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ELECTROMAGNETIC FIELD OF A HORIZONTAL ELECTRIC DIPOLE BURIED IN A FOUR-LAYERED REGION

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Abstract—In this paper, the electromagnetic field of a horizontal electric dipole buried in a four-layered region is treated in detail. The region of interest consists of a perfect conductor, coated with the two-layered dielectrics under the air. Because of existing multi-reflections, the final representations of the six field components are much more complex. It is noted that the trapped surface wave and the lateral wave along the boundary between the air and the upper dielectric layer and those along the boundary between the two dielectric layers are included. Analysis and computations have some practical applications in microstrip antenna with super substrate.

1. INTRODUCTION

Almost a century ago, the electromagnetic field radiated by a dipole source in the planar boundary between two different media was first investigated by Sommerfeld [1]. The subsequent works on the electromagnetic field of a dipole source in stratified media have been carried out by many researchers, especially Wait and King [2–20]. In the pioneering works by Wait, detailed analysis was carried out on the electromagnetic field in stratified media by using asymptotic methods, contour integration, and branch cuts [2–5]. In a series works by King et al. the completed formulas for the electromagnetic fields due to horizontal and vertical electric dipoles in the two- and three-layered media were derived and computed [8–13]. Lately, the dyadic Green's function technique is used to examine the electromagnetic field in a four-layered forest environment [14–16].

In the late 1990's, the controversies concerning existence or nonexistence of the trapped surface wave for the electromagnetic field

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of a dipole in a three-layered region had continued for several years and rekindled several investigators to revisit the problem [17–24]. In recent works on this problem, it is concluded that the trapped surface wave, which varies as $\rho^{-1/2}$ along the planar surface in the far-field region, can be excited efficiently by vertical or horizontal electric dipole in the presence of a three-layered region [21–24]. Naturally, similar works are carried out on the electromagnetic field radiated by a vertical or horizontal electric dipole in the presences of a four-layered region [25, 26]. The details of recent research findings on the electromagnetic field in three- and four-layered regions are summarized in the book by Li [27].

If both a dipole and the observation point are buried in a four-layered region, because of existing multi-reflections, the problem is in general more complex. In the proceeding work, the electromagnetic field of a vertical electric dipole buried in a four-layered region was treated [29]. In what follows, we will attempt to outline the completed formulas of the electromagnetic field radiated by a horizontal electric dipole buried in a four-layered region.

2. ELECTROMAGNETIC FIELD OF A HORIZONTAL ELECTRIC DIPOLE BURIED IN A FOUR-LAYERED REGION

The relevant geometry and Cartesian coordinate system are shown in Fig. 1, where a horizontal electric dipole located at $(0, 0, d)$ in the upper dielectric layer. Region 0 ($z > l_1$) is the space above the upper dielectric layer with the air characterized by permeability μ_0 and uniform permittivity ε_0, Region 1 ($0 < z < l_1$) is the upper dielectric layer characterized by permeability μ_0, relative permittivity ε_{r1}, and conductivity σ_1, Region 2 ($-l_2 < z < 0$) is the lower dielectric layer characterized by permeability μ_0, relative permittivity ε_{r2}, and conductivity σ_2, and Region 3 ($z < -l_2$) is the rest space occupied by a perfect conductor. Then, the wave numbers in the four-layered region are

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} \tag{1}$$

$$k_j = \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{rj} + \mathrm{i}\sigma_j/\omega\right)}; \qquad j = 1, 2 \tag{2}$$

$$k_3 \to \infty. \tag{3}$$

Because of $k_3 \to \infty$, it is seen that the surface impedance $\eta_3 = 0$. For mathematical convenience, it is necessary to define the reflection coefficients R_{01}^{TM} and R_{21}^{TM} of electric-type (TM) wave in Region 1, which represents the reflections from the boundary between Regions 0

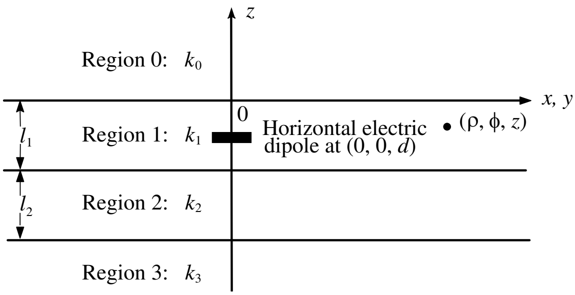

Figure 1. Geometry of a horizontal electric dipole buried in a four-layered region.

and 1 and the boundary between Regions 1 and 2. Correspondingly, we also define the reflection coefficients R_{01}^{TE} and R_{21}^{TE} of magnetic-type (TE) wave from the two boundarys. Let $k_{i\rho} = \lambda$ and $k_{iz} = \sqrt{k_i^2 - \lambda^2} = \gamma_i$, we have

$$R_{01}^{TM} = \frac{\varepsilon_0 k_{1z} - \varepsilon_1 k_{0z}}{\varepsilon_0 k_{1z} + \varepsilon_1 k_{0z}} = \frac{\varepsilon_0 \gamma_1 - \varepsilon_1 \gamma_0}{\varepsilon_0 \gamma_1 + \varepsilon_1 \gamma_0} = \frac{\frac{\gamma_1}{k_1^2} - \frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2} + \frac{\gamma_0}{k_0^2}} \tag{4}$$

$$R_{21}^{TM} = \frac{\frac{\gamma_1}{k_1^2} + i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2} - i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2} \tag{5}$$

$$R_{01}^{TE} = \frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1} \tag{6}$$

$$R_{21}^{TE} = \frac{\gamma_2 + i\gamma_1 \tan\gamma_2 l_2}{\gamma_2 - i\gamma_1 \tan\gamma_2 l_2}. \tag{7}$$

Obviously, the reflected field includes the waves in the directions $+\hat{z}$ and $-\hat{z}$. Considering multiply reflection, the reflection coefficients in the direction $+\hat{z}$ is written in the forms

$$P^{TM} = \frac{\frac{\frac{\gamma_1}{k_1^2}+i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2}-i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2} e^{i\gamma_1 d} - \frac{\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}} e^{i\gamma_1(2l_1-d)} \frac{\frac{\gamma_1}{k_1^2}+i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2}-i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}}{1 - \frac{\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}} \frac{\frac{\gamma_1}{k_1^2}+i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2}-i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2} e^{i2\gamma_1 l}} \tag{8}$$

$$P^{TE} = \frac{\frac{\gamma_2+i\gamma_1\tan\gamma_2 l_2}{\gamma_2-i\gamma_1\tan\gamma_2 l_2} e^{i\gamma_1 d} - \frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1} e^{i\gamma_1(2l_1-d)} \frac{\gamma_2+i\gamma_1\tan\gamma_2 l_2}{\gamma_2-i\gamma_1\tan\gamma_2 l_2}}{1 - \frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1} \frac{\gamma_2+i\gamma_1\tan\gamma_2 l_2}{\gamma_2-i\gamma_1\tan\gamma_2 l_2} e^{i2\gamma_1 l_1}}. \tag{9}$$

Similarly, the reflection coefficients in the direction $-\hat{z}$ is written in the forms

$$Q^{TM} = \frac{\frac{\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}} e^{i\gamma_1(2l_1-d)} - \frac{\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}} e^{i\gamma_1(2l_1+d)} \frac{\frac{\gamma_1}{k_1^2}+i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2}-i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}}{1-\frac{\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}}{\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}} \frac{\frac{\gamma_1}{k_1^2}+i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2}{\frac{\gamma_1}{k_1^2}-i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2} e^{i2\gamma_1 l_1}} \tag{10}$$

$$Q^{TE} = \frac{\frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1} e^{i\gamma_1(2l_1-d)} - \frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1} e^{i\gamma_1(2l_1+d)} \frac{\gamma_2+i\gamma_1\tan\gamma_2 l_2}{\gamma_2-i\gamma_1\tan\gamma_2 l_2}}{1-\frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1}\frac{\gamma_2+i\gamma_1\tan\gamma_2 l_2}{\gamma_2-i\gamma_1\tan\gamma_2 l_2} e^{i2\gamma_1 l_1}}. \tag{11}$$

From (4.7.19) in the book by Kong [29], the integrated formulas for E_{1z} and H_{1z} can be expressed in the following forms.

$$\begin{aligned} E_{1z} &= \frac{i}{8\pi\omega\varepsilon_1}\int_{-\infty}^{+\infty}\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}-Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times H_1^{(1)}(\lambda\rho)\cos\phi\lambda^2 d\lambda; \qquad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \end{aligned} \tag{12}$$

$$\begin{aligned} H_{1z} &= \frac{i}{8\pi\gamma_1}\int_{-\infty}^{+\infty}\left[e^{i\gamma_1|z-d|}+P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times H_1^{(1)}(\lambda\rho)\sin\phi\lambda^2 d\lambda. \end{aligned} \tag{13}$$

