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RESONANT SCATTERING OF ELECTROMAGNETIC WAVE BY STRIPE GRATING BACKED WITH A LAYER OF METAMATERIAL

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Abstract—Scattering of electromagnetic waves by periodic stripe grating backed with a layer of metamaterial is considered. The distinguished feature of the structure is the association of periodicity of two scales: Micro scale that is the scale characteristic to metamaterial of the layer, and the scale of periodic metal stripe grating that is of the scale of wavelength of incident electromagnetic field. Such association gives rise to new type of resonant phenomena such as "crowding" of resonant transmission/reflection peaks in the vicinity of characteristic frequencies of the structure. The study of the problem is performed on the base of rigorous and accurate solution to the diffraction and spectral problems, which guarantees the robustness of numerical algorithm; and allows asymptotical analytical analysis of the problem and prediction of various resonant phenomena.

1. INTRODUCTION

The manufacturing of new composit materials performing unexpected features had achieved considerable successes during recent years [1]. Diverse aspects of possible applications of such materials are in agenda in modern electromagnetic community [2–7].

The investigation of wave processes in open structures (open resonators and waveguides, diffraction gratings and others) containing

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metamaterials, will, undoubtedly, be of great interest for creation of new principles of electromagnetic wave generating, amplifying and energy channeling. The key issue in such investigation is the study of resonant phenomena, arising in interaction of electromagnetic waves with periodic structures comprising metamaterials.

In this paper the results of investigation of resonant phenomena, appearing in interaction of monochromatic electromagnetic waves with periodic stripe grating backed with a layer of metamaterial with frequency dependent effective permittivity are presented. The distinguished feature of the structure is the association of periodicity of two scales: i) Micro scale — characteristic for metamaterial structure, and ii) the scale of the period of metallic stripe grating, which is of the order of the wavelength of incident electromagnetic field. Such association gives rise to the new type of resonant phenomena such as "crowding" of resonant transmission/reflection peaks in the vicinity of characteristic frequencies of the structure.

As a metal stripe grating is rather popular structure in applied electromagnetics thus corresponding diffraction problem for metal stripe grating has its own long term history. Our consideration is based on the ideas relayed on the Riemann Hilbert problem technique, that where published for the first time in 1962, and described in details in [12].

In [15] we have considered the electromagnetic wave scattering by metal stripe grating placed on half space of metamaterial (reflective structure). There we have focused on the solution to diffraction problem and on the analysis of resonant transformation of incident field into eigen surface oscillations.

Here we perform a systematic analysis based on rigorous and accurate solution to the diffraction and spectral problems for the metal grating backed with a layer of metamaterial (semi transparent structure). The numerical algorithm for simulation was constructed as further development of the algorithms and codes described in [15].

The advantages of the rigorous methods for analysis of such problems are particularly clearly seen in the possibility to perform analytical preliminary analysis of electromagnetic features of the problem and to estimate the location of resonance frequencies and their behavior, find out their peculiarities and to obtain the numerical solution with any required accuracy.

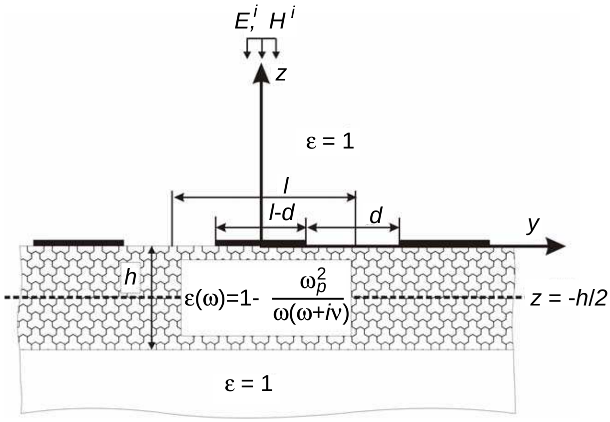


Figure 1. Geometry of the problem.

