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DISPERSION OF ELECTROMAGNETIC WAVES GUIDED BY AN OPEN TAPE HELIX I

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Abstract—The dispersion equation for free electromagnetic waves guided by an anisotropically conducting open tape helix is derived from the exact solution of a homogenous boundary value problem for Maxwell's equations without invoking any apriori assumption about the tape-current distribution. A numerical solution of the dispersion equation for a set of typical parameter values reveals that the tape-helix dispersion curve is virtually indistinguishable from the corresponding dominant-mode sheath helix dispersion curve except within the tape-helix forbidden regions.

1. INTRODUCTION

It is now well established that the tape-helix model gives a better approximation to the slow-wave structure of a TWT amplifier than the sheath-helix model over the entire frequency range of operation. Moreover, there is no possibility of simulating the input and the output ports of the amplifier with the sheath-helix model. Thus, a field-theoretical analysis of the dispersion characteristics of the tape-helix slow-wave structure will be of immense interest to the TWT community.

An indepth study of electromagnetic wave propagation on helical conductors has been performed by Samuel Sensiper way back in 1952 [1]. He has outlined essentially two approaches for analyzing the tape-helix problem. Using the first approach, he has demonstrated the feasibility of an exact solution for the tape helix; unfortunately, he chose to eschew this approach on the ground that "it is of no practical use for obtaining useful numerical results or for determining the detailed character of the solutions" preferring instead a second

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approach that involved an apriori assumption about the current distribution on the tape as a result of which it was possible to satisfy the boundary conditions on the tangential electric field only approximately. Nevertheless, it is this latter approach that has been endorsed by the majority of later generations of research workers in the TWT area mainly because of its tractability. All variants of this second simplified approach are characterized invariably by a common assumption, namely, that the tape current density component perpendicular to the winding direction may be neglected without much error. A notable exception to the practice of satisfying the tangential electric field boundary condition only along the centerline of the tape is the variational formulation developed by Chodorow and Chu [2] for cross-wound twin helices wherein the error in satisfying the tangential electric field boundary condition is minimized for an assumed tape-current distribution by making the average error equal to zero. The rationale behind the approach of Chodorow and Chu for single-wire helix has been outlined by Watkins in his book [3] assuming that (i) the tape current flows only along the winding direction, (ii) it does not vary in phase or amplitude over the width of the tape, and (iii) its phase variation is according to $\beta_0 z$ for z corresponding to a point moving along the centerline of the tape.

The method adopted for the solution of the cold-wave problem for the tape helix in this paper derives from the following fact: If one is willing to neglect in any case the contribution of the perpendicularly directed current density component on the tape then there is neither a need for any apriori assumption regarding the tape-current distribution nor is there any difficulty in satisfying the tangential electric field boundary condition over the entire width of the tape. The hypothesis that the transverse component of the tape-current density is zero may be incorporated explicitly into the model by assuming that the tape helix is made out of an anisotropic material exhibiting infinite conductivity in the winding direction but zero conductivity in the orthogonal direction. This anisotropically conducting model for the tape helix leads to considerable simplification of the solution of the boundary value problem for the guided modes supported by an open helical structure. First of all, the boundary conditions give rise to only a single infinite set of linear homogeneous equations for determining the modal amplitudes of the tape-current density. Moreover, the approximate secular equation, for determining guided-mode propagation constant, resulting from setting the determinant of the coefficient matrix, corresponding to a symmetric truncation of the infinite set of equations, to zero will be in the form of a series whose terms decrease rapidly in magnitude with the order of truncation.

This last feature of the truncated secular equation is quite attractive from a computational point of view since it then becomes possible to secure a fairly accurate estimate of the dispersion characteristic with a reasonably low order of truncation. The entire analysis is of course based on the premise that the transverse component of the tape-current density does not have any significant effect on the value of the propagation constant even for tapes which are not narrow.

2. DERIVATION OF THE DISPERSION EQUATION

A tape helix of infinite length, constant pitch, constant tape width and infinitesimal thickness surrounded by free space is considered. The helix is assumed to be made of an anisotropic material exhibiting infinite conductivity in the direction of the tape winding but zero conductivity in the orthogonal direction.

Since the formulation of the cold-wave problem for the tape helix has become quite standard, we make use of the notation and terminology employed in one of the conventional treatments following Sensiper [1] of the problem as presented in [4] except that we use w, instead of δ, to denote the width of the tape in the axial direction. Accordingly, we take the axis of the helix along the z-coordinate of a cylindrical coordinate system (ρ, φ, z). The radius of the helix is a, the pitch is p, and $\cot\psi = 2\pi a/p$ (Fig. 1).

Periodicity of the infinite helical structure in the z and φ variables permits an expansion of the phasor representation $F(\rho, \varphi, z)$ (corresponding to a radian frequency ω) of any field component in a

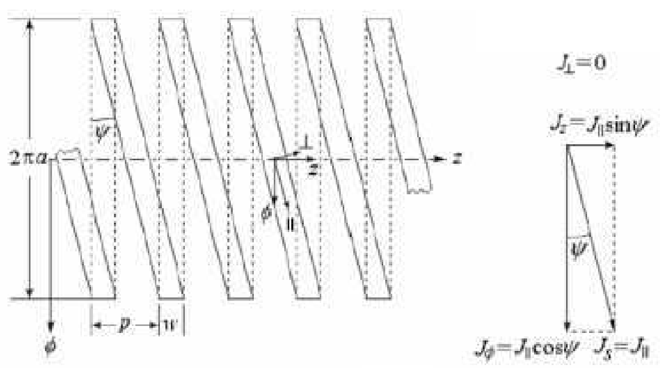

Figure 1. Geometrical relations in a developed tape helix.

double infinite series

$$F\left(\rho,\phi,z\right)=e^{-j\beta_0 z}\sum_{\upsilon=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}F_{\upsilon n}\left(\rho\right)e^{j\upsilon\varphi}e^{-j2\pi nz/p} \tag{1}$$

in view of Floquet's theorem [4, 5] where $\beta_0 = \beta_0(\omega)$ is the guided wave propagation constant at the radian frequency ω. Moreover, the invariance of the infinite helical structure under a translation Δz in the axial direction and a simultaneous rotation by $2\pi\Delta z/p$ around the axis imply that $F_{\upsilon n}(\rho)$ are non zero only if $\upsilon = n$. Thus, the double-series expansion (1) for any field component degenerates to the single-series expansion

$$F(\rho,\phi,z)=\sum_{n=-\infty}^{\infty}F_n\left(\rho\right)e^{j(n\varphi-\beta_n z)} \tag{2}$$

where

$$\beta_n=\beta_0+2\pi n/p \tag{3}$$

Each term in the series-expansion has to satisfy the Helmholtz equation in cylindrical coordinates. Hence, the Borgnis potentials [4] for guided-wave solutions, at the radian frequency ω, may be assumed in the form

$$U=\sum_{n=-\infty}^{\infty}\left[A_n+(C_n-A_n)H(\rho-a)\right]G_n(\tau_n\rho)e^{j(n\varphi-\beta_n z)}, \tag{4a}$$

$$V=\sum_{n=-\infty}^{\infty}\left[B_n+(D_n-B_n)H(\rho-a)\right]G_n(\tau_n\rho)e^{j(n\varphi-\beta_n z)}, \tag{4b}$$

where

$$G_n(\tau_n\rho)\triangleq\begin{array}{l}I_n(\tau_n\rho)\text{ for }0\leq\rho<a,\\ K_n(\tau_n\rho)\text{ for }\rho>a,\end{array}$$