By using the relations in (4.7.9) and (4.7.10) in the book by Kong [29], the rest four field components can be obtained readily. We write

$$\begin{aligned} E_{1\rho} &= -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty}\gamma_1\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}+Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times\left[\lambda H_0^{(1)}(\lambda\rho)-\rho^{-1}H_1^{(1)}(\lambda\rho)\right]d\lambda \\ &\quad -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty}\gamma_1^{-1}\left[e^{i\gamma_1|z-d|}+P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times H_1^{(1)}(\lambda\rho)d\lambda \end{aligned} \tag{14}$$

$$\begin{aligned} E_{1\phi} &= -\frac{\omega\mu_0\sin\phi}{8\pi}\int_{-\infty}^{\infty}\gamma_1^{-1}\left[e^{i\gamma_1|z-d|}+P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times\left[\lambda H_0^{(1)}(\lambda\rho)-\rho^{-1}H_1^{(1)}(\lambda\rho)\right]d\lambda \\ &\quad +\frac{\sin\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty}\gamma_1\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}+Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\ &\quad \times H_1^{(1)}(\lambda\rho)d\lambda \end{aligned} \tag{15}$$

$$\begin{aligned}
H_{1\rho} &= \frac{\sin\phi}{8\pi}\int_{-\infty}^{\infty}\left[\mp e^{i\gamma_1|z-d|}-P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times\left[\lambda H_0^{(1)}(\lambda\rho)-\rho^{-1}H_1(1)(\lambda\rho)\right]d\lambda \\
&\quad-\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}-Q^{TM}r^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix}0\le z\le d\\ d\le z\end{matrix} \qquad (16)
\end{aligned}$$

$$\begin{aligned}
H_{1\phi} &= \frac{\cos\phi}{8\pi}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}-Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times\left[\lambda H_0^{(1)}(\lambda\rho)-\rho^{-1}H_1^{(1)}(\lambda\rho)\right]d\lambda \\
&\quad+\frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}-P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix}0\le z\le d\\ d\le z\end{matrix} \qquad (17)
\end{aligned}$$

It is convenient to express the above formulas of the six field components in the following forms.

$$E_{1\rho} = E_{1\rho}^{(1)}+E_{1\rho}^{(2)}+E_{1\rho}^{(3)} \qquad (18)$$

$$E_{1\phi} = E_{1\phi}^{(1)}+E_{1\phi}^{(2)}+E_{1\phi}^{(3)} \qquad (19)$$

$$E_{1z} = E_{1z}^{(1)}+E_{1z}^{(2)}+E_{1z}^{(3)} \qquad (20)$$

$$H_{1\rho} = H_{1\rho}^{(1)}+H_{1\rho}^{(2)}+H_{1\rho}^{(3)} \qquad (21)$$

$$H_{1\phi} = H_{1\phi}^{(1)}+H_{1\phi}^{(2)}+H_{1\phi}^{(3)} \qquad (22)$$

$$H_{1z} = H_{1z}^{(1)}+H_{1z}^{(2)}+H_{1z}^{(3)} \qquad (23)$$

where

$$\begin{aligned}
E_{1\rho}^{(1)} &= -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}+Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times H_0^{(1)}(\lambda\rho)\gamma_1\lambda d\lambda \qquad (24)
\end{aligned}$$

$$\begin{aligned}
E_{1\rho}^{(2)} &= \frac{\cos\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}+P^{TM}e^{i\gamma_1 z}+Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times H_1^{(1)}(\lambda\rho)\gamma_1 d\lambda \qquad (25)
\end{aligned}$$

$$\begin{aligned}
E_{1\rho}^{(3)} &= -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|}+P^{TE}e^{i\gamma_1 z}+Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \\
&\quad\times H_1^{(1)}(\lambda\rho)\gamma_1^{-1}d\lambda \qquad (26)
\end{aligned}$$

$$E_{1\phi}^{(1)} = \frac{\sin\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|} + P^{TM}e^{i\gamma_1 z} + Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho)\gamma_1 d\lambda \tag{27}$$

$$E_{1\phi}^{(2)} = -\frac{\omega\mu_0\sin\phi}{8\pi}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|} + P^{TE}e^{i\gamma_1 z} + Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \times H_0^{(1)}(\lambda\rho)\gamma_1^{-1}\lambda d\lambda \tag{28}$$

$$E_{1\phi}^{(3)} = \frac{\omega\mu_0\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[e^{i\gamma_1|z-d|} + P^{TE}e^{i\gamma_1 z} + Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho)\gamma_1^{-1} d\lambda \tag{29}$$

$$H_{1\rho}^{(1)} = \frac{\sin\phi}{8\pi}\int_{-\infty}^{\infty}\left[\mp e^{i\gamma_1|z-d|} - P^{TE}e^{i\gamma_1 z} + Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \times H_0^{(1)}(\lambda\rho)\lambda d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{30}$$

$$H_{1\rho}^{(2)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[\mp e^{i\gamma_1|z-d|} - P^{TE}e^{i\gamma_1 z} + Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho) d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{31}$$

$$H_{1\rho}^{(3)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[\pm e^{i\gamma_1|z-d|} + P^{TM}e^{i\gamma_1 z} - Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho) d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{32}$$

$$H_{1\phi}^{(1)} = \frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[\mp e^{i\gamma_1|z-d|} - P^{TE}e^{i\gamma_1 z} + Q^{TE}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho) d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{33}$$

$$H_{1\phi}^{(2)} = \frac{\cos\phi}{8\pi}\int_{-\infty}^{\infty}\left[\pm e^{i\gamma_1|z-d|} + P^{TM}e^{i\gamma_1 z} - Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \times H_0^{(1)}(\lambda\rho)\lambda d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{34}$$

$$H_{1\phi}^{(3)} = -\frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty}\left[\pm e^{i\gamma_1|z-d|} + P^{TM}e^{i\gamma_1 z} - Q^{TM}e^{i\gamma_1(2l_1-z)}\right] \times H_1^{(1)}(\lambda\rho) d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{35}$$

$$E_{1z}^{(1)} = \frac{i\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty}\pm e^{i\gamma_1|z-d|}\lambda^2 H_1^{(1)}(\lambda\rho) d\lambda; \quad \begin{matrix} 0\le z\le d \\ d\le z \end{matrix} \tag{36}$$

$$E_{1z}^{(2)} = \frac{i\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty} P^{TM}e^{i\gamma_1 z}\lambda^2 H_1^{(1)}(\lambda\rho)d\lambda \tag{37}$$

$$E_{1z}^{(3)} = -\frac{i\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty} Q^{TM}e^{i\gamma_1(2l_1-z)}\lambda^2 H_1^{(1)}(\lambda\rho)d\lambda \tag{38}$$

$$H_{1z}^{(1)} = \frac{i\sin\phi}{8\pi}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\lambda^2\gamma_1^{-1} H_1^{(1)}(\lambda\rho)d\lambda \tag{39}$$

$$H_{1z}^{(2)} = \frac{i\sin\phi}{8\pi}\int_{-\infty}^{\infty} P^{TE}e^{i\gamma_1 z}\lambda^2\gamma_1^{-1} H_1^{(1)}(\lambda\rho)d\lambda \tag{40}$$

$$H_{1z}^{(3)} = \frac{i\sin\phi}{8\pi}\int_{-\infty}^{\infty} Q^{TE}e^{i\gamma_1(2l_1-z)}\lambda^2\gamma_1^{-1} H_1^{(1)}(\lambda\rho)d\lambda. \tag{41}$$

By now, it is found that the integrated formulas of the electric and magnetic field components in the directions of $\hat{\rho}$ and $\hat{\phi}$ are still quite complex. In order to evaluate those integrals, it is necessary to divide those integrals into the electric-type terms and magnetic-type terms. We have

$$E_{1\rho}^{(1)} = E_{1\rho}^{(1,1)} + E_{1\rho}^{(1,2)} + E_{1\rho}^{(1,3)} \tag{42}$$

$$E_{1\rho}^{(2)} = E_{1\rho}^{(2,1)} + E_{1\rho}^{(2,2)} + E_{1\rho}^{(2,3)} \tag{43}$$

$$E_{1\rho}^{(3)} = E_{1\rho}^{(3,1)} + E_{1\rho}^{(3,2)} + E_{1\rho}^{(3,3)} \tag{44}$$

$$E_{1\phi}^{(1)} = E_{1\rho}^{(1,1)} + E_{1\rho}^{(1,2)} + E_{1\rho}^{(1,3)} \tag{45}$$

$$E_{1\phi}^{(2)} = E_{1\rho}^{(2,1)} + E_{1\rho}^{(2,2)} + E_{1\rho}^{(2,3)} \tag{46}$$

$$E_{1\phi}^{(3)} = E_{1\rho}^{(3,1)} + E_{1\rho}^{(3,2)} + E_{1\rho}^{(3,3)} \tag{47}$$

$$H_{1\rho}^{(1)} = H_{1\rho}^{(1,1)} + H_{1\rho}^{(1,2)} + H_{1\rho}^{(1,3)} \tag{48}$$

$$H_{1\rho}^{(2)} = H_{1\rho}^{(2,1)} + H_{1\rho}^{(2,2)} + H_{1\rho}^{(2,3)} \tag{49}$$

$$H_{1\rho}^{(3)} = H_{1\rho}^{(3,1)} + H_{1\rho}^{(3,2)} + H_{1\rho}^{(3,3)} \tag{50}$$

$$H_{1\phi}^{(1)} = H_{1\rho}^{(1,1)} + H_{1\rho}^{(1,2)} + H_{1\rho}^{(1,3)} \tag{51}$$

$$H_{1\phi}^{(2)} = H_{1\rho}^{(2,1)} + H_{1\rho}^{(2,2)} + H_{1\rho}^{(2,3)} \tag{52}$$

$$H_{1\phi}^{(3)} = H_{1\rho}^{(3,1)} + H_{1\rho}^{(3,2)} + H_{1\rho}^{(3,3)} \tag{53}$$

where

$$E_{1\rho}^{(1,1)} = -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \tag{54}$$

$$E_{1\rho}^{(2,1)} = \frac{\cos\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{55}$$

$$E_{1\rho}^{(3,1)} = -\frac{\omega\mu_0 \cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)d\lambda \tag{56}$$

$$E_{1\phi}^{(1,1)} = \frac{\sin\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{57}$$