2. MATHEMATICAL MODEL

2.1. Formulation of Diffraction Problem and Method for Its Solving

Consider the following diffraction problem. Let the domain $-h < z < 0$ (see Fig. 1) is filled with isotropic metamaterial with effective permittivity given by the relation

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}, \tag{1}$$

here ω_p is a characteristic frequency, often called plasma frequency, which depends on the structural parameters of the metamaterial, $\nu \geq 0$ is a frequency defining active losses, $\mu = 1$. Such model of metamaterial had been suggested in [8]. The grating of the period l formed by perfectly conducting stripes with the slot of width d at $z = 0$ is placed on the upper face of the metamaterial layer of the thickness h, see Fig. 1. The structure is infinite and homogeneous along x. We presume that the incident wave and diffraction field are independent of coordinate x, the time factor is chosen as $e^{-i\omega t}$. The incident H-polarized plane wave of unit amplitude propagates normally to the xy plane and comes from the upper half space ($z > 0$):

$$H_x^i = e^{-ikz}, \quad E_y^i = e^{-ikz}, \quad E_x^i = E_z^i = H_y^i = H_z^i = 0,$$

here $k = \omega/c$, c is the light velocity in vacuum. The oblique incidence does not bring essential difference into mathematical derivations, but due to the "splitting" of resonances, see [15], it gives more complicated pictures of diffraction characteristics and makes the physical analysis vaguer.

The diffraction field $\left(\vec{E}^{\partial}, \vec{H}^{\partial}\right)$ has to satisfy:

- ∼ homogeneous system of Maxwell equations,
- ∼ Meixner condition,
- ∼ radiation conditions at infinity ($z \to \pm\infty$),
- ∼ the condition of periodicity,

boundary conditions on perfectly conducting stripes of grating and transparency conditions in the boundary of metamaterial, that is in the slots.

One can show that under the given assumptions the components of diffracted field $(\vec{E}^{\partial}, \vec{H}^{\partial})$ are defined via the only nonzero component of magnetic field H_x^{∂}:

$$E_y^{\partial} = -\frac{1}{ik\varepsilon}\frac{\partial H_x^{\partial}}{\partial z}, \quad E_z^{\partial} = \frac{1}{ik\varepsilon}\frac{\partial H_x^{\partial}}{\partial y}.$$

The rest of the field components $(\vec{E}^{\partial}, \vec{H}^{\partial})$ are equal to zero, that is the diffraction field is also H-polarized.

Let us introduce the function $u\,(y, z)$, coinciding with magnetic component of total field $H_x^i + H_x^{\partial}$. As follows from Maxwell equations this function $u\,(y, z)$ has to satisfy Helmholtz equation everywhere outside of the stripes of grating and boundary of metamaterial

$$\begin{aligned} &\Delta u\,(y,z) + k^2\,(z)\,u\,(y,z) = 0; \\ &k^2\,(z) = \begin{cases} k^2, & z > 0,\ z < -h, \\ k^2\varepsilon, & -h < z < 0. \end{cases} \end{aligned} \tag{2}$$

$u\,(y, z)$has to be periodic over y with period l and to meet radiation condition in half space $z > 0$ and $z < -h$. Taking into consideration these requirements we can present the function $u\,(y, z)$ in the form

$$u\,(y,z) = \begin{cases} e^{-i\kappa\frac{2\pi}{l}z} + \sum\limits_n R_n e^{i\frac{2\pi n}{l}y} e^{i\Gamma_{1n}\frac{2\pi}{l}z}, & z > 0, \\ \sum\limits_n e^{i\frac{2\pi n}{l}y}(C_{1n}e^{i\Gamma_{2n}\frac{2\pi}{l}(z+2h)} + C_{2n}e^{-i\Gamma_{2n}\frac{2\pi}{l}z}), & \\ & -h < z < 0, \\ \sum\limits_n T_n e^{i\frac{2\pi n}{l}y} e^{-i\Gamma_{1n}\frac{2\pi}{l}(z+h)}, & z < -h. \end{cases} \tag{3}$$

Here $\Gamma_{1n} = \sqrt{\kappa^2 - n^2}$, $\Gamma_{2n} = \sqrt{\kappa^2\varepsilon - n^2}$, $\kappa = \omega l/2\pi c$. Branches of the square roots are chosen according to the radiation conditions [9, 16], requiring the absence of waves coming from infinity: $\kappa \mathrm{Re}\,(\Gamma_{1n}) \geq 0$, $\mathrm{Im}\,(\Gamma_{1n}) \geq 0$. Note, that in general the choice of the branch of the root $\Gamma_{2n} = \sqrt{\kappa^2\varepsilon - n^2}$ is arbitrary, but, to avoid ambiguity, we presume that $\kappa \mathrm{Re}\,(\Gamma_{2n}) \geq 0$, $\mathrm{Im}\,(\Gamma_{2n}) \geq 0$.

