Progress In Electromagnetics Research, PIER 94, 105–117, 2009

INVESTIGATIONS OF THE ELECTROMAGNETIC PROPERTIES OF THREE-DIMENSIONAL ARBITRARILY-SHAPED CLOAKS

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Abstract—We investigate the electromagnetic properties of three-dimensional (3D) arbitrarily-shaped invisible cloaks based on the analytical field transformation theory instead of the complicated numerical simulations. Very simple closed-form expressions for fields and energy flows have been derived for arbitrarily-shaped 3D cloak, which could help us to investigate the electromagnetic properties of the 3D cloaks rapidly and efficiently. The difference between 2D and 3D cloaks have been compared in detail. Distributions of the fields, power flows and wave polarizations for the 3D case have been discussed inside the cloak. Numerical results have been presented at the cutplanes of cloaks to valid the theoretical analysis, which shows clearly how the incident waves are bent at the inner boundary. In order to further reveal the physical essence of the cloaks, both 3D spherical and ellipsoidal cloaks have been considered based on the analytical method. The common features and the differences for the two structures have been also illustrated in this paper.

1. INTRODUCTION

In the past three years, the method of transformation optics has aroused great interest which offers scientists an effective means to control electromagnetic (EM) fields. The theory of transformation optics was first proposed by Pendry [1], which is based on the

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coordinate transformation and the invariance of Maxwell's equations. If a cylindrical region is compressed into a cylindrical shell, one will obtain an ideal two-dimensional (2D) cloak and the object inside the cylindrical shell will become invisible [1]. A number of 2D cloaks with different shapes have been proposed due to various application environments [2–16, 21–28].

In most of the published papers on 2D cloaks, people use transformation optics to determine the the permittivity and permeability of the transformed medium based on a given coordinate transformation. Then numerical approaches like the finite-element method (or the commercial software Comsol) and the finite-difference time-domain method have been used to simulate the EM properties numerically [2–13, 20–24]. However, the full-wave analysis and simulation of three-dimensional (3D) cloaks are still limited due to the large amount of computational burden. For example, the Comsol software cannot be used to simulate a large 3D cloak since the huge memory requirement and computational time. Hence the study of 3D cloaks is not as popular as that of 2D cloaks, and only some specific research has been conducted. The ray-tracing simulations of 3D spherical cloaks were reported in the geometric-optics limit [1, 7]. A rigorous solution to Maxwell's equations was then given for the spherical cloak [15, 25], confirming the cloaking effect. Using the similar method, the interaction between a spherical cloak and the incident radiation from internal sources was also investigated [16], in which the inner boundary had extraordinary surface voltages to prevent EM waves generated by the internal sources from going out. Later, the discrete dipole approximation method has been applied to simulate the 3D spherical cloaks [17] and irregular 3D cloaks [18] approximately.

In this paper, we have investigated the EM properties of 3D arbitrarily-shaped invisible cloaks from another point of view.The rigorous expressions for fields and power flows inside the arbitrarily-shaped cloaks have been derived for the first time based on the analytical field transformation theory [1, 5, 14], which has been shown to be valid for the transformation optics. Through the analysis of spherical and ellipsoidal cloaks, we can obtain the details of the EM properties inside the cloak, such as the field and power flows in all directions in the transformed space of arbitrary shape. This is a rapid and efficient method for the analysis of these problems, and the numerical results have validated the theoretical predictions.