$$E_{1\phi}^{(2,1)} = -\frac{\omega\mu_0 \sin\phi}{8\pi}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\lambda\gamma_1^{-1}H_0^{(1)}(\lambda\rho)d\lambda \tag{58}$$

$$E_{1\phi}^{(3,1)} = \frac{\omega\mu_0 \sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)d\lambda \tag{59}$$

$$H_{1\rho}^{(1,1)} = \frac{\sin\phi}{8\pi}\int_{-\infty}^{\infty} \mp e^{i\gamma_1|z-d|}\lambda H_0^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{60}$$

$$H_{1\rho}^{(2,1)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} \mp e^{i\gamma_1|z-d|}H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{61}$$

$$H_{1\rho}^{(3,1)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} \pm e^{i\gamma_1|z-d|}H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{62}$$

$$H_{1\phi}^{(1,1)} = \frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} \mp e^{i\gamma_1|z-d|}H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{63}$$

$$H_{1\phi}^{(2,1)} = -\frac{\cos\phi}{8\pi}\int_{-\infty}^{\infty} \pm e^{i\gamma_1|z-d|}\lambda H_0^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{64}$$

$$H_{1\phi}^{(3,1)} = -\frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} \pm e^{i\gamma_1|z-d|}H_1^{(1)}(\lambda\rho)d\lambda; \qquad \begin{matrix} 0 \le z \le d \\ d \le z \end{matrix} \tag{65}$$

$$E_{1\rho}^{(1,2)} = -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty} P^{TM}e^{i\gamma_1 z}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \tag{66}$$

$$E_{1\rho}^{(1,3)} = -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{-\infty}^{\infty} Q^{TM}e^{i\gamma_1(2l_1-z)}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \tag{67}$$

$$E_{1\rho}^{(2,2)} = \frac{\cos\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} P^{TM}e^{i\gamma_1 z}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{68}$$

$$E_{1\rho}^{(2,3)} = \frac{\cos\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} Q^{TM}e^{i\gamma_1(2l_1-z)}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{69}$$

$$E_{1\phi}^{(1,2)} = \frac{\sin\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} P^{TM}e^{i\gamma_1 z}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{70}$$

$$E_{1\phi}^{(1,3)} = \frac{\sin\phi}{8\pi\omega\varepsilon_1\rho}\int_{-\infty}^{\infty} Q^{TM}e^{i\gamma_1(2l_1-z)}\gamma_1 H_1^{(1)}(\lambda\rho)d\lambda \tag{71}$$

$$H_{1\rho}^{(3,2)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TM}e^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)d\lambda \tag{72}$$

$$H_{1\rho}^{(3,3)} = \frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TM} e^{i\gamma_1(2l_1-z)} H_1^{(1)}(\lambda\rho) d\lambda \tag{73}$$

$$H_{1\phi}^{(2,2)} = \frac{\cos\phi}{8\pi}\int_{-\infty}^{\infty} P^{TM} e^{i\gamma_1 z} H_0^{(1)}(\lambda\rho)\lambda d\lambda \tag{74}$$

$$H_{1\phi}^{(2,3)} = -\frac{\cos\phi}{8\pi}\int_{-\infty}^{\infty} Q^{TM} e^{i\gamma_1(2l_1-z)} H_0^{(1)}(\lambda\rho)\lambda d\lambda \tag{75}$$

$$H_{1\phi}^{(3,2)} = -\frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TM} e^{i\gamma_1 z} H_1^{(1)}(\lambda\rho) d\lambda \tag{76}$$

$$H_{1\phi}^{(3,3)} = \frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TM} e^{i\gamma_1(2l_1-z)} H_1^{(1)}(\lambda\rho) d\lambda \tag{77}$$

$$E_{1\rho}^{(3,2)} = -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z}\gamma_1^{-1} H_1^{(1)}(\lambda\rho) d\lambda \tag{78}$$

$$E_{1\rho}^{(3,3)} = -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)}\gamma_1^{-1} H_1^{(1)}(\lambda\rho) d\lambda \tag{79}$$

$$E_{1\phi}^{(2,2)} = -\frac{\omega\mu_0\sin\phi}{8\pi}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z}\lambda\gamma_1^{-1} H_0^{(1)} d\lambda \tag{80}$$

$$E_{1\phi}^{(2,3)} = -\frac{\omega\mu_0\sin\phi}{8\pi}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)}\lambda\gamma_1^{-1} H_0^{(1)} d\lambda \tag{81}$$

$$E_{1\phi}^{(3,2)} = \frac{\omega\mu_0\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z}\gamma_1^{-1} H_1^{(1)}(\lambda\rho) d\lambda \tag{82}$$

$$E_{1\phi}^{(3,3)} = \frac{\omega\mu_0\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)}\gamma_1^{-1} H_1^{(1)}(\lambda\rho) d\lambda \tag{83}$$

$$H_{1\rho}^{(1,2)} = -\frac{\sin\phi}{8\pi}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z}\lambda H_0^{(1)}(\lambda\rho) d\lambda \tag{84}$$

$$H_{1\rho}^{(1,3)} = \frac{\sin\phi}{8\pi}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)} H_0^{(1)}(\lambda\rho)\lambda d\lambda \tag{85}$$

$$H_{1\rho}^{(2,2)} = \frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z} H_1^{(1)}(\lambda\rho) d\lambda \tag{86}$$

$$H_{1\rho}^{(2,3)} = -\frac{\sin\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)} H_1^{(1)}(\lambda\rho) d\lambda \tag{87}$$

$$H_{1\phi}^{(1,2)} = -\frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} P^{TE} e^{i\gamma_1 z} H_1^{(1)}(\lambda\rho) d\lambda \tag{88}$$

$$H_{1\phi}^{(1,3)} = \frac{\cos\phi}{8\pi\rho}\int_{-\infty}^{\infty} Q^{TE} e^{i\gamma_1(2l_1-z)} H_1^{(1)}(\lambda\rho) d\lambda \tag{89}$$

It is seen that (36), (39), and (54)–(65) represent the direct incident wave. They have been evaluated in the book by King, Owens, and Wu. The integrals in (37), (38), and (66)–(77) involving Q^{TM} and P^{TM} are defined as the terms of electric type. The integrals in (40), (41), and (78)–(89) involving Q^{TE} and P^{TE} are defined as the terms of magnetic type. Next, we will attempt to evaluate the integrals in the electric-type and magnetic-type terms.

3. EVALUATION FOR THE ELECTRIC-TYPE FIELD

In this section, we will evaluate the electric-type integral $E_{1\rho}^{(1,2)}$. In order to evaluate the integral $E_{1\rho}^{(1,2)}$, it is necessary to shift the contour around the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The next main tasks are to determine the poles and to evaluate the integrations along the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. First, we will examine the pole equation of the electric-type field. It is

$$f^{TM}(\lambda) = \frac{\gamma_0\gamma_1}{k_0^2k_1^2} - i\frac{\gamma_1\gamma_2}{k_1^2k_2^2}\tan\gamma_2 l_2 - i\frac{\gamma_1^2}{k_1^4}\tan\gamma_1 l_1 - \frac{\gamma_0\gamma_2}{k_0^2k_2^2}\tan\gamma_1 l_1\tan\gamma_2 l_2 = 0. \tag{90}$$

Comparing with the electric-type pole equation of the four-layered cases as addressed in [25–28], it is seen that (90) is same as that addressed in [25–28]. It had been known that the poles may exist in the rang of $k_0 < \lambda < k_2$. The poles can be determined by using Newton's iteration method. Then, we have

$$E_{1\rho}^{(1,2)} = -\frac{i\cos\phi}{4\omega\varepsilon_1}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}\lambda_{jE}^*\gamma_1(\lambda_{jE}^*)H_0^{(1)}(\lambda_{jE}^*\rho) - \frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{\Gamma_0+\Gamma_1+\Gamma_2} P^{TM}e^{i\gamma_1 z}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \tag{91}$$

where

$$P_1^{TM}(\lambda) = \frac{\frac{\gamma_1\cos\gamma_1 d}{k_1^2} + \frac{\gamma_1\tan\gamma_1 l_1\sin\gamma_1 d}{k_1^2} - i\frac{\gamma_0\tan\gamma_1 l_1\cos\gamma_1 d}{k_0^2} + i\frac{\gamma_0\sin\gamma_1 d}{k_0^2}}{[f^{TM}(\lambda)]'} \cdot\left(\frac{\gamma_1}{k_1^2} + i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l_2\right) \tag{92}$$

where λ_{jE}^* is the poles of electric-type wave, and $[f^{TM}(\lambda)]'$ is written in the form

$$[f^{TM}(\lambda)]' = -\frac{\lambda}{k_0^2k_1^2}\left(\frac{\gamma_1}{\gamma_0} + \frac{\gamma_0}{\gamma_1}\right) + \frac{i\lambda}{k_1^2k_2^2}\left(\frac{\gamma_2}{\gamma_1}\tan\gamma_2 l_2 + \frac{\gamma_1}{\gamma_2}\tan\gamma_2 l_2\right.$$

$$+\gamma_1 l_2 \sec^2 \gamma_2 l_2\Big) + \frac{i\lambda}{k_1^4}\left(2\tan\gamma_1 l_1 + \gamma_1 l_1 \sec^2 \gamma_1 l_1\right) + \frac{\lambda}{k_0^2 k_2^2}\left(\frac{\gamma_2}{\gamma_0}\tan\gamma_1 l_1 \tan\gamma_2 l_2 + \frac{\gamma_0}{\gamma_2}\tan\gamma_1 l_1 \tan\gamma_2 l_2 + \frac{\gamma_0\gamma_2 l_1}{\gamma_1}\sec^2\gamma_1 l_1 \tan\gamma_2 l_2 + \gamma_0 l_2 \tan\gamma_1 l_1 \sec^2\gamma_2 l_2\right) \quad (93)$$

$$\gamma_n(\lambda_{jE}^*) = \sqrt{k_n^2 - \lambda^{*2}_{jE}}; \qquad n = 0, 1, 2. \quad (94)$$