Functions $u\,(y,z)$ and $\frac{1}{k^2(z)}\frac{\partial u(y,z}{\partial z}$ have to be continuous in the slots of grating ($z = 0,\ |\frac{2\pi y}{l} + n| > \pi(1 - \frac{d}{l}),\ n = 0, \pm 1, \pm 2, \ldots$) and in the boundary between metamaterial and free space at $z = -h$; on the stripes ($z = 0,\ |\frac{2\pi y}{l} + n| < \pi(1 - \frac{d}{l}),\ n = 0, \pm 1, \pm 2, \ldots$) function $u\,(y,z)$ has to meet conditions

$$\left.\frac{\partial u\,(y,z)}{\partial z}\right|_{z=0+0} = 0, \quad \left.\frac{\partial u\,(y,z)}{\partial z}\right|_{z=0-0} = 0. \tag{4}$$

Using the analytical regularization method, associated with Riemann Gilbert conjugation problem, developed in [10, 11], the problem can be reduced to the solution of the infinite system of linear algebraic equations for unknown amplitudes $(R_n)_{n=-\infty}^{\infty}$ of diffraction field in half space $z > 0$ (see (3)).

The system has the form

$$(1+\varepsilon)\,R = AR + b, \tag{5}$$

where $R = (R_n)_{n=-\infty}^{\infty}$, $b = (b_n)_{n=-\infty}^{\infty}$, $A = \|a_{mn}\|_{m,n=-\infty}^{\infty}$.

A and b are defined as follows:

$$b_m = \begin{cases} \frac{(1+d_0)(1+\varepsilon)}{d_0-1} + \frac{i\kappa W_0(u)(1+\varepsilon)^2}{1-d_0}, & m = 0, \\ -(1+\varepsilon)\,\frac{i\kappa V_{m-1}^{-1}(u)}{m}, & m \neq 0, \end{cases}$$

$$a_{mn} = \begin{cases} \frac{-i\kappa W_0(u)(1+\varepsilon)^2}{1-d_0}, & m = n = 0, \\ \frac{\delta_n |n|(1+\varepsilon)}{(1-d_0)n} V_{n-1}^{-1}(u), & m = 0, \quad n \neq 0, \\ \frac{i\kappa}{m} V_{m-1}^{-1}(u)(1+\varepsilon)^2, & m \neq 0, \quad n = 0, \\ \frac{|n|\delta_n}{m} V_{m-1}^{n-1}(u), & m, n \neq 0, \quad m \neq n, \\ d_m + \varepsilon + \frac{|m|\delta_m}{m} V_{m-1}^{m-1}(u), & m \neq 0, \quad m = n, \end{cases} \tag{6}$$

here $\delta_n = 1 - d_n + \frac{i(1+\varepsilon)}{|n|}\Gamma_{1n}$, $d_n = \frac{\Gamma_{1n}\varepsilon[e^{i2\Gamma_{2n}\bar{h}}(\Gamma_{2n}-\varepsilon\Gamma_{1n})+\Gamma_{2n}+\varepsilon\Gamma_{1n}]}{\Gamma_{2n}[e^{i2\Gamma_{2n}\bar{h}}(\Gamma_{2n}-\varepsilon\Gamma_{1n})-\Gamma_{2n}-\varepsilon\Gamma_{1n}]}$, $\bar{h} = 2\pi h/l$. Functions $W_0(u), V_{m-1}^{n-1}(u)$, where $u = -\cos(\pi d/l)$, are defined in [12] and have the form:

$$V_{m-1}^{n-1}(u) = \frac{1}{2}\begin{cases} \frac{m}{m-n}[P_{m-1}(u)P_n(u) - P_m(u)P_{n-1}(u)], & m \neq n, \\ \sum\limits_{s=0}^{n} \rho_{n-s}(u)P_{s-n}(u), & m = n \geq 0, \end{cases}$$

$$V_{-n-1}^{-n-1}(u) = -V_{n-1}^{n-1}(u),\ n \geq 1, \quad W_0 = -\ln\frac{1+u}{2}.$$

Here $P_n(u)$ are the Legendre polynomials with following relation for negative n $P_n(u) = P_{|n|-1}(u)$; functions $\rho_n(u), n = 0, 1, 2, \dots$ are defined via the Legendre polynomials according:

$$\begin{aligned}\rho_0(u) &= 1, \quad \rho_1(u) = -u,\\ \rho_n(u) &= P_n(u) - 2uP_{n-1}(u) - P_{n-2}(u),\ n = 2, 3, \dots\end{aligned}$$