2. GENERAL THEORY

Due to the metric invariance of Maxwell's equations, we can easily get the general properties of the transformed media. Assume that x^i $(i = 1, 2, 3)$ are the coordinates of the original space, that is, the Cartesian coordinates. Similarly, $x^{i'}$ $(i' = 1, 2, 3)$ are the coordinates of the transformed space, with the metric tensor $g^{i'j'}$. After lengthy mathematic manipulations [7], we derive the permittivity and permeability tensors in the transformation media as

$$\epsilon^{i'j'} = [\det(\wedge_i^{i'})]^{-1} \wedge_i^{i'} \wedge_j^{j'} \epsilon^{ij}, \tag{1}$$

$$\mu^{i'j'} = [\det(\wedge_i^{i'})]^{-1} \wedge_i^{i'} \wedge_j^{j'} \mu^{ij}, \tag{2}$$

where $\wedge_i^{i'}$ are elements of the Jacobian transformation matrix, with

$$\wedge_i^{i'} = \frac{\partial x^{i'}}{\partial x^i}, \tag{3}$$

and $\det(\wedge_i^{i'})$ is the determinant of the Jacobi matrix. Once the transformation matrix is determined, the components of the electric and magnetic fields could be expressed as

$$E_{i'} = \wedge_{i'}^{i} E_i. \tag{4}$$

Therefore the EM fields in the transformed space can be obtained immediately no matter how complicated the defined transformation is:

$$\begin{bmatrix} E_{x'} \\ E_{y'} \\ E_{z'} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial x'} & \frac{\partial y}{\partial x'} & \frac{\partial z}{\partial x'} \\ \frac{\partial x}{\partial y'} & \frac{\partial y}{\partial y'} & \frac{\partial z}{\partial y'} \\ \frac{\partial x}{\partial z'} & \frac{\partial y}{\partial z'} & \frac{\partial z}{\partial z'} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \tag{5}$$

This equation is very useful to solve complicated 3D transformation-optics problems without numerical simulations, which is named field transformation theory by us in this paper. For most of the transformation media, the components of permittivity and permeability tensors vary with the spatial positions. In numerical simulations of complicated 3D problems, large amount of mesh discretizations have to be performed, and hence the memory requirement and computing time will be extremely large. This limitation essentially eliminated the personal computer as a practical tool for simulating complicated 3D cloaks. The analytical expression, Eq. (5), provides an efficient method to observe the field distributions inside the transformation media.

Based on the field transformation theory mentioned above [1, 7, 14], next we further discuss more details of the field properties in the transformed media, such as the wave polarization and power flows, which have not been extensively investigated in the published papers. First we need to emphasize that in the transformed media, the polarization type of the incident waves may be changed. For example, when a TE-polarized wave is incident upon a 3D cloak, the waves inside the cloak will have both TE and TM polarized, as we can see from Eq. (5). Some components of the electric/magnetic fields are discontinuous on the outer boundary of the cloak, but the whole cloak is still reflectionless for incident waves due to the impedance matching between the cloak and the surrounding free space.

An important concept we must remark in the transformation is the relation between the space transformation and the material implementation. As shown in Eqs. (1) and (2), since we use the transformed media, which are composed of anisotropic metamaterials, to control the EM waves, the calculated field components in the new coordinates from Eq. (5) are actually those components in the Cartesian coordinate. That is to say, under the condition of material implementation, the coordinate system always keeps the same, which is very important for people to investigate the properties of EM waves inside the transformed media.

From Eq. (5), we can immediately get the components of the Poynting vector in the original and transformed media. As stated above, the polarization of incident field will be changed and the electric and magnetic fields may have components in all directions. Hence the Poynting vector could be written as

$$S_x = S_{x'} = E_{y'} H^*_{z'} - E_{z'} H^*_{y'}, \tag{6}$$

$$S_y = S_{y'} = E_{z'} H^*_{x'} - E_{x'} H^*_{z'}, \tag{7}$$

$$S_z = S_{z'} = E_{x'} H^*_{y'} - E_{y'} H^*_{x'}. \tag{8}$$

Eqs. (6)–(8) are extremely important for people to observe the power flows at different positions in the transformed media. We can directly get the distributions, directions and amplitudes of the power flows, and decide whether the cloaks or other functional devices based on the optical transformation work or not.