I_n and K_n are nth order modified Bessel functions of the first and second kind respectively, and H is the Heaviside function and where

$$\beta_n^2-\tau_n^2=k_0^2\triangleq\omega^2\mu\varepsilon \tag{5}$$

with μ the permeability and ϵ the permittivity of the ambient space. The explicit expressions for the field components become [4]

$$E_z=\frac{\partial^2 U}{\partial z^2}+k_0^2U=\sum_{n=-\infty}^{\infty}-\tau_n^2\Lambda_n(\rho)G_n(\tau_n\rho)e^{j(n\varphi-\beta_n z)} \tag{6a}$$

$$E_\rho = \frac{\partial^2 U}{\partial\rho\partial z} - \frac{j\omega\mu}{\rho}\frac{\partial V}{\partial\varphi}$$
$$= \sum_{n=-\infty}^{\infty} \left[-j\beta_n\tau_n\Lambda_n(\rho)G_n'(\tau_n\rho) + \omega\mu n\Omega_n(\rho)G_n(\tau_n\rho)/\rho\right] e^{j(n\varphi-\beta_n z)} \quad (6b)$$

$$E_\phi = \frac{\partial^2 U}{\partial\varphi\partial z} - \frac{j\omega\mu}{\rho}\frac{\partial V}{\partial\rho}$$
$$= \sum_{n=-\infty}^{\infty} \left[n\beta_n\Lambda_n(\rho)G_n(\tau_n\rho) + j\omega\mu\tau_n\Omega_n(\rho)G_n'(\tau_n\rho)\right] e^{j(n\varphi-\beta_n z)} \quad (6c)$$

$$H_z = \frac{\partial^2 V}{\partial z^2} + k_0^2 V = \sum_{n=-\infty}^{\infty} -\tau_n^2\Omega_n(\rho)G_n(\tau_n\rho)e^{j(n\varphi-\beta_n z)} \quad (7a)$$

$$H_\rho = \frac{\partial^2 V}{\partial\rho\partial z} + \frac{j\omega\epsilon}{\rho}\frac{\partial U}{\partial\varphi}$$
$$= \sum_{n=-\infty}^{\infty} \left[\omega\varepsilon n\Lambda_n(\rho)G_n(\tau_n\rho)/\rho + j\beta_n\tau_n\Omega_n(\rho)G_n'(\tau_n\rho)\right] e^{j(n\varphi-\beta_n z)} \quad (7b)$$

$$H_\phi = \frac{\partial^2 V}{\partial\varphi\partial z} - \frac{j\omega\epsilon}{\rho}\frac{\partial U}{\partial\rho}$$
$$= \sum_{n=-\infty}^{\infty} \left[-j\omega\varepsilon\tau_n\Lambda_n(\rho)G_n'(\tau_n\rho) + n\beta_n\Omega_n(\rho)G_n(\tau_n\rho)/\rho\right] e^{j(n\varphi-\beta_n z)} \quad (7c)$$

In the expressions (6) and (7) for the field components, G_n' denotes the derivative of the function G_n with respect to its argument and

$$\Lambda_n(\rho) \underline{\underline{\Delta}} A_n + (C_n - A_n)H(\rho - a) \quad (8a)$$
$$\Omega_n(\rho) \underline{\underline{\Delta}} B_n + (D_n - B_n)H(\rho - a) \quad (8b)$$

and where A_n, B_n, C_n and D_n, $n \in \mathbb{Z}$, are (complex) constants to be determined by the tape helix boundary conditions.

In order for (6) and (7) to correspond to guided waves (as opposed to radiation modes) supported by the open helix, we need $\tau_n > 0$ for all $n \in \mathbb{Z}$, that is

$$|\beta_n| > k_0 \text{ for all } n \in \mathbb{Z} \quad (9)$$

which, for $n = 0$, becomes $|\beta_0| > k_0$ i.e., only slow guided waves are supported by the helical structure. The condition (9) for other values of n translates, in view of (5), to

$$|\beta_0 + n\cot\psi/a| > k_0$$

which is equivalent to either

$$\beta_0 > k_0 - n\cot\psi/a \tag{10a}$$

$$\text{or}\quad \beta_0 < -k_0 - n\cot\psi/a \tag{10b}$$

If $\beta_0 > k_0$, the inequality (10a) is automatically satisfied for all $n \geq 0$. We then need the condition (10b) to be satisfied for the remaining values of n, i.e., for $n \leq -1$, that is,

$$-\mid n\mid \cot\psi/a < -(\beta_0 + k_0) \quad \text{for all } n \leq -1 \tag{11}$$

In order for the inequality (11) to hold for all $n \leq -1$, it is sufficient to have

$$-\cot\psi/a < -(\beta_0 + k_0)$$

or

$$-\cot\psi/a > (\beta_0 + k_0) > 2k_0$$

which implies the cut-off condition

$$k_0 a < (1/2)\cot\psi \tag{12}$$

If, on the other hand, $\beta_0 < -k_0$, then the inequality (10b) is automatically satisfied for all $n \leq 0$. Then, the need to satisfy the inequality (10a) for the remaining positive values of n leads to the inequality

$$n\cot\psi/a > k_0 - \beta_0 \quad \text{for all } n \geq 1$$

for which it is sufficient to have

$$-\cot\psi/a > (k_0 - \beta_0) > 2k_0$$

that is, $k_0 < (1/2)\cot\psi$ which is again the cut-off condition (12). Thus, pure guided modes do not exist on an open tape helix for a (radian) frequency $\omega \geq (c/2a)\cot\psi$, where $c = 1/\sqrt{\mu\varepsilon}$ is the speed of light in the ambient space.

The boundary conditions at $\rho = a$ for the anisotropically conducting model of the tape helix are

(i) The tangential electric field is continuous for all values of φ and z.

(ii) The tangential component of the magnetic field parallel to the winding direction is continuous for all φ and z.

(iii) The discontinuity in the tangential component of the magnetic field perpendicular to the winding direction is equal to the surface current density on the tape.

(iv) The tangential component of the electric field parallel to the winding direction is zero on the tape surface.

Thus

$$E_z(a-,\varphi,z) - E_z(a+,\varphi,z) = 0 \tag{13a}$$
$$E_\varphi(a-,\varphi,z) - E_\varphi(a+,\varphi,z) = 0 \tag{13b}$$
$$[H_z(a-,\varphi,z) - H_z(a+,\varphi,z)]\sin\psi + [H_\varphi(a-,\varphi,z) - H_\varphi(a+,\varphi,z)]\cos\psi = 0 \tag{13c}$$
$$[H_z(a-,\varphi,z) - H_z(a+,\varphi,z)]\cos\psi - [H_\varphi(a-,\varphi,z) - H_\varphi(a+,\varphi,z)]\sin\psi = J_s(\varphi,z) \tag{13d}$$
$$[E_z(a,\varphi,z)\sin\psi - E_\varphi(a,\varphi,z)\cos\psi]g(\varphi,z) = 0 \tag{13e}$$

where $J_s(\varphi, z)$ is the surface current density component supported by helix and the function $g(\varphi, z)$, defined in terms of the indicator functions of the disjoint (for the same value of φ) intervals $[(l + \varphi/2\pi)p - w/2, (l + \varphi/2\pi)p + w/2]$, $l \in \mathbb{Z}$, by

$$g(\varphi,z) \triangleq \sum_{l=-\infty}^{\infty} 1_{[(l+\varphi/2\pi)p-w/2,(l+\varphi/2\pi)p+w/2]}(z)$$

will be equal to 1 on the tape surface and 0 elsewhere on the surface of the (infinite) cylinder $\rho = a$. In (13a)–(13d),