In domain $(z < -h)$, and in the layer of metamaterial $(-h < z < 0)$ amplitudes $(T_n)_{n=-\infty}^{\infty}$, $(C_{1n})_{n=-\infty}^{\infty}$, and $(C_{2n})_{n=-\infty}^{\infty}$ of diffraction harmonics may be expressed via amplitudes $(R_n)_{n=-\infty}^{\infty}$ of the reflected diffraction field

$$\begin{aligned}T_n &= \frac{\varepsilon\Gamma_{1n}(R_n - \delta_{0n})}{i\Gamma_{2n}\sin(\Gamma_{2n}\bar{h}) - \varepsilon\Gamma_{1n}\cos(\Gamma_{2n}\bar{h})},\\ C_{1n} &= \frac{\varepsilon\Gamma_{1n}(R_n - \delta_{0n})\left(\Gamma_{2n} - \varepsilon\Gamma_{1n}\right)}{[e^{i2\Gamma_{2n}\bar{h}}\left(\Gamma_{2n} - \varepsilon\Gamma_{1n}\right) - \Gamma_{2n} - \varepsilon\Gamma_{1n}]\Gamma_{2n}},\\ C_{2n} &= \frac{\varepsilon\Gamma_{1n}(R_n - \delta_{0n})\left(\Gamma_{2n} + \varepsilon\Gamma_{1n}\right)}{[e^{i2\Gamma_{2n}\bar{h}}\left(\Gamma_{2n} - \varepsilon\Gamma_{1n}\right) - \Gamma_{2n} - \varepsilon\Gamma_{1n}]\Gamma_{2n}}.\end{aligned} \tag{7}$$

δ_{0n} — is Kroneker delta.

Using asymptotical estimates for $\delta_n = O(\kappa^2/n^2)$ and for $V_{m-1}^{n-1}(u)$ (see [12]) one can prove that matrix A of the system (5) gives in $l_2 = \{(R_n)_{n=-\infty}^{\infty} : \sum\limits_n |R_n|^2 < \infty\}$ space the kernel type operator, see in the book [13]. Hence, the numerical solution to the system (5) can be obtained by means of truncation method with any required accuracy. So, finding out from the system (5) unknown coefficients $(R_n)_{n=-\infty}^{\infty}$ and using relations (3) and (7) we obtain the solution to original diffraction problem. In the case of lossless metamaterial $(\nu = 0)$, one can obtain the solution to original diffraction problem from (5) for any value of frequencies except $\omega = \omega_p/\sqrt{2}$, that is equivalent to$\varepsilon = -1$. This value of frequency is, in certain sense, the essential singular point for the system (5). In particular, when $\omega \to \omega_p/\sqrt{2}$ the operator $[(\varepsilon + 1)\, I - A]$ of the system (5) becomes a kernel type operator, and consequently, it has no bounded inverse operator. That is why in numerical analysis the direct truncation of (5) is impossible. As a solution to our problem in this special case we treat the limit of values $(R_n)_{n=-\infty}^{\infty}$(the solution to the system (5)), acquired by them when the losses in metamaterial tend to zero $(\nu \to 0)$.

2.2. Spectral Problem

Spectral problem describes the singularities of analytical continuation of diffraction problem into the domain of complex valued frequencies.

The mathematical formulation of this problem differs from that one for diffraction problem in two issues: First, there is no incident wave; second, the presentation (3) for function $u(y,z)$, accounting radiation conditions, has to be analytically continued from the domain of real valued frequency into domain of complex valued frequency, that is into the infinitely sheeted Riemann surface (see for example [13]).

We consider the spectral problem for eigen frequencies and eigen oscillations of periodic stripe grating backed with a layer of metamaterial with effective permittivity given by (1). We choose frequency (or normalized frequency $\kappa = \omega l/2\pi c$) as a spectral parameter to be found out; and we seek for ω (or κ) in infinitely sheeted Riemann surface. Further on, we consider the lossless metamaterial ($\nu = 0$), thus the expression for effective permittivity takes the form

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \tag{8}$$

We seek for spectral parameter ω (or κ) in the first "physical" † sheet of Riemann surface defined for various situations in [9, 15, 16].