3. FIELD TRANSFORMATION FOR SPHERICAL CLOAKS

Now we investigate two kinds of 3D cloaks using the field transformation theory. First we consider spherical cloaks. As proposed in [1], in order to conceal the object inside a sphere, we should compress a spherical region into a spherical shell. Let the inner and outer radii of the spherical shell be a and b, the coordinates in the transformed space be (x', y', z'), the coordinates in the original space be (x, y, z), $r = \sqrt{x^2 + y^2 + z^2}$, and $r' = \sqrt{x'^2 + y'^2 + z'^2}$. Then the transformation equation could be written as

$$r' = \frac{b-a}{b} r + a, \qquad (r \leq b), \tag{9}$$

$$r' = r, \qquad (r > b). \tag{10}$$

We assume that the incident plane wave is TE-polarized to the y direction with the electric field $E_x = e^{iky}$ and the magnetic field $H_z = -e^{iky}/\eta_0$, where k and η_0 are the wave number and wave impedance of free space, respectively. According to the transformation invariance, the unit vectors in the original and transformed spaces must be equal:

$$\frac{x}{r} = \frac{x'}{r'}, \quad \frac{y}{r} = \frac{y'}{r'}, \quad \frac{z}{r} = \frac{z'}{r'}. \tag{11}$$

Hence the Jacobian transformation matrix could be expressed as

$$\wedge_{i'}^{i} = \begin{bmatrix} \frac{b}{b-a}\left(1-\frac{a(y'^2+z'^2)}{r'^3}\right) & \frac{abx'y'}{(b-a)r'^3} & \frac{abx'z'}{(b-a)r'^3} \\ \frac{abx'y'}{(b-a)r'^3} & \frac{b}{b-a}\left(1-\frac{a(x'^2+z'^2)}{r'^3}\right) & \frac{aby'z'}{(b-a)r'^3} \\ \frac{abx'z'}{(b-a)r'^3} & \frac{aby'z'}{(b-a)r'^3} & \frac{b}{b-a}\left(1-\frac{a(x'^2+y'^2)}{r'^3}\right) \end{bmatrix}. \tag{12}$$

From Eq. (4), we easily obtain the electric fields inside the spherical shell as

$$E_x = E_{x'} = \frac{b}{b-a}\left(1 - \frac{a(y'^2 + z'^2)}{r'^3}\right) e^{iky}, \tag{13}$$

$$E_y = E_{y'} = \frac{abx'y'}{(b-a)r'^3} e^{iky}, \tag{14}$$

$$E_z = E_{z'} = \frac{abx'z'}{(b-a)r'^3} e^{iky}. \tag{15}$$

Outside the spherical cloak, we have

$$E_x = E_{x'}, \quad E_y = E_{y'} = 0, \quad E_z = E_{z'} = 0. \tag{16}$$

The magnetic field inside the cloak could be obtained similarly. We remark that the spherical shell region is the transformed space. Hence the cartesian coordinates (x, y, z) for any position inside the shell are actually new coordinates (x', y', z') in the curved space used in Eq. (12). One should be careful on the relation between the original and transformed spaces during the whole field transformation.

From Eqs. (13)–(15), we obtain very simple closed-form expressions for 3D spherical cloak, which are much simpler than other analytical methods. From Eqs. (13)–(15) and the resulting power-flow formulas, one can easily reveal the physical insights of the 3D spherical cloak. We remark that Eqs. (13)–(15) can provide exactly the same results as the series expansion, as shown in the later simulations [15].

From Eq. (12), it is obvious that a TM-to-y wave has appeared inside the cloak due to the emergence of E_y and E_z. Therefore two kinds of polarized waves co-exist within the spherical shell, which is different from the the 2D case, where $\wedge^x_{z'} = \wedge^y_{z'} = 0$ and $\wedge^z_{z'} = 1$, as shown in Eq. (25) in [7]. For the 2D cylindrical cloak, the wave polarization inside the cloak relies on the polarization of incident wave. When the electric field of the incident wave is polarized in the z direction, the polarization keeps unchanged inside the cloak since there is no spatial transformation in that direction in the 2D case. However, when the incident wave is polarized in other directions, both TE and TM waves also exist in the 2D cloak.