$$F(a\pm,\varphi,z) \triangleq \lim_{\delta\downarrow 0} F(a\pm\delta,\varphi,z)$$

for any field component $F(\rho, \varphi, z)$. In the case of an helical conductor of nonzero thickness, $\rho = a-$ and $\rho = a+$ correspond respectively to the inner and the outer surface of the tape. The functional form of the surface current density component $J_s(\varphi, z)$, which is confined only to the two-dimensional region occupied by the tape-helix material, is restricted by the periodicity and the symmetry conditions imposed by the helix geometry. Accordingly, $J_s(\varphi, z)$ admits the representation

$$J_s(\rho,\varphi,z) = \left(\sum_{n=-\infty}^{\infty} J_n e^{j(n\varphi-\beta_n z)}\right) g(\varphi,z) \tag{14}$$

where the (complex) constants J_n, $n \in \mathbb{Z}/\{0\}$, in the expansion (14) are to be determined in terms of the arbitrary constant J_0 by the boundary conditions. An explicit representation for the surface current density on the helix as in (14) does not appear to have been made use of in any of the previous analysis of the cold wave problem for the tape helix. In the terms of the new (independent) variables ξ and ζ, defined by

$$\xi \triangleq \varphi\sqrt{p^2 + (2\pi a)^2}/2\pi, \qquad \zeta \triangleq z - \varphi p/2\pi$$

on the cylindrical surface $\rho = a$ containing tape helix, we have

$$e^{j(n\phi - \beta_n z)} = e^{-j\beta_0 \xi \sin\psi} e^{-j\beta_n \zeta}$$

since $z = \zeta + \xi \sin\psi$ and $\varphi = 2\pi\xi \sin\psi/p$. Thus, the surface current density component $J_s(\varphi, z)$, when expressed in terms of the variables ξ and ζ, becomes

$$\tilde{J}_s(\xi, \zeta) = e^{-j\beta_0(\zeta + \xi \sin\psi)} f(\zeta) = e^{-j\beta_0 z} f(\zeta) \tag{15}$$

where

$$f(\zeta) = \sum_{l=-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} J_n e^{-j2\pi n\zeta/p} \right) 1_{[lp - w/2, lp + w/2]}(\zeta) \tag{16}$$

The function f, being periodic in ζ with period p, may be expanded in a Fourier series

$$f(\zeta) = \sum_{k=-\infty}^{\infty} \tilde{J}_k e^{-j2\pi k\zeta/p}$$

where the Fourier coefficients $\tilde{J}_k$, $k \in \mathbb{Z}$, are given by

$$\tilde{J}_k = (1/p) \int_{-p/2}^{p/2} f(\zeta) e^{j2\pi k\zeta/p} d\zeta = \hat{w} \sum_{n=-\infty}^{\infty} J_n \operatorname{sinc}(n - k)\hat{w} \tag{17}$$

In (17), $\hat{w} \triangleq w/p$ and $\operatorname{sinc} X \triangleq \sin \pi X/\pi X$. Thus

$$\tilde{J}_s(\xi, \zeta) = \hat{w} e^{-j\beta_0(\zeta + \xi \sin\psi)} \sum_{k=-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} J_n \operatorname{sinc}(n - k)\hat{w} \right) e^{-j2\pi k\zeta/p}$$

Reverting back to the original variables φ and z, we have

$$J_s(\varphi, z) = \sum_{n=-\infty}^{\infty} \Gamma_n e^{j(n\varphi - \beta_n z)} \tag{18}$$

where

$$\Gamma_n \triangleq \hat{w} \sum_{k=-\infty}^{\infty} J_k \operatorname{sinc}(k - n)\hat{w} \tag{19}$$

We are now ready to tackle the boundary conditions. First, we introduce the following convenient abbreviations for the modified Bessel functions and their derivatives evaluated at $\rho = a$:

$$I_{na} \triangleq I_n(\tau_n a), \qquad K_{na} \triangleq K_n(\tau_n a),$$

$$I'_{na} \triangleq I'_n(\tau_n a), \qquad K'_{na} \triangleq K'_n(\tau_n a),$$

The boundary conditions (9a)–(9c) immediately give the following relations among the four sets of coefficients A_n, B_n, C_n and D_n, $n \in \mathbb{Z}$:

$$C_n = (I_{na}/K_{na})A_n \tag{20a}$$

$$D_n = (I'_{na}/K'_{na})B_n \tag{20b}$$

$$B_n = (j\omega\varepsilon a\tau_n K'_{na} \cos\psi)A_n/K_{na}(a\tau_n^2 \sin\psi - n\beta_n \cos\psi) \tag{20c}$$

The fourth boundary condition (13d) together with the relations (20) and the expression (18) for $J_s(\varphi, z)$, in turn, relates A_n to Γ_n as

$$A_n = K_{na}[a\tau_n^2 \sin\psi - n\beta_n \cos\psi]\Gamma_n/j\omega\varepsilon\tau_n^2 \tag{20d}$$

Finally, the enforcement of the homogeneous boundary condition on the tangential electric field component parallel to the winding direction leads to the set of equations

$$e^{-j\beta_0 z} \sum_{l=-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} \sigma_n \Gamma_n e^{-j2\pi n\zeta/p} \right) 1_{[lp-w/2, lp+w/2]}(\zeta) = 0 \tag{21}$$

on cancellation of the non-zero constant factor $(j/\omega\varepsilon a)$ where

$$\sigma_n \triangleq K_{na} I_{na} (a\tau_n^2 \sin\psi - n\beta_n \cos\psi)^2/\tau_n^2 + (k_0 a)^2 I'_{na} K'_{na} \cos^2\psi \tag{22}$$

Since $e^{-j\beta_0 z} \neq 0$, (21) implies that each Fourier coefficient of the periodic function

$$h(\zeta) \triangleq \sum_{l=-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} \sigma_n \Gamma_n e^{-j2\pi n\zeta/p} \right) 1_{[lp-w/2, lp+w/2]}(\zeta)$$

of ζ (with period p) must vanish, that is,

$$\sum_{n=-\infty}^{\infty} \sigma_n \Gamma_n \mathrm{sinc}(n-k)\hat{w} = 0 \qquad \text{for } k \in \mathbb{Z} \tag{23}$$

Substituting for Γ_n from (19), the condition (23) may be put in the form

$$\sum_{q=-\infty}^{\infty} \alpha_{kq} J_q = 0 \qquad \text{for } k \in \mathbb{Z} \tag{24}$$

where

$$\alpha_{kq} = \sum_{n=-\infty}^{\infty} \sigma_n \mathrm{sinc}(k-n)\hat{w}\,\mathrm{sinc}(q-n)\hat{w} \tag{25}$$

For a nontrivial solution of the infinite set of linear homogenous equations (24) for J_q, $q \in \mathbb{Z}$, it is necessary that the determinant of the coefficient matrix $\mathbf{A} \underline{\underline{\triangle}} [\alpha_{kq}]$, $k, q \in \mathbb{Z}$ is zero, that is,

$$|\mathbf{A}| = 0 \tag{26}$$

The determinantal equation (26) gives the dispersion relation for the cold wave modes supported by an open anisotropically conducting tape helix. It is to be emphasized that, unlike similar treatments of the cold-wave problem that neglect the transverse component of the tape current density, the present derivation of the dispersion equation is based neither on any apriori assumption regarding the tape-current distribution nor on any approximation of the helix boundary conditions. In this sense, the derivation is exact within the assumed model for the tape helix.