The spectral problem under consideration is equivalent to the Equation (5) under condition $b = 0$ (no incident wave); it is necessary to consider the matrix entries in (6) as functions of spectral parameter κ that is varying in "physical" sheet of Riemann surface. Such problem is the problem for characteristic numbers and eigen vectors of operator function $I - B(\kappa)$, here $B(\kappa) = (1+\varepsilon)^{-1}A(\kappa)$. Relying on the results [13], it has been proved that matrix operator $B(\kappa)$ is a finite meromorphic kernel operator function of frequency parameter κ allover "physical" sheet of Riemann surface except points $\kappa = 0$ and $\kappa = \pm\kappa_p/\sqrt{2}$, and branching points $\tilde{\kappa}_n, n = 0, \pm1, \pm2, \ldots$. The poles of operator function $I - B(\kappa)$ coincide with point $\kappa = 0$ and roots of the equations

$$\begin{aligned} &e^{i\Gamma_{20}\bar{h}}(\Gamma_{20} - \varepsilon\Gamma_{10}) \pm (\Gamma_2 + \varepsilon\Gamma_1) = 0 \\ &e^{i2\Gamma_{2n}\bar{h}}(\Gamma_{2n} - \varepsilon\Gamma_{1n}) - \Gamma_{2n} - \varepsilon\Gamma_{1n} = 0,\ n = 1, 2, \ldots \end{aligned} \tag{9}$$

Thus, characteristic numbers and, consequently eigen frequencies are the roots of the equation

$$\det(I - B(\kappa)) = 0. \tag{10}$$

It is clear that function $\det(I - B(\kappa))$ is meromorphic in any bounded domain, not containing points $\kappa = 0$, $\kappa = \pm\kappa_p/\sqrt{2}$ and branching

† We mean conventional definition of "physical" sheet of Riemann surface, providing physically meaningful limits of propagation constants with $\kappa'' \to 0$.

points $\kappa = \tilde{\kappa}_n$, $n = 0, \pm 1, \pm 2, \ldots$. Thus, set of the roots of the Equation (10) is the countable set in "physical" sheet of Riemann surface with possible accumulation points at $\kappa = 0$, $\kappa = \pm\kappa_p/\sqrt{2}$, or with poles determined from (9).

Consider now several qualitative properties of the set of roots of the Equation (10). Suppose that the stripes are small enough, that is, $1 - \frac{d}{l} \ll 1$ (when $\frac{d}{l} = 1$ the grating vanishes). Present now the operator function $B(\kappa)$ as a sum of two operator functions $B(\kappa) = B_1(\kappa) + B_2(\kappa)$, where $B_1(\kappa) = \|\delta_m^n \gamma_m\|_{m,n=-\infty}^{\infty}$, $\gamma_0 = 0$, $\gamma_m = \frac{d_m+\varepsilon}{1+\varepsilon}$, $m \neq 0$, then for operator-function $B_2(\kappa)$, relying on the asymptotic estimates of $W_0(u)$, $V_{m-1}^{n-1}(u)$ for $\frac{d}{l} \to 1$ (see, for example, [12]), one can prove

$$\|B_2(\kappa)\| < const\ \sin\pi\left(1 - \frac{d}{l}\right), \tag{11}$$

$\|\ldots\|$ is a norm of operator function $B_2(\kappa)$ in l_2 space. Hence for $\frac{d}{l} \to 1$ the norm of $B_2(\kappa)$ tends to zero in any bounded domain of values of spectral parameter κ, not containing points $\kappa = 0$, $\kappa = \pm\kappa_p/\sqrt{2}$, branching points and poles. It is easy to show, that characteristic numbers of operator function $I - B_1(\kappa)$ coincide with the roots of equation

$$e^{i\Gamma_{2n}\bar{h}}(\Gamma_{2n} - \varepsilon\Gamma_{1n}) \pm (\Gamma_{2n} + \varepsilon\Gamma_{1n}) = 0,\ n = 1, 2, \ldots. \tag{12}$$

One can show that in the domain of values κ, where $\mathrm{Re}(\varepsilon) < 0$ there are real valued roots of Equation (12).

In Fig. 2, the numerical solution to (12), computed for various values of index $n = 1, 2, \ldots$ (asterisks correspond to the sign minus in (12), the square dots — to the sign plus) are presented. With $n \to \infty$ these roots tend asymptotically to the value $\kappa = \kappa_p/\sqrt{2}$ (dot line in Fig. 2). So, the point $\kappa = \kappa_p/\sqrt{2}$ is the accumulation point.