As an example, the distributions of electric fields E_x at different

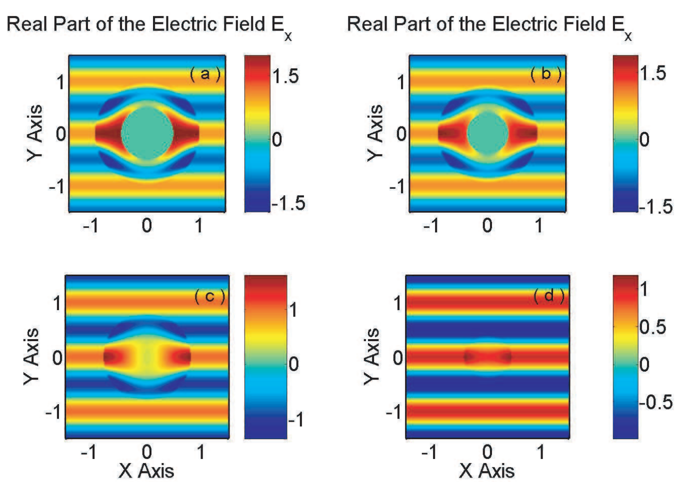

Figure 1. The distributions of electric fields E_x on different cut-planes inside and outside the 3D spherical cloak, where $a = 0.5\,\mathrm{m}$, $b = 1\,\mathrm{m}$, and $f = 300\,\mathrm{MHz}$. (a) $z = 0$. (b) $z = 0.3\,\mathrm{m}$. (c) $z = 0.6\,\mathrm{m}$. (d) $z = 0.9\,\mathrm{m}$.

cut-planes inside and outside a spherical cloak are plotted in Figs. 1(a)–1(d), in which $a = 0.5$ m and $b = 1$ m. In order to compare the simulation result with that in [15], we choose the operating frequency as 300 MHz. We select the four cut-planes at $z = 0$, 0.3 m, 0.6 m, and 0.9 m. The field distribution at $z = 0$ plane is exactly the same as that in Fig. 2 in [15], which was obtained by the extended Mie scattering theory. From Fig. 1, we observe that the electric field E_x is enhanced in the left and right sides of the cloak, and it is discontinuous at the outer boundary since it is not a tangential component. There is no field perturbation outside the cloak due to the matched impedance of the transformed media. When the cut-plane approaches the outer boundary of the spherical cloak, the electric field tends to be the plane wave, as expected.

It is interesting to observe the distributions of other components of

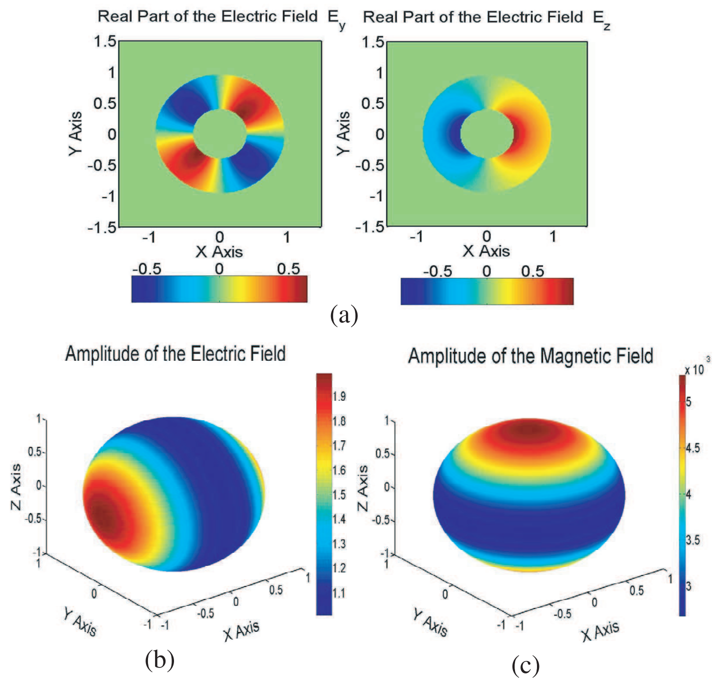

Figure 2. The distributions of electric fields inside and outside the 3D spherical cloak on the cut-plane $z = 0.3$ m, where $a = 0.5$ m, $b = 1$ m, and $f = 300$ MHz. (a) The distributions of $|E_y|$ and $|E_z|$. (b) The magnitude of the total electric field on the outer boundary. (c) The magnitude of the total magnetic field on the outer boundary.