It may be observed from (25) that the diagonal entries of the coefficient matrix $\mathbf{A}$ are given by

$$\alpha_{kk} = \sum_{n=-\infty}^{\infty} \sigma_n \mathrm{sinc}^2(k-n)\hat{w}, \qquad k \in \mathbb{Z} \tag{27}$$

Since the function $\mathrm{sinc}^2 X$ decreases fairly rapidly with $|X|$, an estimate of α_{kk} to any required order of accuracy δ may be obtained by truncating the infinite series (27) to the neighborhood $A_{k,\delta}$ of k where

$$A_{k,\delta} \underline{\underline{\triangle}} \left\{ n \in \mathbb{Z} : \sigma_n \mathrm{sinc}^2(k-n)\hat{w} > \delta \right\}$$

However, for a given order of accuracy δ, the cardinality of the set $A_{k,\delta}$ decreases only slowly with k since σ_n is only of order $1/|n|$ for $|n| \geq 1$. Unfortunately, this also means that a fairly large number of diagonal entries needs to be retained in any truncation of the coefficient matrix if an accurate estimate of the dispersion characteristic is desired. However, a redeeming feature is the presence of the factor $\mathrm{sinc}(k-n)\hat{w}$ $\mathrm{sinc}(q-n)\hat{w}$ in the expression for the matrix entry α_{kq}; this implies that the off-diagonal entries will be quite small in magnitude compared to the nearest diagonal entry.

Before continuing with any further discussion of the dispersion equation (26) for the tape-helix model, let us consider two limiting, nevertheless interesting, special cases:

Case 1: The tape width w (in the axial direction) approaches the helix pitch p.

In this case

$$\alpha_{kq} \rightarrow \sum_{n=-\infty}^{\infty} \sigma_n \delta_{kn} \delta_{qn} = \sigma_k \delta_{kq}$$

and the dispersion equation (26) degenerates to

$$\prod_{k=-\infty}^{\infty} \sigma_k = 0$$

that is,

$$\sigma_n = 0, \qquad n = 0, \pm 1, \pm 2, \ldots \tag{28}$$

The relation $\sigma_n = 0$ may be recognized as the dispersion equation for the nth mode of a sheath helix made of the same anisotropically conducting material as the tape helix provided β_n, as a whole, is interpreted as the propagation constant of the nth sheath-helix mode. With this interpretation, we see that $\sigma_n = \sigma_{-n}$, $\forall\, n \in \mathbb{N}$, so that if β_n is a root of the equation $\sigma_n = 0$ so is $-\beta_n$. We may therefore set $\beta_{-n} = -\beta_n$. This is a restatement of the well-known fact that all the sheath-helix modes, except the zeroth, are two-fold degenerate. Thus, the sheath-helix dispersion equation simplifies to

$$\prod_{n=0}^{\infty} \sigma_n = 0 \tag{29}$$

which may be satisfied by any one of the sheath-helix modes, which is again a reiteration of the fact that each mode of the sheath helix satisfies all the sheath-helix boundary conditions individually. This is in marked contrast to the case of the tape helix, which requires all of the so called 'space harmonics' to satisfy the boundary conditions. This clear demarcation between the roles of the space harmonics in the two different contexts of the sheath helix and the tape helix does not appear to have been adequately emphasized in the literature.

Case 2: The infinite-order coefficient matrix $\mathbf{A}$ is truncated to the 1×1 matrix $[\alpha_{00}]$.

Under this truncation, the dispersion equation reduces to

$$\alpha_{00} = \sum_{n=-\infty}^{\infty} \sigma_n \,\mathrm{sinc}^2 n\hat{w} = 0 \tag{30}$$

In spite of the drastic nature of this truncation, the resulting grossly approximate dispersion equation (30) is similar in appearance to the approximate eigenvalue equation

$$\sum_{n=-\infty}^{\infty} \sigma_n \, R_n = 0 \tag{31}$$

derived in the literature [3–5] on the basis of an assumed tape-current distribution which forces the tangential electric field boundary condition to be satisfied only along the centerline of the tape. The value of the multiplying factor R_n in (31) is decided by the type of assumption made regarding the tape-current distribution, and irrespective of the particular assumption made, the decay of R_n with respect to n turns out to be no better than $|n|^{-1}$ except for the case of the one-term approximation to the tape-current distribution made by Chodorow and Chu [2] and Watkins [3]. In fact, the approximate dispersion equations derived by Chodorow and Chu and Watkins have a form identical to that of (30). Thus, the terms of the series in (30) and those in [2] and [3] decrease rapidly enough with $|n|$ (due to the presence of $(\mathrm{sinc})^2$ factors) to enable the infinite series to be symmetrically truncated to a low order without appreciable error. Even though such a truncated series does not appear to be anything like a reasonable approximation to the actual dispersion equation, it will be shown in the sequel that the zeroth term σ_0 in the infinite-series representation (30) of α_{00} does indeed serve as the leading order term in a numerical scheme for getting better and better approximations by successive addition of higher order correction terms.

In order to test how good (30) is as an approximation for the dispersion equation for narrow tapes, the infinite-series representation of α_{00} is symmetrically truncated to the order $W \underline{\Delta} \lfloor p/w \rfloor$ so as to retain all the terms falling within the 'main lobe' of the $(\mathrm{sinc})^2$ functions in the truncated series. The truncated version of (30) is then put in the fixed-point format

$$k_{0a} = \left\{ \beta_{0a}^2 / Q_{sa} + F_0^{(W)} / Q_{sa} I_{0a} \sin^2 \psi \right\}^{1/2} \tag{32}$$

where

$$k_{0a} = k_0 a, \quad \beta_{0a} = \beta_0 a, \quad F_0^{(W)} \underline{\Delta} \sum_{n=1}^{W} (\sigma_n + \sigma_{-n}) \mathrm{sinc}^2 n\hat{w}$$

and $Q_{sa}\underline{\Delta}1 - I'_{0a}K'_{0a}\cot^2\psi/I_{0a}K_{0a}$. An attempt to solve (32) for the choice of $\hat{w} = 0.1/\pi$ (prototypical value used in the literature for a narrow tape) and $\psi = 10°$ leads to the unexpected conclusion that the truncated version of (30), which has been projected for so long in the literature as a reasonable approximation to the dispersion equation for narrow tapes, does not possess any real solution for $k_{0a}(\beta_{0a})$ beyond $\beta_{0a} = 1.543$. An increase of the truncation order does not improve the situation; in fact the interval of existence shrinks slightly with an increase in the order truncation beyond W for a fixed value of $\hat{w}$. For larger values of $\hat{w}$, however, the truncated version of (30) is seen to possess real solutions for $k_{0a}(\beta_{0a})$ for progressively larger values of β_{0a} in the complement of the forbidden regions. In the limit $\hat{w} \to 1$, equation (30) (and its truncated version) degenerates to the dominant-mode sheath-helix dispersion equation which is of course known to possess a unique real solution $k_{0a}(\beta_{0a})$ for every real value of β_{0a}.