Next, it is necessary to evaluate the integrals along the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. It is easily proved that the integrations along the branch line $\lambda = k_2$ is zero for the integrals in (91). Subject to the conditions of the far-field of $k_0\rho$ and $z + d \ll \rho$, it is seen that the dominant contributions of the integrations along the branch lines at $\lambda = k_0$ and $\lambda = k_1$ come from the vicinity of k_0 and that of k_1, respectively. First we will treat the integral along the branch line at $\lambda = k_1$. Let $\lambda = k_1(1 + i\tau^2)$, at the vicinity of k_1, the following values are approximated as

$$H_1^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_1 \rho}} e^{i(k_1\rho - \frac{3}{4}\pi)} \cdot e^{-k_1\rho\tau^2} \quad (95)$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx k_1 e^{i\frac{3}{4}\pi}\sqrt{2}\tau \quad (96)$$

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx i\sqrt{k_1^2 - k_0^2} = \gamma_{01} \quad (97)$$

$$\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \sqrt{k_2^2 - k_1^2} = \gamma_{21}. \quad (98)$$

Considering the case of interest that both l_1 and l_2 are not very large, we arrive at the following approximations.

$$\cos\gamma_1 d \approx 1; \quad \sin\gamma_1 d \approx \gamma_1 d; \quad \tan\gamma_1 l_1 \approx \gamma_1 l_1. \quad (99)$$

Neglecting the high-order terms of γ_1, the reflection coefficient P^{TM} is simplified as

$$P^{TM} = (\tau + A_{p1})B_{p1} \quad (100)$$

where

$$A_{p1} = \frac{k_1}{\sqrt{2}k_2^2} e^{-i\frac{\pi}{4}} \gamma_{21} \tan\gamma_{21} l_2 \quad (101)$$

$$B_{p1} = \frac{\sqrt{2}\left(\frac{1}{k_1^2} - i\frac{\gamma_{01} l_1}{k_0^2} + i\frac{\gamma_{01} d}{k_0^2}\right) e^{i\frac{3}{4}\pi}}{k_1\left(\frac{\gamma_{01}}{k_0^2 k_1^2} - i\frac{\gamma_{21}\tan\gamma_{21} l_2}{k_1^2 k_2^2} - \frac{\gamma_{01}\gamma_{21} l_1}{k_0^2 k_2^2}\tan\gamma_{21} l_2\right)}. \quad (102)$$

Considering the condition $\rho \gg z$, we find

$$e^{ik_1\rho + \frac{ik_1 z^2}{2\rho}} \approx e^{ik_1\sqrt{\rho^2 - z^2}}. \tag{103}$$

With the change of variable $\tau = \frac{e^{i\frac{5}{4}\pi}}{\sqrt{2}\rho} z - t$, and use is made of the following integrals,

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \tag{104}$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}} \tag{105}$$

the evaluation along the branch line $\lambda = k_1$ for the integral in (91) can be obtained readily. We write

$$\begin{aligned} I_1 &= -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{\Gamma_1} P^{TM} e^{i\gamma_1 z}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \\ &= \frac{\cos\phi B_{p1} k_1^2}{2\pi\omega\varepsilon_1\rho} e^{ik_1\sqrt{\rho^2+z^2}}\left[\frac{1}{2k_1\rho}\left(A_{p1} + \frac{3z}{\sqrt{2}\rho}e^{i\frac{5}{4}\pi}\right)\right. \\ &\quad \left. + \left(\frac{z^3}{2\sqrt{2}\rho^3}e^{-i\frac{\pi}{4}} + \frac{iz^2}{2\rho^2}A_{p1}\right)\right]. \end{aligned} \tag{106}$$

Similarly, the integral in (91) along the branch line $\lambda = k_0$ can also be evaluated by using the same manner. Let $\lambda = k_0(1 + i\tau^2)$, at the vicinity of k_0, the following values are approximated as

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0 e^{i\frac{3}{4}\pi}\sqrt{2}\tau \tag{107}$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \sqrt{k_1^2 - k_0^2} = \gamma_{10} \tag{108}$$

$$\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \sqrt{k_2^2 - k_0^2} = \gamma_{20}. \tag{109}$$

Then, the reflection coefficient P^{TM} can be expressed as follows:

$$P^{TM} = C_1\left(1 + \frac{B_{p2}}{\tau - A_{p2}}\right) \tag{110}$$

where

$$C_1 = \frac{-i\left(\frac{\gamma_{10}}{k_1^2} + i\frac{\gamma_{20}}{k_2^2}\tan\gamma_{20}l_2\right)(\tan\gamma_{10}l_1\cos\gamma_{10}d - \sin\gamma_{10}d)}{\frac{\gamma_{10}}{k_1^2} - \frac{\gamma_{20}}{k_2^2}\tan\gamma_{10}l_1\tan\gamma_{20}l_2} \tag{111}$$

$$A_{p2} = \frac{i\frac{\gamma_{10}\gamma_{20}}{k_1^2 k_2^2}\tan\gamma_{20}l_2 + i\frac{\gamma_{10}^2}{k_1^4}\tan\gamma_{10}l_1}{\frac{\sqrt{2}e^{i\frac{3}{4}\pi}}{k_0}\left(\frac{\gamma_{10}}{k_1^2} - \frac{\gamma_{20}}{k_2^2}\tan\gamma_{10}l_1\tan\gamma_{20}l_2\right)} \tag{112}$$

$$B_{p2} = A_{p2} + \frac{\frac{\gamma_{10}}{k_1^2}\cos\gamma_{10}d + \frac{\gamma_{10}}{k_1^2}\tan\gamma_{10}l_1\sin\gamma_{10}d}{\frac{\sqrt{2}e^{i\frac{\pi}{4}}}{k_0}\left(\tan\gamma_{10}l_1\cos\gamma_{10}d - \sin\gamma_{10}d\right)}. \tag{113}$$

Then, the integral in (91) along the branch line $\lambda = k_0$ can be evaluated readily. We write

$$\begin{aligned} I_2 &= -\frac{\cos\phi}{8\pi\omega\varepsilon_1}\int_{\Gamma_0} P^{TM}\lambda\gamma_1 H_0^{(1)}(\lambda\rho)d\lambda \\ &= -\frac{\cos\phi k_0^2}{2\pi\omega\varepsilon_1}C_1 B_{p2}\gamma_{10}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho+\frac{\pi}{4})} \\ &\quad\cdot\left[\frac{1}{2}\sqrt{\frac{\pi}{k_0\rho}} + i\frac{\pi}{2}A_{p2}e^{-k_0\rho A_{p2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \end{aligned} \tag{114}$$

Combined with (91), (106) and (114), we have

$$\begin{aligned} E_{1\rho}^{(1,2)} &= -\frac{i\cos\phi}{4\omega\varepsilon_1}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}\lambda_{jE}^*\gamma_1(\lambda_{jE}^*)H_0^{(1)}(\lambda_{jE}^*\rho) \\ &\quad+\frac{\cos\phi B_{p1}k_1^2}{2\pi\omega\varepsilon_1\rho}e^{ik_1\sqrt{\rho^2+z^2}}\left[\frac{1}{2k_1\rho}\left(A_{p1} + \frac{3z}{\sqrt{2}\rho}e^{i\frac{5}{4}\pi}\right)\right. \\ &\quad\left.+\left(\frac{z^3}{2\sqrt{2}\rho^3}e^{-i\frac{\pi}{4}} + \frac{iz^2}{2\rho^2}A_{p1}\right)\right] \\ &\quad-\frac{\cos\phi k_0^2}{2\pi\omega\varepsilon_1}C_1 B_{p2}\gamma_{10}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho+\frac{\pi}{4})} \\ &\quad\times\left[\frac{1}{2}\sqrt{\frac{\pi}{k_0\rho}} + i\frac{\pi}{2}A_{p2}e^{-k_0\rho A_{p2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right]. \end{aligned} \tag{115}$$

With the similar method, the rest terms can also be evaluated readily.