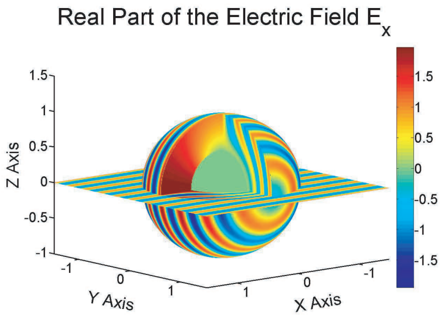

Figure 3. The distributions of electric field E_x on some cut-planes and the surface of the 3D spherical cloak, where $a = 0.5\,\text{m}$, $b = 1\,\text{m}$, and $f = 1\,\text{GHz}$.

electric field, as shown in Fig. 2(a), where E_y and E_z on the cut-plane $z = 0.3$ m are plotted. Both components have some kinds of symmetry, which can be inferred from the field expressions, Eqs. (14) and (15). It is clear that the amplitude of E_y is antisymmetric to both x and y axes. However, the amplitude of E_z is only antisymmetric to y axis. The field symmetry is the direct result of the space transformation. The amplitudes of total electric field $|E|$ and magnetic field $|H|$ on the outer boundary of the cloak are illustrated in Figs. 2(b) and 2(c). We notice that the electric field is enhanced on the two ends of sphere parallel to the polarization of incident wave, and the magnetic field is enhanced on the two ends normal to the polarization direction.

In order to show more details of the interactions between electromagnetic waves and the 3D cloak, we consider a high-frequency case, where $a = 0.5\,\text{m}$, $b = 1\,\text{m}$, and $f = 1\,\text{GHz}$. The wave is still incident from the y direction, and the distributions of the electric field E_x around the cloak are illustrated in Fig. 3, where some cutplanes have been chosen for readers to observe the field properties more clearly. It is obvious that the incident waves are bent inside the cloak and propagated along the transformed media with penetrating into the internal regions. Similarly, we can get the field distributions in other transformation media quickly and efficiently based on the field transformation method, as we will address in the following section.

4. FIELD TRANSFORMATION FOR ARBITARILY-SHAPED 3D CLOAKS

The analytical method is also applicable to any other complicated transformation. As an example, we consider a 3D ellipsoidal cloak, which is an extension of 2D elliptical cloaks [11, 12]. Assume that the semi-axes of the inner ellipsoid are a, b and c in the x, y and z directions, and semi-axes of the outer ellipsoid are a_1, b_1 and c_1, which satisfy $a_1/a = b_1/b = c_1/c$. Let $r = \sqrt{x^2 + y^2 + z^2}$ and $r' = \sqrt{x'^2 + y'^2 + z'^2}$. We define the transformation equation as

$$r' = \frac{a_1 - a}{a_1} r + R(r), \tag{17}$$

where $R(r)$ describes the distance between a point on the inner boundary of the cloak and the cloak center:

$$R(r) = \frac{abcr}{\sqrt{x^2b^2c^2 + y^2a^2c^2 + z^2a^2b^2}} = \frac{abcr'}{\sqrt{x'^2b^2c^2 + y'^2a^2c^2 + z'^2a^2b^2}}. \tag{18}$$