3. A NUMERICAL SCHEME

We now present an approach for improving upon the approximate dispersion equation (30) resulting from the single-entry truncation of the coefficient matrix $\mathbf{A}$ on the basis of the decay properties of the matrix entries α_{kq}. We start out with a symmetric truncation of the infinite-order coefficient matrix $\mathbf{A}$ to the $(2N + 1) \times (2N + 1)$ matrix $[\alpha_{kq}]_{-N\leq k,\ q\leq N}$. Our objective is to study, for a specific value of the ratio $\hat{w} = w/p$, the behavior of the dispersion characteristic with respect to the truncation order N, and arrive at a compromise value of N that gives a reasonably good approximation to the actual dispersion curve within the confines of the assumed model for the tape helix. It is readily seen from (25) that only the main 'lobes' of the *sinc* functions contribute significantly to the value of α_{kq}. We may therefore restrict the range of values of n in the summation for α_{kq} to

$$\max(k, q) - p/w < n < \min(k, q) + p/w \tag{33}$$

For the specific choice of $w/p = 1/2$, (33) becomes

$$\max(k, q) - 1 \leq n \leq \min(k, q) + 1 \tag{34}$$

Since (34) implies that $0 \leq |k - q| \leq 2$, the $(2N + 1) \times (2N + 1)$ coefficient matrix reduces to a banded symmetric matrix with nonzero entries only along the main diagonal and the four subdiagonals (two each on either side of the main diagonal) adjacent to the main diagonal.

Thus, the infinite series for α_{kq} gets truncated to

$$a_{kq} \underline{\triangle} \hat{\alpha}_{kq} = \sum_{n=\max(k,q)-1}^{\min(k,q)+1} \sigma_n \mathrm{sinc}\frac{(k-n)}{2}\,\mathrm{sinc}\frac{(q-n)}{2}, \qquad -N \le k, q \le N,\ 0 \le |k-q| \le 2 \tag{35}$$

It will be demonstrated in the sequel how this banded structure of the coefficient matrix can be exploited to give an effective algorithm (which may be programmed easily on a computer) for the computation of its determinant. Denoting the $(2N+1) \times (2N+1)$ truncated coefficient matrix by $\hat{\mathbf{A}}$ and the corresponding $(2N+1)$-dimensional null-space vector by $\hat{\mathbf{J}} = [\hat{J}_{-N}, \hat{J}_{-N+1}, \ldots, \hat{J}_{-1}, \hat{J}_0, \hat{J}_1, \ldots, \hat{J}_{N-1}, \hat{J}_N]^T$ the truncated version of (24) becomes

$$\hat{\mathbf{A}}\hat{\mathbf{J}} = \mathbf{0} \tag{36}$$

The convention of denoting a negative index by an overbar will be adopted in the sequel in the interests of brevity; thus, $\hat{J}_{-n}$, for example, will be denoted by $\hat{J}_{\overline{n}}$. When the contributions from the main lobes of the *sinc* functions only are retained in the expression for a_{kq}, $-N \le k, q \le N$, there will only be three types of non-zero entries in the $(2N+1) \times (2N+1)$ symmetric matrix $\hat{\mathbf{A}}$ for the choice $\hat{w} = w/p = 1/2$, viz.,

$$a_{kk} = \sigma_k + (2/\pi)^2(\sigma_{k-1} + \sigma_{k+1}) \qquad -N \le k \le N,$$

$$\begin{aligned} a_{k,k+1} &= a_{k+1,k} = (2/\pi)(\sigma_k + \sigma_{k+1}) && -N \le k \le N-1, \\ a_{k,k+2} &= a_{k+2,k} = (2/\pi)^2\sigma_{k+1} && -N \le k \le N-2. \end{aligned}$$

Assume $N > 2$. Since $\hat{J}_{\overline{N}}$ and $\hat{J}_N$ appear respectively in the first three and the last three equations in the set (36), we may solve for $\hat{J}_{\overline{N}}$ (respectively $\hat{J}_N$) from the first (respectively the last) equation in terms of $\hat{J}_{\overline{N-1}}$ and $\hat{J}_{\overline{N-2}}$ (respectively $\hat{J}_{N-1}$ and $\hat{J}_{N-2}$) and substitute for $\hat{J}_{\overline{N}}$ and $\hat{J}_N$ in the next two equations to eliminate $\hat{J}_{\overline{N}}$ and $\hat{J}_N$ from the set (36) thereby reducing the order of the matrix $\hat{\mathbf{A}}$ by two from $(2N+1) \times (2N+1)$ to $(2N-1) \times (2N-1)$ at the first stage of the reduction process. Continuing this process of successive substitution and elimination on the resulting matrix (of the same symmetry and band structure), we see that the order of the coefficient matrix gets reduced by two at each of the succeeding stages. At the ith stage (count the starting stage as the 0th stage), we will have a set of $2(N-i)+1$ equations of the form

$$\hat{\mathbf{A}}^{(i)}\hat{\mathbf{J}}^{(i)} = 0$$

where

$$\hat{\mathbf{J}}^{(i)} = [\hat{J}_{\overline{N-i}}, \ldots\ldots \hat{J}_{N-i}]^T$$

and the entries in the 2×2 submatrix in the right bottom corner (respectively in the left top corner) of $\hat{\mathbf{A}}^{(i)}$ are given in terms of the corresponding entries of $\hat{\mathbf{A}}^{(i-1)}$ by

$$\begin{aligned} a^{(i)}_{N-i,N-i} &= a^{(i-1)}_{N-i,N-i} - \left(a^{(i-1)}_{N-i,N-i+1}\right)^2 / a^{(i-1)}_{N-i+1,N-i+1} \\ a^{(i)}_{N-i,N-i-1} &= a^{(i)}_{N-i-1,N-i} = a_{N-i,N-i-1} \\ &\quad - a^{(i-1)}_{N-i,N-i+1} a_{N-i+1,N-i-1} / a^{(i-1)}_{N-i+1,N-i+1} \\ a^{(i)}_{N-i-1,N-i-1} &= a_{N-i-1,N-i-1} - \left(a_{N-i-1,N-i+1}\right)^2 / a^{(i-1)}_{N-i+1,N-i+1} \end{aligned} \tag{37}$$

and by an identical set of three relations with an overbar over the suffixes so long as $1 \leq i \leq N-2$. The remaining entries of $\hat{\mathbf{A}}^{(i-1)}$ are not affected by the reduction process. At the $(N-2)$th stage we have the following set of five equations:

$$\begin{aligned} a^{(N-2)}_{\bar{2}\bar{2}}\hat{J}_{\bar{2}} + a^{(N-2)}_{\bar{2}\bar{1}}\hat{J}_{\bar{1}} + a_{\bar{2}0}\hat{J}_0 &= 0 \\ a^{(N-2)}_{\bar{1}\bar{2}}\hat{J}_{\bar{2}} + a^{(N-2)}_{\bar{1}\bar{1}}\hat{J}_{\bar{1}} + a_{\bar{1}0}\hat{J}_0 + a_{\bar{1}1}\hat{J}_1 &= 0 \\ a_{0\bar{2}}\hat{J}_{\bar{2}} + a_{0\bar{1}}\hat{J}_{\bar{1}} + a_{00}\hat{J}_0 + a_{01}\hat{J}_1 + a_{02}\hat{J}_2 &= 0 \\ a_{1\bar{1}}\hat{J}_{\bar{1}} + a_{10}\hat{J}_0 + a^{(N-2)}_{11}\hat{J}_1 + a^{(N-2)}_{12}\hat{J}_2 &= 0 \\ a_{20}\hat{J}_0 + a^{(N-2)}_{21}\hat{J}_1 + a^{(N-2)}_{22}\hat{J}_2 &= 0 \end{aligned} \tag{38}$$