$$\begin{aligned} E_{1\rho}^{(1,3)} &= -\frac{i\cos\phi}{4\omega\varepsilon_1}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}\lambda_{jE}^*\gamma_1(\lambda_{jE}^*)H_0^{(1)}(\lambda_{jE}^*\rho) \\ &\quad+\frac{\cos\phi}{2\pi\omega\varepsilon_1\rho}B_{q1}k_1^2 e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\ &\quad\times\left\{\frac{1}{2k_1\rho}\left[\frac{3(2l_1-z)^3 e^{i\frac{5}{4}\pi}}{\sqrt{2}\rho} + A_{q1}\right]\right. \\ &\quad\left.+\left[\frac{(2l_1-z)^3 e^{-i\frac{\pi}{4}}}{2\sqrt{2}\rho^3} + \frac{i(2l_1-z)^2 A_{q1}}{2\rho^2}\right]\right\} \\ &\quad-\frac{\cos\phi}{4\pi\omega\varepsilon_1}C_2 B_{q2}k_0^2\gamma_{10}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho+\frac{\pi}{4})} \\ &\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \end{aligned} \tag{116}$$

$$\begin{aligned}
E_{1\rho}^{(2,2)} &= \frac{i\cos\phi}{4\omega\varepsilon_1\rho}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}\gamma_1(\lambda_{jE}^*)H_1^{(1)}(\lambda_{jE}^*\rho) \\
&+\frac{i\cos\phi}{4\pi\omega\varepsilon_1\rho^3}B_{p1}k_1e^{ik_1\sqrt{\rho^2+z^2}} \\
&\times\left[\left(\frac{3e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p1}\right)\frac{1}{k_1}+\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p1}\right)\frac{iz^2}{\rho}\right] \\
&+\frac{\cos\phi k_0\gamma_{10}}{4\pi\omega\varepsilon_1\rho}C_1B_{p2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (117)
\end{aligned}$$

$$\begin{aligned}
E_{1\rho}^{(2,3)} &= \frac{i\cos\phi}{4\omega\varepsilon_1\rho}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}\gamma_1(\lambda_{jE}^*)H_0^{(1)}(\lambda_{jE}^*\rho) \\
&+\frac{i\cos\phi}{4\pi\omega\varepsilon_1\rho^3}B_{q1}k_1e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\
&\times\left[\left(\frac{3e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q1}\right)\frac{1}{k_1}\right. \\
&\left.+\left(\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q1}\right)\frac{i(2l_1-z)^2}{\rho}\right] \\
&+\frac{\cos\phi k_0\gamma_{10}}{4\pi\omega\varepsilon_1\rho}C_2B_{q2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (118)
\end{aligned}$$

$$\begin{aligned}
E_{1\phi}^{(1,2)} &= \frac{i\sin\phi}{4\omega\varepsilon_1\rho}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}\gamma_1(\lambda_{jE}^*)H_1^{(1)}(\lambda_{jE}^*\rho) \\
&+\frac{i\sin\phi}{4\pi\omega\varepsilon_1\rho^3}B_{p1}k_1e^{ik_1\sqrt{\rho^2+z^2}}\left[\left(\frac{3e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p1}\right)\frac{1}{k_1}\right. \\
&\left.+\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p1}\right)\frac{iz^2}{\rho}\right] \\
&+\frac{\sin\phi k_0\gamma_{10}}{4\pi\omega\varepsilon_1\rho}C_1B_{p2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (119)
\end{aligned}$$

$$\begin{aligned}
E_{1\phi}^{(1,3)} &= \frac{i\sin\phi}{4\omega\varepsilon_1\rho}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}\gamma_1(\lambda_{jE}^*)H_0^{(1)}(\lambda_{jE}^*\rho) \\
&\quad+\frac{i\sin\phi}{4\pi\omega\varepsilon_1\rho^3}B_{q1}k_1e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\
&\quad\times\left[\left(\frac{3e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q1}\right)\frac{1}{k_1}\right. \\
&\quad\left.+\left(\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q1}\right)\frac{i(2l_1-z)^2}{\rho}\right] \\
&\quad+\frac{\sin\phi k_0\gamma_{10}}{4\pi\omega\varepsilon_1\rho}C_2B_{q2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (120)
\end{aligned}$$

$$\begin{aligned}
H_{1\rho}^{(3,2)} &= -\frac{i\sin\phi}{4\rho}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}H_1^{(1)}(\lambda_{jE}^*\rho) \\
&\quad-\frac{\sin\phi}{8\pi\rho^3}B_{p1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p1}\right) \\
&\quad-\frac{\sin\phi}{8\pi\rho}C_1B_{p2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\quad\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (121)
\end{aligned}$$

$$\begin{aligned}
H_{1\rho}^{(3,3)} &= \frac{i\sin\phi}{4\rho}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}H_1^{(1)}(\lambda_{jE}^*\rho) \\
&\quad+\frac{\sin\phi}{8\pi\rho^3}B_{q1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\
&\quad\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q1}\right) \\
&\quad+\frac{\sin\phi}{8\pi\rho}C_2B_{q2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (122)
\end{aligned}$$

$$H_{1\phi}^{(2,2)} = \frac{i\cos\phi}{4}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}\lambda_{jE}^*H_0^{(1)}(\lambda_{jE}^*\rho)$$

$$
\begin{aligned}
&+\frac{\cos\phi k_1}{8\pi\rho^2}B_{p1}e^{i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p1}\right)\\
&-\frac{\sin\phi}{8\pi}C_1B_{p2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho+\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (123)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\phi}^{(2,3)} = &-\frac{i\cos\phi}{4}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}\lambda_{jE}^*H_0^{(1)}(\lambda_{jE}^*\rho)\\
&-\frac{\cos\phi k_1}{8\pi\rho^2}B_{q1}e^{i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\\
&\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q1}\right)\\
&+\frac{\sin\phi}{8\pi}C_2B_{q2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho+\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (124)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\phi}^{(3,2)} = &-\frac{i\cos\phi}{8\pi\rho}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}H_1^{(1)}(\lambda_{jE}^*\rho)\\
&-\frac{\cos\phi}{8\pi\rho^3}B_{p1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p1}\right)\\
&-\frac{\cos\phi}{8\pi\rho}C_1B_{p2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (125)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\phi}^{(3,3)} = &\frac{\cos\phi}{8\pi\rho}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}H_1^{(1)}(\lambda_{jE}^*\rho)\\
&+\frac{\cos\phi}{8\pi\rho^3}B_{q1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\\
&\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q1}\right)\\
&+\frac{\cos\phi}{8\pi\rho}C_2B_{q2}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (126)
\end{aligned}
$$

$$\begin{aligned}
E_{1z}^{(2)} &= -\frac{\cos\phi}{4\omega\varepsilon_1}\sum_j P_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)z}(\lambda_{jE}^*)^2 H_1^{(1)}(\lambda_{jE}^*\rho) \\
&\quad+\frac{i\cos\phi}{8\pi\rho^2\omega\varepsilon_1}k_1^2 B_{p1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p1}\right) \\
&\quad+\frac{i\cos\phi}{8\pi\omega\varepsilon_1}k_0^3 C_1 B_{p2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p2}e^{-k_0\rho A_{p2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \qquad (127)
\end{aligned}$$

$$\begin{aligned}
E_{1z}^{(3)} &= \frac{\cos\phi}{4\omega\varepsilon_1}\sum_j Q_1^{TM}(\lambda_{jE}^*)e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}(\lambda_{jE}^*)^2 H_1^{(1)}(\lambda_{jE}^*\rho) \\
&\quad-\frac{i\cos\phi}{8\pi\rho^2\omega\varepsilon_1}k_1^2 B_{p1}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\
&\quad\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2_1-z)A_{q1}\right) \\
&\quad-\frac{i\cos\phi}{8\pi\omega\varepsilon_1}k_0^3 C_2 B_{q2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q2}e^{-k_0\rho A_{q2}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right] \qquad (128)
\end{aligned}$$

where

$$Q_1^{TM}(\lambda)=\frac{\left(\frac{\gamma_1}{k_1^2}\cos\gamma_1 d-\frac{\gamma_2}{k_2^2}\sin\gamma_1 d\tan\gamma_2 l_2\right)(1-i\tan\gamma_1 l_1)}{[f^{TM}(\lambda)]'}\times\left(\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}\right) \qquad (129)$$

$$A_{q1}=-\frac{\gamma_{01}}{\sqrt{2}e^{i\frac{3}{4}\pi}k_0^2 k_1\left(\frac{1}{k_1^2}+\frac{i\gamma_{01}l_1}{k_0^2}\right)} \qquad (130)$$

$$B_{q1}=\frac{\sqrt{2}e^{i\frac{3}{4}\pi}k_1\left(\frac{1}{k_1^2}+\frac{i\gamma_{01}l_1}{k_0^2}\right)\left(\frac{1}{k_1^2}-\frac{\gamma_{21}d\tan\gamma_{21}l_2}{k_2^2}\right)}{\frac{\gamma_{01}}{k_0^2k_1^2}-i\frac{\gamma_{21}}{k_2^2}\tan\gamma_{21}l_2\left(\frac{1}{k_1^2}-i\frac{\gamma_{01}l_1}{k_0^2}\right)} \qquad (131)$$

$$C_2=-\frac{\left(\frac{\gamma_{10}}{k_1^2}\cos\gamma_{10}d-\frac{\gamma_{20}}{k_2^2}\sin\gamma_{10}d\tan\gamma_{20}l_2\right)(1-i\tan\gamma_{10}l_1)}{\frac{\gamma_{10}}{k_1^2}-\frac{\gamma_{20}}{k_2^2}\tan\gamma_{10}l_1\tan\gamma_{20}l_2} \qquad (132)$$

$$A_{q2}=i\frac{\frac{\gamma_{10}^2}{k_1^4}\tan\gamma_{10}l_1+\frac{\gamma_{10}\gamma_{20}}{k_1^2k_2^2}\tan\gamma_{20}l_2}{\sqrt{2}k_0e^{i\frac{3}{4}\pi}\left(\frac{\gamma_{10}}{k_0^2k_1^2}-\frac{\gamma_{20}}{k_0^2k_2^2}\tan\gamma_{10}l_1\tan\gamma_{20}l_2\right)} \qquad (133)$$

$$B_{q2}=A_{q2}-\frac{k_0}{\sqrt{2}}e^{-i\frac{3}{4}\pi}\frac{\gamma_{10}}{k_1^2} \tag{134}$$

4. EVALUATION FOR THE MAGNETIC-TYPE FIELD

As an example, $E_{1\rho}^{(3,2)}$ is chosen to be evaluated specifically. In order to evaluate $E_{1\rho}^{(3,2)}$, we shift the contour around the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. Next, it is necessary to determine the poles and to evaluate the integrations along the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The pole equation of the magnetic-type terms is written in the following form.

$$\begin{aligned} f^{TE}(\lambda) &= \gamma_1\gamma_2 - i\gamma_0\gamma_2\tan\gamma_1 l_1 - i\gamma_0\gamma_1\tan\gamma_2 l_2 \\ &\quad -\gamma_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2 = 0. \end{aligned} \tag{135}$$