Following similar procedures as stated above, we have the Jacobian transformation matrix

$$\wedge_{i'}^{i} = \frac{a_1}{a_1 - a} \begin{bmatrix} 1 - abc(t^2 - x'^2b^2c^2)/t^3 & a^3bc^3x'y'/t^3 & a^3b^3cx'z'/t^3 \\ ab^3c^3x'y'/t^3 & 1 - abc(t^2 - y'^2a^2c^2)/t^3 & a^3b^3cy'z'/t^3 \\ ab^3c^3x'z'/t^3 & a^3bc^3y'z'/t^3 & 1 - abc(t^2 - z'^2a^2b^2)/t^3 \end{bmatrix} \tag{19}$$

where $t = \sqrt{x'^2b^2c^2 + y'^2a^2c^2 + z'^2a^2b^2}$. For the incident plane wave: $E_x = e^{iky}$, we obtain the electric field inside the transformed space as

$$E_x = E_{x'} = [1 - abc(t^2 - x'^2b^2c^2)/t^3]e^{iky}, \tag{20}$$

$$E_y = E_{y'} = (ab^3c^3x'y'/t^3)e^{iky}, \tag{21}$$

$$E_z = E_{z'} = (ab^3c^3x'z'/t^3)e^{iky}. \tag{22}$$

When the semi-axes of the ellipsoid in three directions are equal, that is, $a = b = c$ and $a_1 = b_1 = c_1$, the ellipsoidal cloak turns into a spherical cloak. Then Eqs. (20)–(22) are exactly the same as Eqs. (12)–(15).

Figure 4 illustrates the distributions of electric field E_x on different cut-planes inside and outside the 3D ellipsoidal cloak when $a = 1\,\mathrm{m}$,

$b = 0.5$ m, $c = 0.5$ m, $a_1 = 1.5$ m, $b_1 = 0.75$ m, $c_1 = 0.75$ m, and the operating frequency is $f = 300$ MHz. Four cut-planes at $z = 0.0$, 0.25 m, 0.55 m, and 0.7 m are selected to observe the field distributions around the cloak, as shown in Figs. 4(a)–4(d). Comparing Fig. 4 to Fig. 1, the field distributions on the cut-planes are similar to those of

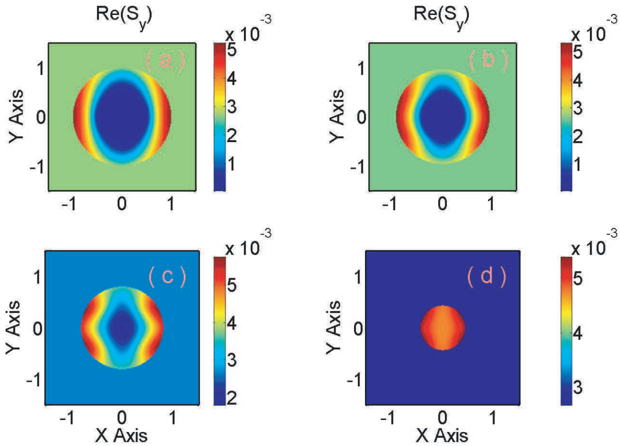

Figure 4. The distributions of S_y on different cut-planes inside and outside the 3D spherical cloak, where $a = 0.5$ m, $b = 1$ m, and $f = 300$ MHz. (a) $z = 0$. (b) $z = 0.3$ m. (c) $z = 0.6$ m. (d) $z = 0.9$ m.