After eliminating $\hat{J}_{\bar{2}}$ and $\hat{J}_2$ from (38), we have a set of three equations for $\hat{J}_{\bar{1}}$, $\hat{J}_0$ and $\hat{J}_1$ at the $(N-1)$th stage:

$$\begin{aligned} a^{(N-1)}_{\bar{1}\bar{1}}\hat{J}_{\bar{1}} + a^{(N-1)}_{\bar{1}0}\hat{J}_0 + a_{\bar{1}1}\hat{J}_1 &= 0 \\ a^{(N-1)}_{0\bar{1}}\hat{J}_{\bar{1}} + a^{(N-1)}_{00}\hat{J}_0 + a^{(N-1)}_{01}\hat{J}_1 &= 0 \\ a_{1\bar{1}}\hat{J}_{\bar{1}} + a^{(N-1)}_{10}\hat{J}_0 + a^{(N-1)}_{11}\hat{J}_1 &= 0 \end{aligned} \tag{39}$$

Solving for $\hat{J}_{\bar{1}}$ and $\hat{J}_1$ from the first and the third equations in (39) in terms of $\hat{J}_0$ and substituting for $\hat{J}_0$ in the second equation of (39), we finally have at the Nth stage

$$\left[a^{(N-1)}_{00} - a^{(N-1)}_{0\bar{1}}a^{(N)}_{\bar{1}0} - a^{(N-1)}_{01}a^{(N)}_{10}\right]\hat{J}_0 = 0 \tag{40}$$

where

$$\begin{aligned} a_{00}^{(N-1)} &= a_{00} - \left(a_{\overline{2}0}\right)^2 / a_{\overline{2}\overline{2}}^{(N-2)} - \left(a_{20}\right)^2 / a_{22}^{(N-2)}, \qquad (41) \\ a_{\overline{1}0}^{(N)} &= \left(a_{\overline{1}0}^{(N-1)} a_{11}^{(N-1)} - \gamma_0 \sigma_0 a_{10}^{(N-1)}\right) / \left(a_{\overline{1}\overline{1}}^{(N-1)} a_{11}^{(N-1)} - \gamma_0^2 \sigma_0^2\right), \\ a_{10}^{(N)} &= \left(a_{10}^{(N-1)} a_{\overline{1}\overline{1}}^{(N-1)} - \gamma_0 \sigma_0 a_{\overline{1}0}^{(N-1)}\right) / \left(a_{11}^{(N-1)} a_{\overline{1}\overline{1}}^{(N-1)} - \gamma_0^2 \sigma_0^2\right) \end{aligned}$$

and where $\gamma_0 = (2/\pi)^2$. The first two relations in (37) and their two counterparts without the overbar over the suffixes continue to be valid for $i = N - 1$ also. Thus, the approximate dispersion equation for a truncation order of N becomes

$$\begin{aligned} &\sigma_0 + \gamma_0 \left(\sigma_{-1} + \sigma_1\right) \\ &- \left\{ \frac{\left[a_{11}^{(N-1)} \left(a_{\overline{1}0}^{(N-1)}\right)^2 + a_{\overline{1}\overline{1}}^{N-1} \left(a_{01}^{(N-1)}\right)^2 - 2\gamma_0 \sigma_0 a_{\overline{1}0}^{(N-1)} a_{01}^{(N-1)}\right]}{\left(a_{\overline{1}\overline{1}}^{(N-1)} a_{11}^{(N-1)} - \gamma_0^2 \sigma_0^2\right)} \right\} \\ &- \gamma_0^2 \left(\sigma_{-1}^2 / a_{\overline{2}\overline{2}}^{(N-2)} + \sigma_1^2 / a_{22}^{(N-2)}\right) = 0 \qquad (42) \end{aligned}$$

Once a solution $\hat{\beta}_{\mathbf{0}}$ of the approximate dispersion equation (42) for $\beta_{\mathbf{0}}$ is obtained it is a simple matter to work backwards expressing successively $\hat{J}_i$ and $\hat{J}_{-i}$ for $i = 1, 2, 3, \ldots, N$ in terms of $\hat{J}_0$ to determine the $(2N + 1) \times 1$ mode vector $\hat{\mathbf{J}}$, corresponding to $\hat{\beta}_0$, for the tape current density. A documentation of the mode-vector expression is not attempted here since an open tape helix is only of academic interest as a slow-wave structure for travelling wave tubes. However, a field analysis of the cold-wave (including the contribution of the transverse tape-current density if found significant) problem for a dielectric-loaded tape helix enclosed in a perfectly conducting coaxial cylindrical shell, which serves as a good model for the TWT slow-wave structure, is proposed to be taken up in the near future.

The approximate dispersion equation of the tape helix for any order of truncation N is of the form

$$\sigma_0 + \gamma_0 F_N(\sigma_{\pm 1}, \sigma_{\pm 2}, \ldots, \sigma_{\pm(N+1)}) = 0 \qquad (43)$$

where $F_N = G_N / H_N$ is a symmetric rational function of the $2(N + 1)$ arguments $\sigma_{\pm 1}, \sigma_{\pm 2}, \ldots, \sigma_{\pm(N+1)}$. The expressions for the G_N and the H_N, which are homogeneous functions of their arguments, are given below:

$$\begin{aligned}
G_0 &= \sigma_1 + \sigma_{\bar{1}} \\
G_1 &= \Delta_1^{(1)}\Delta_{\bar{1}}^{(1)}\left[\sigma_1 + \sigma_{\bar{1}} - \gamma_0\left(\sigma_1^2/\Delta_1^{(1)} + \sigma_{\bar{1}}^2/\Delta_{\bar{1}}^{(1)}\right)\right] \\
G_2 &= \Delta_2^{(2)}\Delta_{\bar{2}}^{(2)}\left[\sigma_1 + \sigma_{\bar{1}} - \gamma_0\left(\sigma_1^2/\Delta_1^{(2)} + \sigma_{\bar{2}}^2/\Delta_{\bar{1}}^{(2)}\right)\right] \\
&\quad -\Delta_2^{(2)}\left(\Delta_{\bar{3}}^{(2)}\right)^2 - \Delta_{\bar{2}}^{(2)}\left(\Delta_3^{(2)}\right)^2 \qquad (44) \\
G_N &= \Delta_{2N-2}^{(N)}\Delta_{\overline{2N-2}}^{(N)}\left[\sigma_1 + \sigma_{\bar{1}} - \gamma_0\left(\sigma_1^2/\Delta_{2N-4}^{(N)} + \sigma_{\bar{1}}^2/\Delta_{\overline{2N-4}}^{(N)}\right)\right] \\
&\quad -\Delta_{2N-2}^{(N)}\left(\Delta_{\overline{2N-1}}^{(N)}\right)^2 - \Delta_{\overline{2N-2}}^{(N)}\left(\Delta_{2N-1}^{(N)}\right)^2 \qquad \text{for } N \geq 3 \\
H_0 &= 1 \\
H_1 &= \Delta_1^{(1)}\Delta_{\bar{1}}^{(1)} - 2\gamma_0\left(\sigma_1\Delta_{\bar{1}}^{(1)} + \sigma_{\bar{1}}\Delta_1^{(1)}\right) \\
&\quad +\gamma_0^2\left[(\sigma_1 + \sigma_{\bar{1}})\left(\Delta_1^{(1)}\Delta_{\bar{1}}^{(1)}\right) - (\sigma_1 - \sigma_{\bar{1}})^2\right] \\
H_2 &= \Delta_2^{(2)}\Delta_{\bar{2}}^{(2)} - 2\gamma_0\left(\Delta_2^{(2)}\Delta_{\bar{3}}^{(2)} + \Delta_{\bar{2}}^{(2)}\Delta_3^{(2)}\right) \\
&\quad -\gamma_0^2\left(\Delta_3^{(2)} - \Delta_{\bar{3}}^{(2)}\right)^2 \\
&\quad +\gamma_0^2\left(\Delta_2^{(2)} + \Delta_{\bar{2}}^{(2)}\right)\left[\sigma_1 + \sigma_{\bar{1}} - \gamma_0\left(\sigma_1^2/\Delta_1^{(2)} + \sigma_{\bar{1}}^2/\Delta_{\bar{1}}^{(2)}\right)\right] \qquad (45) \\
H_N &= \Delta_{2N-2}^{(N)}\Delta_{\overline{2N-2}}^{(N)} - 2\gamma_0\left(\Delta_{2N-2}^{(N)}\Delta_{\overline{2N-1}}^{(N)} + \Delta_{\overline{2N-2}}^{(N)}\Delta_{2N-1}^{(N)}\right) \\
&\quad -\gamma_0^2\left(\Delta_{2N-1}^{(N)} - \Delta_{\overline{2N-1}}^{(N)}\right)^2 + \gamma_0^2\left(\Delta_{2N-2}^{(N)} + \Delta_{\overline{2N-2}}^{(N)}\right) \\
&\quad \left[\sigma_1 + \sigma_{\bar{1}} - \gamma_0\left(\sigma_1^2/\Delta_{2N-4}^{(N)} + \sigma_{\bar{1}}^2/\Delta_{\overline{2N-4}}^{(2)}\right)\right] \qquad \text{for } N \geq 3
\end{aligned}$$