It follows from the Equation (12), that there is certain "characteristic" value of metamaterial parameter $\kappa_p^* = x_0\frac{l}{2\pi h}$, with $x_0 \approx 1.325$, found out from the equation

$$xe^{-0.5\sqrt{x^2+4}} + 2 - \sqrt{x^2+4} = 0.$$

For the case when $\kappa_p \leq \kappa_p^*$ there is only finite number of roots κ_n that are greater than $\kappa_p/\sqrt{2}$ (see Fig. 2(a)); when $\kappa_p > \kappa_p^*$ all the roots of (12) are less than $\kappa_p/\sqrt{2}$ (see Fig. 2(b)).

Let now the value $1 - \frac{d}{l}$ be small and κ_0 be a characteristic number of operator function $I - B_1(\kappa)$, that is the root of equation

$\det(I - B_1(\kappa_0)) = 0$. Obviously, at the boundary of the circle of sufficiently small radius with the center at the point κ_0 operator $(I - B_1(\kappa))^{-1}$ is bounded. Then from (12) follows, that for sufficiently small $1 - \frac{d}{l}$ the inequality

$$\left\|(I - B_1(\kappa))^{-1}B_2(\kappa)\right\| < 1$$

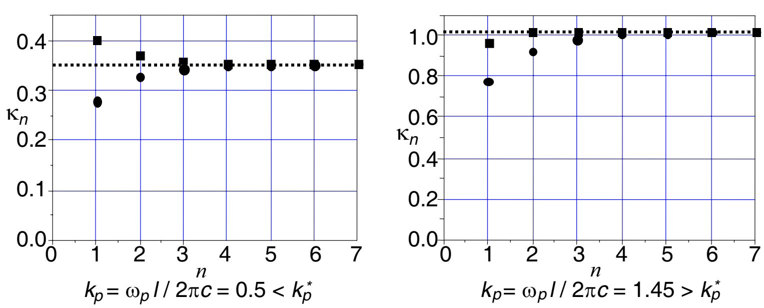
holds along the boundary of the circle.

Using the operator generalization of Rouché's theorem [14] we arrive to the conclusion that in the above mentioned circle there is a characteristic number of operator function $I - B_1(\kappa) - B_2(\kappa)$. It allows to state at least in qualitative level that the set of characteristic numbers of operator function $I - B(\kappa)$, and consequently, the set of eigen frequencies of electromagnetic problem considered (see Fig. 1) has the accumulation point at $\kappa = \kappa_p/\sqrt{2}$.

In order to calculate eigen frequencies for arbitrary values of parameter $0 < \frac{d}{l} \leq 1$ the algebraic scheme suggested in [15] was successfully modified and applied here.

3. NUMERICAL RESULTS

We have performed numerical simulation relying on preliminary analytical analysis and asymptotical estimates, predicting certain unusual resonant phenomena. Several of the results of electromagnetic simulation are discussed in this section.



(a) (b)

Figure 2. Roots κ_n of Equation (12) for various indexes n; $h/l = 1/2\pi$ dotted lines correspond to the value $\kappa_p/\sqrt{2}$. The following notations have been used:

- Vertical even modes ($-$ sign in (12))
- Vertical odd modes ($+$ sign in (12)).

3.1. Spectral Problem

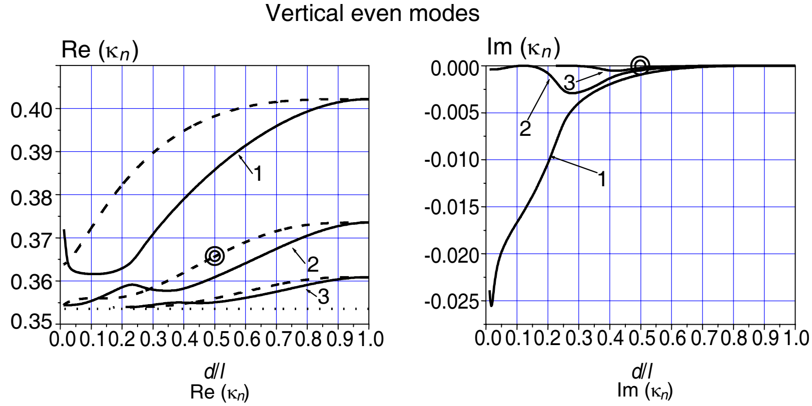
It is reasonable to perform the study within the domain of spectral parameter $\mathrm{Re}\,(\kappa) > 0$. From (6) follows

$$\det\left(I - B\left(\kappa\right)\right) = \overline{\det(I - B(-\bar{\kappa}))},$$

here the dash means complex conjugation. Hence, if κ is eigen frequency with $\mathrm{Re}\,(\kappa) > 0$, then $-\bar{\kappa}$ is an eigen frequency with $\mathrm{Re}\,(\kappa) < 0$ (see for details [9, 16]).

