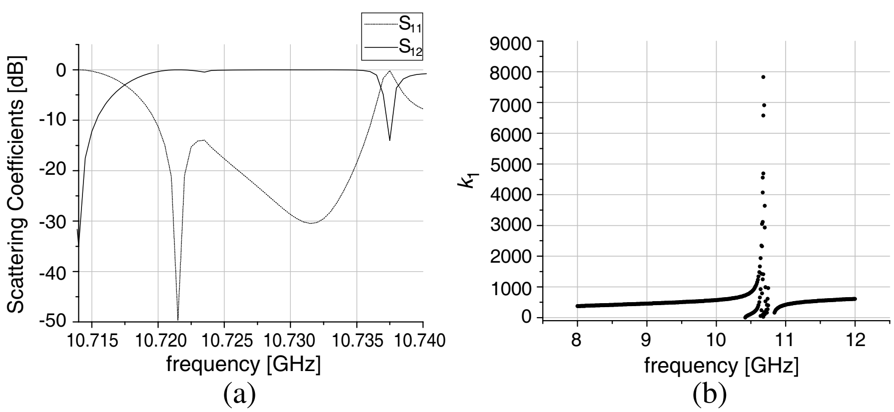


Figure 12. (a) Scattering coefficients of the wave at the ports 1 and 2. The SNG load performs as shown in Fig. 10. Size of the load: $t = 1\,\mathrm{s}$ and $L = 2\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 1$. (b) The propagation constants inside the hybrid area (I, II, III).

3.3. Loads with $\varepsilon_{\mathit{eff}} < 0$ and $\mu_{\mathit{eff}} < 0$

In this case the central area of the waveguide is loaded with a composite medium of a dielectric material with $\varepsilon_h = 10$ and $\mu_h = 1$ which hosts a lattice of cubic cells with two types of spherical particles with radii $r_\ell = 0.7\,\mathrm{mm}$ and $r_s = 0.494\,\mathrm{mm}$, dielectric constant $\varepsilon_p = 800$ and relative permeability $\mu_p = 1$. The size of cell is $s = 2.588\,\mathrm{mm}$, and it means that the distance between the spheres is very small. The sizes of the particles were properly selected in order the larger ones to resonate in the range $10.69\,\mathrm{GHz} < f < 10.75\,\mathrm{GHz}$ at the E_{111} mode and the smaller ones in the range $10.69\,\mathrm{GHz} < f < 10.825\,\mathrm{GHz}$ at the H_{111} mode. The performance of this bispherical material, as a function of frequency, is depicted in Fig. 10. It is shown that both $\varepsilon_{\mathit{eff}} < 0$ and $\mu_{\mathit{eff}} < 0$ in the range $10.69\,\mathrm{GHz} < f < 10.75\,\mathrm{GHz}$ whereas for $10.75\,\mathrm{GHz} < f < 10.825\,\mathrm{GHz}$ solely $\mu_{\mathit{eff}} < 0$. If the waveguide is loaded with this material, it was ascertained that when both $\varepsilon_{\mathit{eff}}$ and μ_{eff} are negative, propagation constants k_1 exist, and the transmission of the wave is cut off solely in a range Bf_w, from $10.75\,\mathrm{GHz} < f < 10.825\,\mathrm{GHz}$, namely when only the μ_{eff} is negative.

The value of Bf_w is variable and depends, as previously, on the width t of the loaded area and the dielectric constants $\varepsilon_{r\mathrm{I}} = \varepsilon_{r\mathrm{III}}$. The results are presented in Fig. 11, and the explanation from side of view of physics is similar to those of the previous cases. Fig. 12 depicts