The recursive formulae for the $\Delta_{2K-1}, \Delta_{\overline{2K-1}}, \Delta_{2K-2}$, and $\Delta_{\overline{2K-2}}$, $1 \leq K \leq N$, appearing in (44) and (45) are

$$\begin{aligned}
\Delta_1^{(N)} &= \sigma_N + \gamma_0(\sigma_{N+1} + (1 - \delta_{0,N-1})\sigma_{N-1}) \qquad \text{for } N \geq 1 \\
\Delta_2^{(N)} &= \sigma_{N-1} + \gamma_0(\sigma_N + (1 - \delta_{0,N-2})\sigma_{N-2}) \\
&\quad -\gamma_0(\sigma_N + \sigma_{N-1})^2/\Delta_1^{(N)} \qquad \text{for } N \geq 2 \\
\Delta_3^{(N)} &= \sigma_{N-1} + (1 - \delta_{0,N-2})\sigma_{N-2}) \\
&\quad -\gamma_0(\sigma_N + \sigma_{N-1})\sigma_{N-1}/\Delta_1^{(N)} \qquad \text{for } N \geq 2 \\
\Delta_4^{(N)} &= \sigma_{N-2} + \gamma_0(\sigma_{N-1} + (1 - \delta_{0,N-3})\sigma_{N-3})
\end{aligned}$$

$$-\gamma_0^2\sigma_{N-1}^2/\Delta_1^{(N)} - \gamma_0\left(\Delta_3^{(N)}\right)^2/\Delta_2^{(N)} \qquad \text{for } N \geq 3$$

$$\begin{aligned}\Delta_{2K-1}^{(N)} &= \sigma_{N-K+1} + (1-\delta_{0,N-K})\sigma_{N-K} \\ &\quad -\gamma_0\sigma_{N-K+1}\Delta_{2K-3}^{(N)}/\Delta_{2K-4}^{(N)} \qquad \text{for } N \geq K \geq 3 \\ \Delta_{2K-2}^{(N)} &= \sigma_{N-K+1} + \gamma_0(\sigma_{N-K+2} + (1-\delta_{0,N-K})\sigma_{N-K}) \\ &\quad -\gamma_0^2\sigma_{N-K+2}^2/\Delta_{2K-6}^{(2)} - \gamma_0\left(\Delta_{2K-3}^{(N)}\right)^2/\Delta_{2K-4}^{(N)} \\ &\qquad \text{for } N \geq K \geq 4\end{aligned}$$

together with a corresponding set of formulae with an overbar over the suffixes.

For the purpose of studying the behavior of the dispersion characteristic as a function of the truncation order, it is convenient to introduce the nondimensional parameter

$$\tau_{na} \triangleq \tau_n a = \left\{(\beta_{0a} + n\cot\psi)^2 - k_{0a}^2\right\}^{1/2}$$

in addition to $\beta_{0a} \triangleq \beta_0 a$, and $k_{0a} \triangleq k_0 a$. In terms of the nondimensional quantities, the expression (22) for σ_n becomes

$$\begin{aligned}\sigma_n &= \left[\left(\beta_{0a}^2 - k_{0a}^2\right)\sin\psi + n\beta_{0a}\cos\psi\right]^2 I_{na}K_{na}/\tau_{na}^2 \\ &\quad + k_{0a}^2 I'_{na}K'_{na}\cos^2\psi, \qquad n \in \mathbb{Z} \qquad (46)\end{aligned}$$

The relation $\sigma_0 = 0$ may be recognized as the sheath-helix dispersion equation for the dominant mode ($n = 0$). Making use the expression (46) for σ_0, the dispersion equation (43) may be put in the fixed-point format

$$\begin{aligned}k_{0a} &= G(k_{0a};\beta_{0a}) \\ &\triangleq \left\{\beta_{0a}^2/Q_{sa} + \frac{\gamma_0 F_N\left(\sigma_{\pm 1},\ldots,\sigma_{\pm(N+1)}\right)}{Q_{sa}I_{0a}K_{0a}\sin^2\psi}\right\}^{1/2} \qquad (47)\end{aligned}$$

The right member of (47) may be viewed as an 'operator' G that maps k_{0a} into $G(k_{0a};\beta_{0a})$ for a fixed β_{0a}. The symmetric dependence of the function F_N on $\sigma_{\pm i}$, $1 \leq i \leq N$, implies that if β_{0a} satisfies the dispersion equation (47) for a given k_{0a} so does $-\beta_{0a}$ for the same k_{0a}. Therefore, solution of the dispersion equation (47) for $k_{0a}(\beta_{0a})$ need only be sought for nonnegative values of β_{0a}. Thus, equation (47) may be solved numerically for $k_{0a}(\beta_{0a})$, $\beta_{0a} \geq 0$, by the method of successive substitutions to find any fixed point of the operator G for

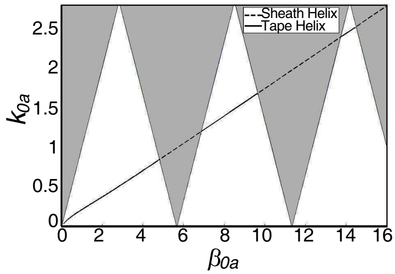

Figure 2. Dispersion characteristic of tape helix for truncation order $N = 2$.

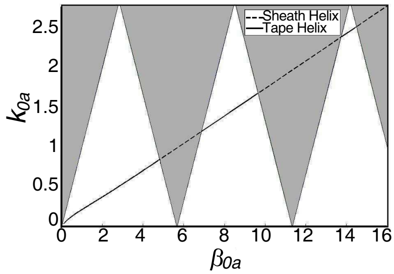

Figure 3. Dispersion characteristic of tape helix for truncation order $N = 3$.