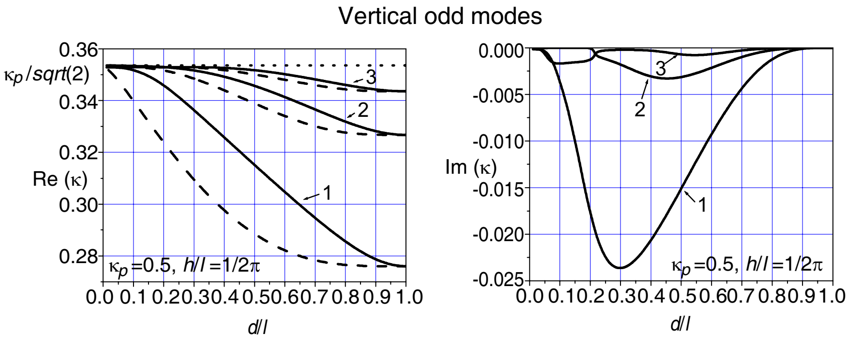
The configuration of grating with layer makes possible to introduce following classification of modes. The symmetry of the structure under consideration with respect to $y = 0$ (see Fig. 1) allows extraction of two classes of eigen oscillations: Odd and even with respect to y. Besides, in the absence of metal stripes, the layer of metamaterial is also symmetric with respect to the plane $z = -h/2$ and, consequently, in such case there are also two types of solutions even and odd with respect to the plane $z = -h/2$. The solutions of the Equation (12) with signs minus and plus correspond to these types of oscillations.

In Fig. 3 and Fig. 4, the behavior of $\mathrm{Re}\,(\kappa)$ and $\mathrm{Im}\,(\kappa)$ when grating factor d/l is varying are presented. The numerical data is obtained from the solution to (10).



(a) (b)

Figure 3. Normalized eigen frequencies $\kappa_n = \omega_n l/2\pi c$ of vertical even modes (symmetric with respect to the plane $z = -h/2$) as a function of the relative slot width d/l, $\kappa_p = 0.5$; $h/l = 1/2\pi$. Solid lines correspond to horizontal even modes (symmetric with respect to the plane $y = 0$) and dashed lines, to horizontal odd modes (anti-symmetric with respect to the plane $y = 0$), for which $\mathrm{Im}\,(\kappa_n) = 0$.



(a) (b)

Figure 4. Normalized eigen frequencies $\kappa_n = \omega_n l/2\pi c$ of vertical odd modes (anti-symmetric with respect to the plane $z = -h/2$) as a function of the relative slot width d/l, $\kappa_p = 0.5$; $h/l = 1/2\pi$. Solid lines correspond to horizontal even modes (symmetric with respect to the plane $y = 0$) and dashed lines, to horizontal odd modes (anti-symmetric with respect to the plane $y = 0$), for which $\mathrm{Im}\,(\kappa_n) = 0$.

The eigen frequencies of oscillations, that are in the limit case $d/l = 1$ symmetric with respect to the plane $z = -h/2$ (sign "−" in (12)) are presented in Fig. 3. Eigen frequencies of antisymmetric (sign "+" in (12)) relatively the plane $z = -h/2$ oscillations are depicted in Fig. 4. Solid lines correspond to the eigen frequencies of oscillations symmetric respectively plane $y = 0$, and dashed lines — to the antisymmetric ones. As magnitudes of imaginary parts of these oscillations are close to zero, in Fig. 3(b) and Fig. 4(b) they are not presented. The dotted lines in Fig. 3(a) and Fig. 4(a) correspond to the value $\mathrm{Re}(\kappa) = \kappa_p/\sqrt{2}$. As one can see from Fig. 3 and Fig. 4, when grating disappears, that is $d/l = 1$, the imaginary parts of eigen frequencies vanish (there are no losses in metamaterial $\nu = 0$).