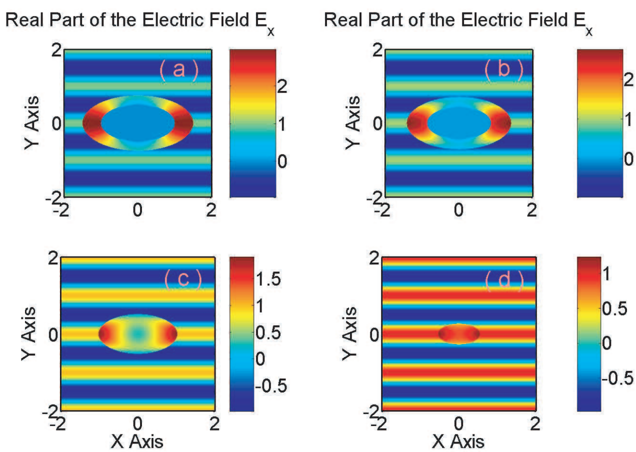

Figure 5. The distributions of electric fields $E^{z'}$ at different cut-planes inside and outside the 3D ellipsoidal cloak, where $a = 1$ m, $b = 0.5$ m, $c = 0.5$ m, $a_1 = 1.5$ m, $b_1 = 0.75$ m, $c_1 = 0.75$ m, and $f = 300$ MHz. (a) $z = 0$. (b) $z = 0.25$ m. (c) $z = 0.55$ m. (d) $z = 0.7$ m.

the spherical cloak. However, the electric fields are more concentrated on the two ends of the ellipsoid and the peak value of amplitude is larger in the ellipsoidal cloak. This is due to the larger compression ratio in the x direction in the ellipsoidal cloak than in the spherical cloak. The bending of EM waves inside the cloak is quite obvious, which validate the analytical results in Eqs. (20)–(22). From these equations, both TE and TM waves coexist inside the ellipsoidal cloak. Note for arbitrarily-shaped cloaks we could see that the mathematics would be a little more complicated [26], due to the complexity of the descriptions on the geometries of the shapes.

5. CONCLUSION

In conclusions, the electromagnetic properties of the arbitrarily-shaped 3D cloaks have been investigated in detail based on the analytical field transformation theory. The expressions for the fields and expressions inside the cloaks have been presented, which offers us an accurate method to understand the physics of the 3D cloaks. The difference between 2D and 3D cloaks has also been considered, and the changes of wave polarizations inside the cloak has also be discussed. Both spherical and ellipsoidal cloaks have been compared to show their common features and differences. Numerical simulations have been made to show the field distributions around the cloak, which validate the theoretical predictions presented in this paper.

ACKNOWLEDGMENT

This work was supported in part by a key project of National Science Foundation of China, in part by the National Basic Research Program (973) of China under Grant No. 2004CB719802, in part by the Natural Science Foundation of Jiangsu Province under Grant No. BK2008031, in part by the 111 Project under Grant No. 111-2-05, and in part by the National Science Foundation of China under Grant Nos. 60871016, 60671015, and 60621002. Email: qiangcheng@emfield.org and tjcui@seu.edu.cn.

REFERENCES

1. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780–1782, 2006.
2. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies, " *Science*, Vol. 314, 977–980, 2006.

3. Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nature Photonics*, Vol. 1, 224–227, 2007.
4. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, "Full-wave simulations of electromagnetic cloaking structures," *Phys. Rev. E*, Vol. 74, 036621, 2006.
5. Milton, G. W., M. Briane, and J. R. Willis, "On cloaking for elasticity and physical equations with a transformation invariant form," *New J. Phys.*, Vol. 8, 248–267, 2006.
6. Alu, A. and N. Engheta, "Multifrequency optical cloaking with layered plasmonic shells," *Phys. Rev. Lett.*, Vol. 100, 113901, 2008.
7. Schurig, D., J. B. Pendry, and D. R. Smith, "Calculation of material properties and ray tracing in transformation media," *Opt. Express*, Vol. 14, 9794–9804, 2006.
8. Jiang, W. X., J. Y. Chin, Z. Li, Q. Cheng, R. Liu, and T. J. Cui, "Analytical design of conformally invisible cloaks for arbitrarily shaped objects," *Phys. Rev. E*, Vol. 77, 066607, 2008.
9. Ruan, Z. C., M. Yan, C. W. Neff, and M. Qiu, "Ideal cylindrical cloak: Perfect but sensitive to tiny perturbations," *Phys. Rev. Lett.*, Vol. 99, 113903, 2007.
10. Zhang, B., H. Chen, B.-I. Wu, Y. Luo, L. Ran, and J. A. Kong, "Response of a cylindrical invisibility cloak to electromagnetic waves," *Phys. Rev. B*, Vol. 76, 121101(R), 2007.
11. Kwon, D.-H. and D. H. Werner, "Two-dimensional eccentric elliptic electromagnetic cloaks," *Appl. Phys. Lett.*, Vol. 92, 013505, 2008.
12. Jiang, W. X., T. J. Cui, G. X. Yu, X. Q. Lin, Q. Cheng, and J. Y. Chin, "Arbitrarily elliptical-cylindrical invisible cloaking," *J. Phys. D: Appl. Phys.*, Vol. 41, 085504, 2008.
13. Rahm, M., D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, "Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations," *Phot. Nano. Fund. Appl.*, Vol. 6, 87–95, 2008.
14. Zharova, N. A., I. V. Shadrivov, A. A. Zharov, and Y. S. Kivshar, "Ideal and nonideal invisibility cloaks," *Opt. Express*, Vol. 16, 21369–21374, 2008.
15. Chen, H., B.-I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic wave interactions with a metamaterial cloak," *Phys. Rev. Lett.*, Vol. 99, 063903, 2007.
16. Zhang, B., H. Chen, B.-I. Wu, and J. A. Kong, "Extraordinary