Clearly, the poles may exist in the range of $k_0 < \lambda < k_2$, and k_1 is a removable pole. The poles can be determined by using Newton's iteration method. Then, we have

$$\begin{aligned} E_{1\rho}^{(3,2)} &= -\frac{i\omega\mu_0\cos\phi}{4\rho}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}\gamma_1^{-1}(\lambda_{jB}^*)H_1^{(1)}(\lambda_{jB}^*\rho) \\ &\quad -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{\Gamma_0+\Gamma_1+\Gamma_2} P^{TE}e^{i\gamma_1 z}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)d\lambda \end{aligned} \tag{136}$$

where λ_{jB}^* is the pole of magnetic-type wave.

$$\begin{aligned} P_1^{TE}(\lambda) &= \frac{\gamma_2 + i\gamma_1\tan\gamma_2 l_2}{[f^{TE}(\lambda)]'}(\gamma_1\cos\gamma_1 d + \gamma_1\tan\gamma_1 l_1\sin\gamma_1 d \\ &\quad -i\gamma_0\tan\gamma_1 l_1\cos\gamma_1 d + i\gamma_0\sin\gamma_1 d) \end{aligned} \tag{137}$$

$$\begin{aligned} [f^{TE}(\lambda)]' &= -\frac{\lambda\gamma_2}{\gamma_1} - \frac{\lambda\gamma_1}{\gamma_2} + i\frac{\lambda\gamma_2}{\gamma_0}\tan\gamma_1 l_1 + i\frac{\lambda\gamma_0}{\gamma_2}\tan\gamma_1 l_1 \\ &\quad +i\frac{\gamma_0\gamma_2\lambda l_1}{\gamma_1}\sec^2\gamma_1 l_1 + i\frac{\lambda\gamma_1}{\gamma_0}\tan\gamma_2 l_2 + i\frac{\lambda\gamma_2}{\gamma_1}\tan\gamma_2 l_2 \\ &\quad +i\frac{\gamma_0\gamma_1\lambda l_2}{\gamma_2}\sec^2\gamma_2 l_2 + 2\lambda\tan\gamma_1 l_1\tan\gamma_2 l_2 \\ &\quad +\gamma_1\lambda l_1\sec^2\gamma_1 l_1\tan\gamma_2 l_2 + \frac{\gamma_1^2\lambda l_2}{\gamma_2}\tan\gamma_1 l_1\sec^2\gamma_2 l_2 \end{aligned} \tag{138}$$

$$\gamma_n(\lambda_{jB}^*) = \sqrt{k_n - {\lambda_{jB}^*}^2}; \qquad j = 0, 1, 2. \tag{139}$$

Similar to the case of the electric-type terms, it is seen that the integration along the branch line $\lambda = k_2$ is zero. It is necessary to evaluate the integrations along the branch lines $\lambda = k_0$ and . We first exam the integration along the branch line $\lambda = k_1$ in (136). Then, we write

$$P^{TE} = (\tau + A_{p3})B_{p3} \tag{140}$$

where

$$A_{p3} = \frac{\gamma_{21}}{i\sqrt{2}k_1 e^{i\frac{3}{4}\pi}\tan\gamma_{21}l_2} \tag{141}$$

$$B_{p3} = \frac{i\sqrt{2}k_1 e^{i\frac{3}{4}\pi}\tan\gamma_{21}l_2(1 - i\gamma_{01}l_1 + i\gamma_{01}d)}{\gamma_{21} - i\gamma_{01}\gamma_{21}l_1 - i\gamma_{01}\tan\gamma_{21}l_2}. \tag{142}$$

Subject to $k_0\rho \gg 1$ and $z + d \ll \rho$, we have

$$\begin{aligned} I_3 &= -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{\Gamma_1} P^{TE} e^{i\gamma_1 z}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)d\lambda \\ &= \frac{\omega\mu_0\cos\phi}{4\pi k_1\rho^2}B_{p3}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho} + A_{p3}\right). \end{aligned} \tag{143}$$

Next, we exam the integration along the branch line $\lambda = k_0$ in (136). Then, P^{TE} is expressed as follows:

$$P^{TE} = C_3\left(1 + \frac{B_{p4}}{\tau - A_{p4}}\right) \tag{144}$$

where

$$C_3 = \frac{(\tan\gamma_{10}l_1\cos\gamma_{10}d - \sin\gamma_{10}d)(\gamma_{20} + i\gamma_{10}\tan\gamma_{20}l_2)}{\gamma_{20}\tan\gamma_{10}l_1 + \gamma_{10}\tan\gamma_{20}l_2} \tag{145}$$

$$A_{p4} = \frac{\gamma_{10}\gamma_{20} - \gamma_{20}^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}{i\sqrt{2}k_0 e^{i\frac{3}{4}\pi}(\gamma_{20}\tan\gamma_{10}l_1 + \gamma_{10}\tan\gamma_{20}l_2)} \tag{146}$$

$$B_{p4} = A_{p4} + \frac{\gamma_{10}\cos\gamma_{10}d + \gamma_{10}\tan\gamma_{10}l_1\sin\gamma_{10}d}{-i\sqrt{2}k_0 e^{i\frac{3}{4}\pi}(\tan\gamma_{10}l_1\cos\gamma_{10}d - \sin\gamma_{10}d)}. \tag{147}$$

Similarly, we have

$$\begin{aligned} I_4 &= -\frac{\omega\mu_0\cos\phi}{8\pi\rho}\int_{\Gamma_0} P^{TE} e^{i\gamma_1 z}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)d\lambda \\ &= -\frac{\omega\mu_0\cos\phi}{4\pi\rho\gamma_{10}}k_0 C_3 B_{p4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i(\gamma_{10}z + k_0\rho - \frac{\pi}{4})} \\ &\quad \cdot\left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right]. \end{aligned} \tag{148}$$

With (136), (143), and (148), the final expression of $E_{1\rho}^{(3,2)}$ is written in the following form.

$$\begin{aligned} E_{1\rho}^{(3,2)} &= -\frac{i\omega\mu_0 \cos\phi}{4\rho} \sum_j P_1^{TE}(\lambda_{jB}^*) e^{i\gamma_1(\lambda_{jB}^*)z} \gamma_1^{-1}(\lambda_{jB}^*) H_1^{(1)}(\lambda_{jB}^*\rho) \\ &\quad + \frac{\omega\mu_0 \cos\phi}{4\pi k_1 \rho^2} B_{p3} e^{ik_1\sqrt{\rho^2+z^2}} \left(\frac{e^{i\frac{5}{4}\pi} z}{\sqrt{2}\rho} + A_{p3} \right) \\ &\quad - \frac{\omega\mu_0 \cos\phi}{4\pi\rho\gamma_{10}} k_0 C_3 B_{p4} \sqrt{\frac{2}{\pi k_0 \rho}} e^{i(\gamma_{10} z + k_0\rho - \frac{\pi}{4})} \\ &\quad \times \left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{p4} e^{-k_0\rho A_{p4}^2} \operatorname{erfc}\left(\sqrt{-k_0\rho A_{p4}^2} \right) \right]. \end{aligned} \tag{149}$$

With similar manner, the rest terms of the magnetic-type field components wave can be derived readily. We write

$$\begin{aligned} E_{1\rho}^{(3,3)} &= -\frac{i\omega\mu_0 \cos\phi}{4\rho} \sum_j \frac{Q_1^{TE}(\lambda_{jB}^*) e^{i\gamma_1(\lambda_{jB}^*)(2l_1 - z)} H_1^{(1)}(\lambda_{jB}^*\rho)}{\gamma_1(\lambda_{jB}^*)} \\ &\quad + \frac{\omega\mu_0 \cos\phi}{4\pi k_1 \rho^2} B_{q3} e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \left[\frac{e^{i\frac{5}{4}\pi}(2l_1 - z)}{\sqrt{2}\rho} + A_{q3} \right] \\ &\quad - \frac{\omega\mu_0 \cos\phi}{4\pi\rho\gamma_{10}} k_0 C_4 B_{q4} \sqrt{\frac{2}{\pi k_0 \rho}} e^{i\gamma_{10}(2l_1 - z)} e^{i(k_0\rho - \frac{\pi}{4})} \\ &\quad \times \left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{q4} e^{-k_0\rho {A_{q4}}^2} \operatorname{erfc}\left(\sqrt{-k_0\rho {A_{q4}}^2} \right) \right] \end{aligned} \tag{150}$$

$$\begin{aligned} E_{1\phi}^{(2,2)} &= -\frac{i\omega\mu_0 \sin\phi}{4} \sum_j P_1^{TE}(\lambda_{jB}^*) e^{i\gamma_1(\lambda_{jB}^*)z} \lambda_{jB}^* \gamma_1^{-1}(\lambda_{jB}^*) H_0^{(1)}(\lambda_{jB}^*) \\ &\quad + \frac{i\omega\mu_0 \sin\phi}{4\pi\rho} B_{p3} e^{ik_1\sqrt{\rho^2+z^2}} \left(\frac{e^{i\frac{5}{4}\pi} z}{\sqrt{2}\rho} + A_{p3} \right) \\ &\quad - \frac{i\omega\mu_0 \sin\phi}{4\pi\gamma_{10}} k_0^2 C_3 B_{p4} \sqrt{\frac{2}{\pi k_0 \rho}} e^{i(\gamma_{10} z + k_0\rho - \frac{\pi}{4})} \\ &\quad \times \left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{p4} e^{-k_0\rho {A_{p4}}^2} \operatorname{erfc}\left(\sqrt{-k_0\rho {A_{p4}}^2} \right) \right] \end{aligned} \tag{151}$$