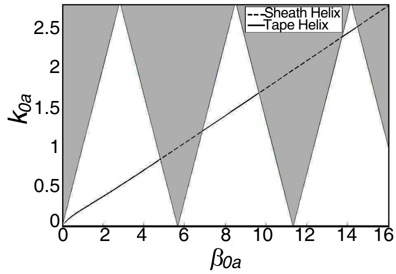

Figure 4. Dispersion characteristic of tape helix for truncation order $N = 4$.

k_{0a} in the range $0 < k_{0a} < (1/2) \cot \psi$. The resulting family of tape-helix dispersion curves for the choice of the pitch angle $\psi = 10^\circ$ are plotted in Figs. 2–4 for truncation orders of 2, 3 and 4 respectively. The dominant mode dispersion curve of the sheath helix (for the same value of $\psi = 10^\circ$) is also plotted in the figures for comparison. Tape-helix dispersion curves for the truncation orders of 0 and 1 are not shown because the iterations for these two cases could not be continued (to convergence) beyond $\beta_{0a} = 4.64$ to yield real values for $k_{0a}(\beta_{0a})$ in the complement of the forbidden regions. A portion of Fig. 2 magnified several fold to make the minute difference between the dispersion curves discernible is shown in Fig. 5. It may be seen from Fig. 5 that the phase speed for the tape-helix model is lower than that for the sheath-helix model for the same value of ω in the complement of the forbidden regions. However, the dispersion curves of Fig. 5 are so closely spaced that it may not be appropriate to draw any inference based on an inspection of Fig. 5.

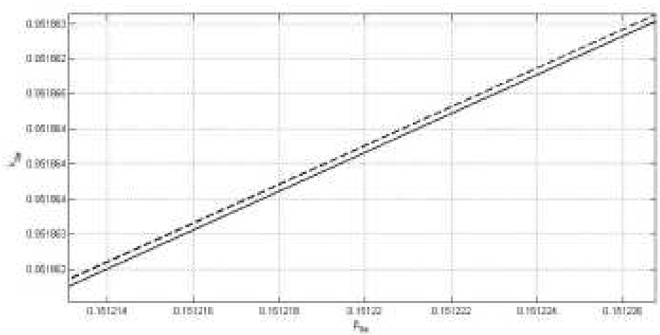

Figure 5. Blown-up portion from Fig. 2.

4. CONCLUSION

It may be observed from the plots of Figs. 2–4 that the tape-helix dispersion curves for $N = 2, 3$ and 4 follow the dominant-mode sheath helix dispersion curve very closely everywhere within the complement of the forbidden regions (shown shaded in the figures). The solution for $k_{0a}(\beta_{0a})$ within the forbidden regions acquires a small imaginary part (on the order of 10^{-5}) on account of τ_{-1a}, τ_{-2a} and τ_{-3a} becoming purely imaginary within the 1st, the 2nd and the 3rd forbidden region respectively where the nth forbidden region for $n \in \mathbb{Z}$, is taken to be the portion of the β_{0a}-k_{0a} plane inside the (inverted) triangle formed by the straight lines $k_{0a} = -\beta_{0a} + n \cot\psi$, $k_{0a} = \beta_{0a} - n \cot\psi$ and $k_{0a} = (1/2)\cot\psi$. It is thus seen that the mode constant τ_{-na} of the $-n$th space-harmonic contribution to the total field becomes imaginary in the nth forbidden region, and that the resultant Poynting vector acquires a small radial component in order to account for the radiation of power from the $-n$th space harmonic. Since the dispersion curves for $N = 2, 3$ and 4 are virtually indistinguishable from one another, a truncation order as low as 2 is adequate to deliver an accurate estimate of the tape-helix dispersion characteristic (at least within the validity limits of the assumed model for the parameter values used in the numerical computations). However, a fairly large number of modal 'amplitudes' $J_n, |n| \geq 0$, is needed in the infinite-series representation (14) for the surface current density component $J_s(\varphi, z)$ so as to ensure a reasonably good approximation for the tape-current density, and hence for the electromagnetic field vectors. The main conclusion that may be drawn from the present study is the following: The dominant-mode dispersion characteristic of the sheath helix is an excellent approximation to that of the tape helix in the complement of the forbidden regions *provided* that the neglect of the transverse component of the tape-current density does not give rise to any appreciable error even for tapes which are not narrow. Whether such an hypothesis is true or false can be ascertained only through an analysis of guided electromagnetic wave propagation that fully accounts for the transverse component of the tape-current density.

Based on the outcome of such a study (which is currently under progress), it is proposed to extend the method adopted for the derivation of the tape-helix dispersion equation to a full field analysis of the practically important case of a dielectric-loaded helical slow-wave structure enclosed in a coaxial metal cylindrical shell and supported by azimuthally symmetrically placed dielectric rods. The effect of the dielectric support rods will have to be modeled by a homogeneous dielectric the effective dielectric constant of which can be determined in terms of the geometric arrangement of the support rods and the actual dielectric constant of the rod material.This process of homogenization is equivalent to replacing the azimuthally nonhomogeneous dielectric constant of the annular region between the helix and the outer conductor by its azimuthal average, which becomes a constant independent of the radial coordinate, for azimuthally symmetrically placed wedge-type support rods. In any case, it is necessary to smooth out any kind of azimuthal nonhomogenity before attempting a solution of the cold-wave problem by an extension of the method introduced in this paper because any axial asymmetry of the slow-wave structure would be inconsistent with the property of geometrical invariance under simultaneous translation and rotation exhibited by an infinite helical structure.

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Errata to DISPERSION OF ELECTROMAGNETIC WAVES GUIDED BY AN OPEN TAPE HELIX I by N. Kalyanasundaram and G. Naveen Babu, in *Progress In Electromagnetics Research B*, Vol. 16, pp. 311–331, 2009

(i) Page 315, 3rd line from top: Correct '$\frac{\partial^2 U}{\partial\varphi\partial z} - \frac{j\omega\mu}{\rho}\frac{\partial V}{\partial\rho}$' to '$\frac{1}{\rho}\frac{\partial^2 U}{\partial\varphi\partial z} + j\omega\mu\frac{\partial V}{\partial\rho}$'.

(ii) Page 315, 4th line from top: Insert '$/\rho$' after '$G_n(\tau_n\rho)$'.

(iii) Page 315, 7th line from top: Insert '$-$' between '$\sum_{n=-\infty}^{\infty}$' and '['.

(iv) Page 315, 8th line from top: Correct '$\frac{\partial^2 V}{\partial\varphi\partial z} - \frac{j\omega\epsilon}{\rho}\frac{\partial U}{\partial\rho}$' to '$\frac{1}{\rho}\frac{\partial^2 V}{\partial\varphi\partial z} - j\omega\epsilon\frac{\partial U}{\partial\rho}$'.

(v) Page 316, 12th line from top: Remove '$-$' before '$\cot\psi/a$'.

(vi) Page 316, 14th line from bottom: Remove '$-$' before '$\cot\psi/a$'.

(vii) Page 316, 13th line from bottom: After 'that is,' replace 'k_0' by 'k_0a'.

(viii) Page 317, 10th line from top: Change '$-$' sign to '$+$' sign.

(ix) Page 317, 6th line from bottom: Correct '$J_s(\rho,\varphi,z)$' to '$J_s(\varphi,z)$'.

(x) Page 319, 6th line from top: Replace '(9a)–(9c)' by '(13a)–(13c)'.

(xi) Page 322, Equation (32): Insert 'k_{0a}' between 'I_{0a}' and '$\sin^2\psi$' in the 2nd term within the curly brackets.

(xii) Page 325, 5th line from bottom: Replace the coefficient '$a_{\overline{11}}$' by '$a_{1\overline{1}}$'.

(xiii) Page 326, 6th line from top: After 'counterparts' replace 'without' by 'with'.

(xiv) Page 327, 3rd line from top: Delete the 'γ_0' in front of the parenthesis.

(xv) Page 327, 4th line from top: Replace 'σ_2^2' by 'σ_1^2'.

(xvi) Page 327, 10th line from top: Insert '$+$' in between '$\Delta_1^{(1)}$' and '$\Delta_{\overline{1}}^{(1)}$'.