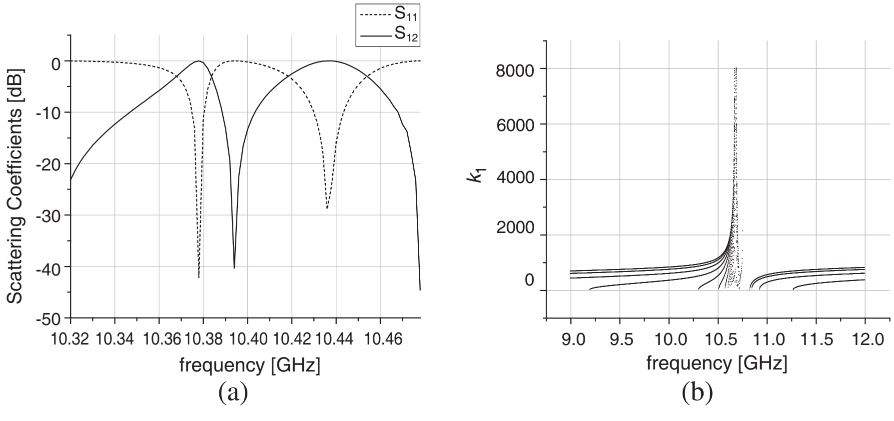
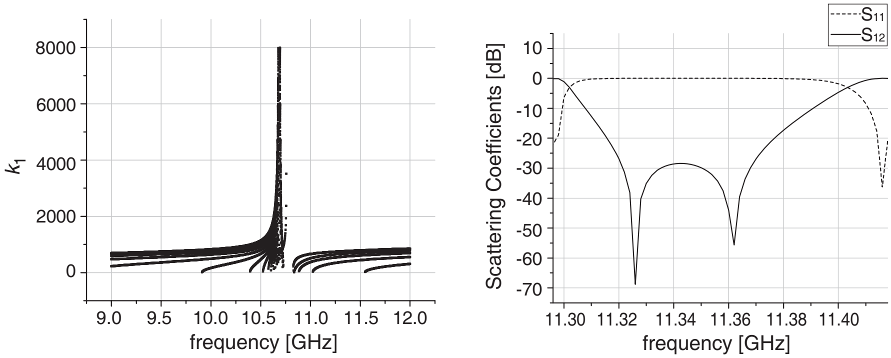


Figure 13. (a) Scattering coefficients of the wave at the ports 1 and 2. The performance of the SNG load is shown in Fig. 10. Size of the load: $t = 5\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 3$. (b) The propagation constants inside the hybrid area $(\mathrm{I}, \mathrm{II}, \mathrm{III})$.



(a) (b)

Figure 14. (a) Scattering coefficients of the wave at the ports 1 and 2. The SNG load has the performance shown in Fig. 10. Size of the load: $t = 6\,\mathrm{s}$ and $L = 4\,\mathrm{s}$. Dielectric constant of the adjoin dielectrics: $\varepsilon_{r\mathrm{I}} = 16$. (b) The propagation constants inside the hybrid area $(\mathrm{I}, \mathrm{II}, \mathrm{III})$.

results for the scattering coefficients S_{11} and S_{12} at the ports 1 and 2 for the frequency range in which the load exhibits DNG performance. Figs. 13 and 14 show results received in frequency bands adjacent to the areas of DNG and cut off performance. All of the examples show the ability of ideal filtering when hybrid loads are embedded in the waveguide.

4. CONCLUSION

The paper presents the investigation of filtering ability of waveguides loaded with SNG and DNG composite dielectric materials. Ideal filtering properties were ascertained. The materials were implemented by periodic lattices of spherical dielectric particles of high dielectric constant embedded inside o dielectric with low dielectric constant. The spheres are of equal or different radii, and when they resonate at E_{111} and H_{111} mode they lead the material to exhibit a negative relative permittivity or/and a negative relative permeability. When an SNG dielectric component loads a specific area of the waveguide, the wave propagation is cut off in the frequency band of the SNG performance, and the structure operates as an ideal band stop filter. The frequency bandwidth of this filtering is generally narrow, so the filters would be termed as notch filters. The bandwidth can be controlled by the size of the SNG dielectric components and the positive dielectric constant of the neighbouring areas of the waveguide. Furthermore ideal filtering with flat response in the pass band and sharp variation of the signal at the bounds of the band were observed in frequency ranges adjacent to the cut off band. Similar is the performance when the waveguide is DNG loaded. It is pointed out that in this case the waves propagate through the loaded area even in the frequency range in which the load performs as a DNG structure, and ideal filtering can also be obtained.

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