$$E_{1\phi}^{(2,3)} = -\frac{i\omega\mu_0 \sin\phi}{4} \sum_j \frac{Q_1^{TE}(\lambda_{jB}^*) e^{i\gamma_1(\lambda_{jB}^*)(2l_1 - z)} \lambda_{jB}^* H_0^{(1)}(\lambda_{jB}^*\rho)}{\gamma_1(\lambda_{jB}^*)}$$

$$+\frac{i\omega\mu_0\sin\phi}{4\pi\rho}B_{q3}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\left(\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q3}\right)$$
$$-\frac{i\omega\mu_0\sin\phi}{4\pi\gamma_{10}}k_0^2C_4B_{q4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})}$$
$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho {A_{q4}}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho {A_{q4}}^2}\right)\right] \quad (152)$$

$$E_{1\phi}^{(3,2)} = \frac{i\omega\mu_0\sin\phi}{4\rho}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}\gamma_1^{-1}(\lambda_{jB}^*)H_1^{(1)}(\lambda_{jB}^*\rho)$$
$$-\frac{\omega\mu_0\sin\phi}{4\pi k_1\rho^2}B_{p3}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p3}\right)$$
$$+\frac{\omega\mu_0\sin\phi}{4\pi\rho\gamma_{10}}k_0C_3B_{p4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i(\gamma_{10}z+k_0\rho-\frac{\pi}{4})}$$
$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \quad (153)$$

$$E_{1\phi}^{(3,3)} = \frac{i\omega\mu_0\sin\phi}{4\rho}\sum_j Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)(2l_1-z)}\gamma_1^{-1}(\lambda_{jB}^*)H_1^{(1)}(\lambda_{jB}^*\rho)$$
$$-\frac{\omega\mu_0\sin\phi}{4\pi k_1\rho^2}B_{q3}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\left[\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q3}\right]$$
$$+\frac{\omega\mu_0\sin\phi}{4\pi\rho\gamma_{10}}k_0C_4B_{q4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})}$$
$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho {A_{q4}}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho {A_{q4}}^2}\right)\right] \quad (154)$$

$$H_{1\rho}^{(1,2)} = -\frac{i\sin\phi}{4}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}\lambda_{jB}^*H_0^{(1)}(\lambda_{jB}^*\rho)$$
$$-\frac{\sin\phi k_1}{8\pi\rho^2}B_{p3}e^{i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p3}\right)$$
$$-\frac{\sin\phi}{8\pi}C_3B_{p4}k_0^2\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho+\frac{\pi}{4})}$$
$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \quad (155)$$

$$H_{1\rho}^{(1,3)} = \frac{i\sin\phi}{4}\sum_j Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)(2l_1-z)}\lambda_{jB}^*H_0^{(1)}(\lambda_{jB}^*\rho)$$

$$
\begin{aligned}
&+\frac{\sin\phi k_1}{8\pi\rho^2}B_{q3}e^{i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\\
&\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q3}\right)\\
&+\frac{\sin\phi}{8\pi}C_4B_{q4}k_0^2\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho+\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho A_{q4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q4}^2}\right)\right] \qquad (156)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\rho}^{(2,2)} &= \frac{i\sin\phi}{4\rho}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}H_1^{(1)}(\lambda_{jB}^*\rho)\\
&+\frac{\sin\phi}{8\pi\rho^3}B_{p3}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p3}\right)\\
&+\frac{\sin\phi}{8\pi\rho}C_3B_{p4}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \qquad (157)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\rho}^{(2,3)} &= -\frac{i\sin\phi}{4\rho}\sum_j Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}H_1^{(1)}(\lambda_{jB}^*\rho)\\
&-\frac{\sin\phi}{8\pi\rho^3}B_{q3}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\\
&\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q3}\right)\\
&-\frac{\sin\phi}{8\pi\rho}C_4B_{q4}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho A_{q4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q4}^2}\right)\right] \qquad (158)
\end{aligned}
$$

$$
\begin{aligned}
H_{1\phi}^{(1,2)} &= -\frac{i\cos\phi}{4\rho}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}H_1^{(1)}(\lambda_{jB}^*\rho)\\
&-\frac{\cos\phi}{8\pi\rho^3}B_{p3}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{1}{\sqrt{2}k_1}+\frac{iz^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}zA_{p3}\right)\\
&-\frac{\cos\phi}{8\pi\rho}C_3B_{p4}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}z}e^{i(k_0\rho-\frac{\pi}{4})}\\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \qquad (159)
\end{aligned}
$$

$$\begin{aligned} H_{1\phi}^{(1,3)} &= \frac{i\cos\phi}{4\rho}\sum_j Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}H_1^{(1)}(\lambda_{jB}^*\rho) \\ &\quad+\frac{\cos\phi}{8\pi\rho^3}B_{q3}e^{-i\frac{\pi}{4}}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\ &\quad\times\left(\frac{1}{\sqrt{2}k_1}+\frac{i(2l_1-z)^2}{\sqrt{2}\rho}+e^{i\frac{5}{4}\pi}(2l_1-z)A_{q3}\right) \\ &\quad+\frac{\cos\phi}{8\pi\rho}C_4B_{q4}k_0\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\ &\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho A_{q4}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q4}^2}\right)\right] \end{aligned} \tag{160}$$

$$\begin{aligned} H_{1z}^{(2)} &= -\frac{\sin\phi}{4}\sum_j P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}(\lambda_{jB}^*)^2\gamma_1^{-1}(\lambda_{jB}^*)H_1^{(1)}(\lambda_{jB}^*\rho) \\ &\quad-\frac{i\sin\phi}{4\pi\rho}k_1B_{p3}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p3}\right) \\ &\quad+\frac{i\sin\phi}{4\pi\gamma_{10}}k_0^3C_3B_{p4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i(\gamma_{10}z+k_0\rho-\frac{\pi}{4})} \\ &\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \end{aligned} \tag{161}$$

$$\begin{aligned} H_{1z}^{(3)} &= -\frac{\sin\phi}{4}\sum_j \frac{Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)(2l_1-z)}(\lambda_{jB}^*)^2H_1^{(1)}(\lambda_{jB}^*\rho)}{\gamma_1(\lambda_{jB}^*)} \\ &\quad-\frac{i\sin\phi}{4\pi\rho}k_1B_{q3}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\left(\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q3}\right) \\ &\quad+\frac{i\sin\phi}{4\pi\gamma_{10}}k_0^3C_4B_{q4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\ &\quad\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho A_{q4}^2}\text{erfc}\left(\sqrt{-k_0\rho A_{q4}^2}\right)\right] \end{aligned} \tag{162}$$

where

$$Q_1^{TE}(\lambda)=\frac{i(\gamma_1-\gamma_0)(1-i\tan\gamma_1l_1)(\gamma_1\tan\gamma_2l_2\cos\gamma_1d+\gamma_2\sin\gamma_1d)}{[f^{TE}(\lambda)]'} \tag{163}$$

$$A_{q3}=-\frac{\gamma_{01}}{\sqrt{2}k_1e^{i\frac{3}{4}\pi}(1+i\gamma_{01}l_1)} \tag{164}$$

$$B_{q3}=\frac{i(\tan\gamma_{21}l_2+\gamma_{21}d)\sqrt{2}k_1e^{i\frac{3}{4}\pi}(1+i\gamma_{01}l_1)}{\gamma_{21}-i\gamma_{01}\tan\gamma_{21}l_2-i\gamma_{01}\gamma_{21}l_1} \tag{165}$$

$$C_4 = \frac{(1 - i\tan\gamma_{01}l_1)(\gamma_{10}\tan\gamma_{20}l_2\cos\gamma_{10}d + \gamma_{20}\sin\gamma_{10}d)}{\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1} \tag{166}$$

$$A_{q4} = \frac{\gamma_{10}\gamma_{20} - \gamma_{10}^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}{i\sqrt{2}k_0 e^{i\frac{3}{4}\pi}(\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1)} \tag{167}$$

$$B_{q4} = A_{q4} - \frac{\gamma_{10}}{\sqrt{2}k_0 e^{i\frac{3}{4}\pi}}. \tag{168}$$

5. FINAL FORMULAS FOR THE FIELD COMPONENTS

With the above results and those for the direct field addressed in the book by King, Owens, and Wu [8], the final formulas for the six components of the electromagnetic field can be obtained readily. The completed formulas of the vertical electric field $E_{1z}(\rho, \phi, z)$ and the vertical magnetic field $H_{1z}(\rho, \phi, z)$ can be expressed as follows:

$$\begin{aligned}
E_{1z} \; = \; & \frac{i\cos\phi}{4\pi\omega\varepsilon_1} e^{ik_1\gamma_1} \left(\frac{\rho}{r_1}\right)\left(\frac{z-d}{r_1}\right)\left(-\frac{k_1^2}{r_1} - \frac{3ik_1}{r_1^2} + \frac{3}{r_1^3}\right) \\
& -\frac{\cos\phi}{4\omega\varepsilon_1}\sum_j P_1^{TM}(\lambda_{jE}^*) e^{i\gamma_1(\lambda_{jE}^*)z}(\lambda_{jE}^*)^2 H_1^{(1)}(\lambda_{jE}^*\rho) \\
& +\frac{\cos\phi}{4\omega\varepsilon_1}\sum_j Q_1^{TM}(\lambda_{jE}^*) e^{i\gamma_1(\lambda_{jE}^*)(2l_1-z)}(\lambda_{jE}^*)^2 H_1^{(1)}(\lambda_{jE}^*\rho) \\
& +\frac{i\cos\phi}{8\pi\rho^2\omega\varepsilon_1} k_1^2 B_{p1} e^{-i\frac{\pi}{4}} e^{ik_1\sqrt{\rho^2+z^2}} \left(\frac{1}{\sqrt{2}k_1} + \frac{iz^2}{\sqrt{2}\rho} + e^{i\frac{5}{4}\pi} z A_{p1}\right) \\
& +\frac{i\cos\phi}{8\pi\omega\varepsilon_1} k_0^3 C_1 B_{p2}\sqrt{\frac{2}{\pi k_0\rho}} e^{i\gamma_{10}z} e^{i(k_0\rho-\frac{\pi}{4})} \\
& \times\left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{p2} e^{-k_0\rho A_{p2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p2}^2}\right)\right] \\
& -\frac{i\cos\phi}{8\pi\rho^2\omega\varepsilon_1} k_1^2 B_{p1} e^{-i\frac{\pi}{4}} e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}} \\
& \times\left(\frac{1}{\sqrt{2}k_1} + \frac{i(2l_1-z)^2}{\sqrt{2}\rho} + e^{i\frac{5}{4}\pi}(2_1 - z)A_{q1}\right) \\
& -\frac{i\cos\phi}{8\pi\omega\varepsilon_1} k_0^3 C_2 B_{q2}\sqrt{\frac{2}{\pi k_0\rho}} e^{i\gamma_{10}(2l_1-z)} e^{i(k_0\rho-\frac{\pi}{4})} \\
& \times\left[\sqrt{\frac{\pi}{k_0\rho}} + i\pi A_{q2} e^{-k_0\rho A_{q2}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q2}^2}\right)\right]
\end{aligned} \tag{169}$$

$$
\begin{aligned}
H_{1z} =& -\frac{i\sin\phi}{4\pi}e^{ik_1r_1}\left(\frac{\rho}{r_1}\right)\left(\frac{k_1}{r_1}+\frac{i}{r_1^2}\right) \\
&-\frac{\sin\phi}{4}\sum_j\frac{P_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)z}(\lambda_{jB}^*)^2H_1^{(1)}(\lambda_{jB}^*\rho)}{\gamma_1(\lambda_{jB}^*)} \\
&-\frac{\sin\phi}{4}\sum_j\frac{Q_1^{TE}(\lambda_{jB}^*)e^{i\gamma_1(\lambda_{jB}^*)(2l_1-z)}(\lambda_{jB}^*)^2H_1^{(1)}(\lambda_{jB}^*\rho)}{\gamma_1(\lambda_{jB}^*)} \\
&-\frac{i\sin\phi}{4\pi\rho}k_1B_{p3}e^{ik_1\sqrt{\rho^2+z^2}}\left(\frac{e^{i\frac{5}{4}\pi}z}{\sqrt{2}\rho}+A_{p3}\right) \\
&+\frac{i\sin\phi}{4\pi\gamma_{10}}k_0^3C_3B_{p4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i(\gamma_{10}z+k_0\rho-\frac{\pi}{4})} \\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{p4}e^{-k_0\rho A_{p4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{p4}^2}\right)\right] \\
&-\frac{i\sin\phi}{4\pi\rho}k_1B_{q3}e^{ik_1\sqrt{\rho^2+(2l_1-z)^2}}\left(\frac{e^{i\frac{5}{4}\pi}(2l_1-z)}{\sqrt{2}\rho}+A_{q3}\right) \\
&+\frac{i\sin\phi}{4\pi\gamma_{10}}k_0^3C_4B_{q4}\sqrt{\frac{2}{\pi k_0\rho}}e^{i\gamma_{10}(2l_1-z)}e^{i(k_0\rho-\frac{\pi}{4})} \\
&\times\left[\sqrt{\frac{\pi}{k_0\rho}}+i\pi A_{q4}e^{-k_0\rho A_{q4}^2}\mathrm{erfc}\left(\sqrt{-k_0\rho A_{q4}^2}\right)\right] \qquad (170)
\end{aligned}
$$

Evidently, the completed formulas for the rest four components can also be written directly. In this paper, those formulas not listed one by one.

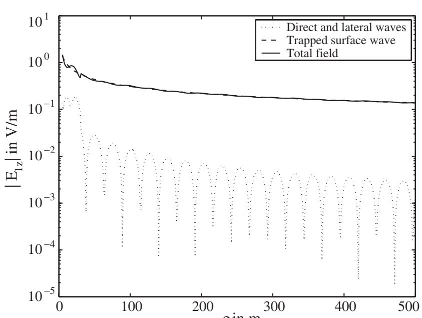

Figure 2. Vertical electric field $E_{0\rho}(\rho, 0, z)$ in V/m at $k_1l_1 = k_2l_2 = 0.5$ with $f = 100\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $z = d = 0\,\mathrm{m}$.

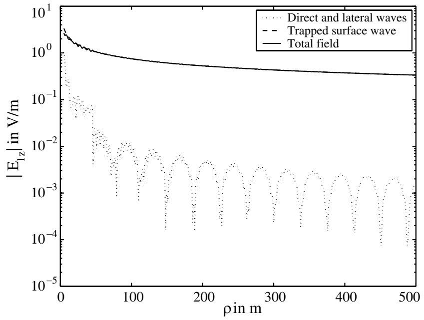

Figure 3. Vertical electric field $E_{0\rho}(\rho, 0, z)$ in V/m at $k_1 l_1 = k_2 l_2 = 1.5$ with $f = 100\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $z = d = 0\,\mathrm{m}$.

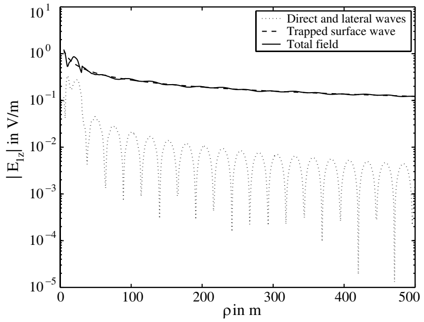

Figure 4. Vertical electric field $E_{0\rho}(\rho, 0, z)$ in V/m at $k_1 d = 0.3$ and $k_1 z = 0.4$ with $f = 100\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $k_1 l_1 = k_2 l_2 = 1.5$.

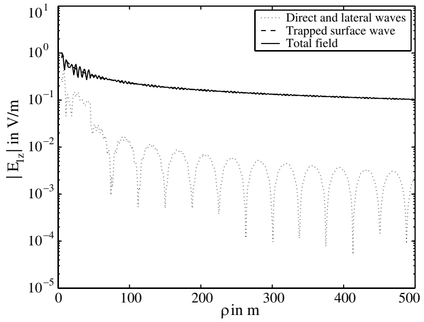

Figure 5. Vertical electric field $E_{0\rho}(\rho, 0, z)$ in V/m at $k_1 d = 1.2$ and $k_1 z = 1.4$ with $f = 100\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $k_1 l_1 = k_2 l_2 = 1.5$.

6. COMPUTATION AND CONCLUSION

For the trapped surface waves, both the wave numbers λ^*_{jE} and λ^*_{jB} are in the range from k_0 to k_2. When both λ^*_{jE} and λ^*_{jB} are in the range from k_0 to k_1, both $\gamma_1(\lambda^*_{jE}) = \sqrt{k_1^2 - {\lambda^*_{jE}}^2}$ and $\gamma_1(\lambda^*_{jB}) = \sqrt{k_1^2 - {\lambda^*_{jB}}^2}$ are positive real numbers, that is to say, the trapped surface waves along the boundaries $z = 0$ and $z = l_1$ have not an attenuated factor in the $\hat{z}$ direction. When both λ^*_{jE} and λ^*_{jB} are in the range from k_1 to k_2, both $\gamma_1(\lambda^*_{jB}) = i\sqrt{{\lambda^*_{jE}}^2 - k_1^2}$ and $\gamma_1(\lambda^*_{jB}) = i\sqrt{{\lambda^*_{jB}}^2 - k_1^2}$ are positive imagine numbers. The trapped surface waves attenuates exponentially as $e^{-\sqrt{{\lambda^*_{jB}}^2 - k_1^2}z}$ along the boundary $z = 0$ in the $\hat{z}$ direction and attenuates exponentially as $e^{-\sqrt{{\lambda^*_{jB}}^2 - k_1^2}(2l_1 - z)}$ along the boundary $z = l_1$ in the $\hat{z}$ direction. In this paper, both the poles λ^*_{jE} of electric type and the poles λ^*_{jB} can be determined by using Newton's iteration method.

The lateral wave consists of two parts. The first lateral wave propagates in Region 0 along the boundary $z = l_1$ between Regions 0 and 1 with the wave number k_0. The second lateral wave propagates in Region 1 along the boundary $z = 0$ between Regions 1 and 2 with the wave number k_1.

With $f = 100\,\text{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $z = d = 0\,\text{m}$, for vertical electric field $E_{0\rho}(\rho, 0, z)$, the total field, the terms of direct wave and lateral wave, and the trapped-surface-wave term are computed at $k_1 l_1 = k_2 l_2 = 0.5$ and $k_1 l = k_2 l_2 = 1.5$ shown in Figs. 2 and 3, respectively. With $f = 100\,\text{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r1} = 4$, and $k_1 l_1 = k_2 l_2 = 1.5$, the corresponding results are computed at $k_1 d = 0.3$, $k_1 z = 0.4$ and $k_1 d = 1.2$ and $k_1 z = 1.4$ and shown in Figs. 4 and 5, respectively. It is noted that, because of multi-reflections, the term of the ideal reflected wave cannot be separated to the lateral-wave terms. The results obtained may have useful practical applications in microstrip antenna with super substrate.

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