surface voltage effect in the invisibility cloak with an active device inside," *Phys. Rev. Lett.*, Vol. 100, 063904, 2008.

17. You, Y., G. W. Kattawar, P.-W. Zhai, and P. Yang, "Zero-backscatter cloak for aspherical particles using a generalized DDA formalism," *Opt. Express*, Vol. 16, 2068–2079, 2008.
18. You, Y., G. W. Kattawar, P.-W. Zhai, and P. Yang, "Invisibility cloaks for irregular particles using coordinate transformations," *Opt. Express*, Vol. 16, 6134–6145, 2008.
19. Liu, R., T. J. Cui, D. Huang, B. Zhao, and D. R. Smith, "Description of electromagnetic behaviors in artificial metamaterials based on effective medium theory," *Phys. Rev. E*, Vol. 76, 026606, 2007.
20. Lagarkov, A. N., V. N. Kisel, and V. N. Semenenko, "Wide-angle absorption by the use of a metamaterial plate," *Progress In Electromagnetics Research Letters*, Vol. 1, 35–44, 2008.
21. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering of two or more incident plane waves by a perfect electromagnetic conductor cylinder coated with a metamaterial," *Progress In Electromagnetics Research B*, Vol. 10, 75–90, 2008.
22. Zhang, J. J., Y. Luo, H. Chen, and B.-I. Wu, "Sensitivity of transformation cloak in engineering," *Progress In Electromagnetics Research*, PIER 84, 93–104, 2008.
23. Silva-Macedo, J. A., M. A. Romero, and B.-H. V. Borges, "An extended FDTD method for the analysis of electromagnetic field rotators and cloaking devices," *Progress In Electromagnetics Research*, PIER 87, 183–196, 2008.
24. Cheng, X., H. Chen, B.-I. Wu, and J. A. Kong, "Cloak for bianisotropic and moving media," *Progress In Electromagnetics Research*, PIER 89, 199–212, 2009.
25. Luo, Y., J. Zhang, H. Chen, and B.-I. Wu, "Full-wave analysis of prolate spheroidal and hyperboloidal cloaks," *J. Phys. D: Appl. Phys.*, Vol. 41, 235101, 2008.
26. Ozgun, O. and M. Kuzuoglu, "Electromagnetic metamorphosis: Reshaping scatters via conformal anisotropic metamaterial coatings," *Microw. and Opt. Tech. Letts.*, Vol. 49, 2386–2392, 2008.
27. Leonhardt, U., "Optical conformal mapping," *Science*, Vol. 312, 1777–1780, 2006.
28. Leonhardt, U., "General relativity in electrical engineering," *New Journal of Physics*, Vol. 8, 1777–1780, 2006.