The real parts of eigen frequencies of even and odd vertical modes (with respect to the plane $z = -h/2$ for given values of $\kappa_p < \kappa_p^* = 1.325l/2\pi h$ are, as we have already mentioned, located at one or other side of the line $\kappa = \kappa_p/\sqrt{2}$. When $d/l = 1$ they coincide with the roots κ_n of the (12) depicted in Fig. 2(a).

In Fig. 5, we have presented the patterns for $|H_x| = const$ and for $\arg(H_x) = const$ of several eigen oscillations, relevant to eigen frequencies depicted in Fig. 3 and Fig. 4.

In Figs. 5(a) and (b), one can see that horizontal odd mode with eigen frequency $\kappa = 0.36565 - i \cdot 0$ (circled points in Figs. 5(a) and (b))

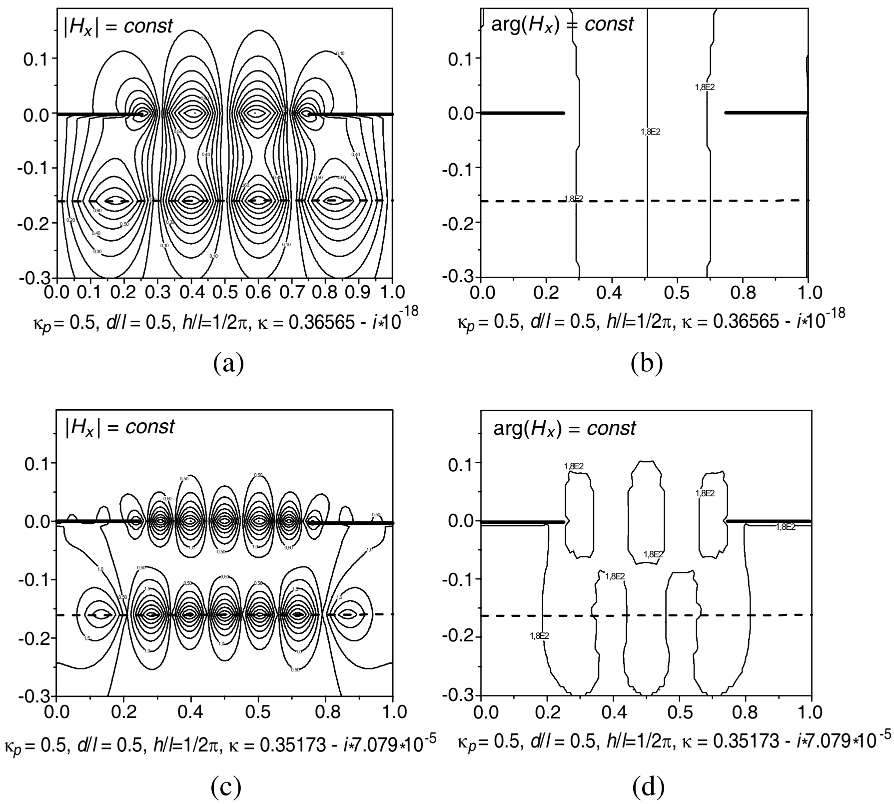


Figure 5. Patterns of eigen fields for several eigen frequencies.

is antisymmetric with respect to the plane $y = 0$ and cophasal (or symmetric) with respect to the plane $z = -h/2$ when $d/l = 1$ (the field of this vertical even mode has the same phase at both boundaries of metamaterial, see Fig. 5(b)). The oscillation with eigen frequency $\kappa = 0.33517 - i \cdot 7.079 \cdot 10^{-5}$, (circular point in Figs. 4(a) and (b)), the patterns are symmetric relative to the plane $y = 0$ and antiphased when $d/l = 1$ relative to $z = -h/2$ (see Fig. 5).

It is worth to be noted that when $d/l \to 0$ (the grating turns into the perfectly conduction plane) the real parts of certain eigen frequencies tend to the poles of operator functions $I - B(\kappa)$, that are the roots of Equation (10); their imaginary parts tend to zero. It is rather difficult to prove these numerical results analytically as entries of operator function $A(\kappa)$ and, consequently, $B(\kappa) = (1 + \varepsilon)^{-1} A(\kappa)$, (see (6)), depend in singular way on parameter d/l when $d/l = 0$.

Let us now investigate the behavior of eigen frequencies when the

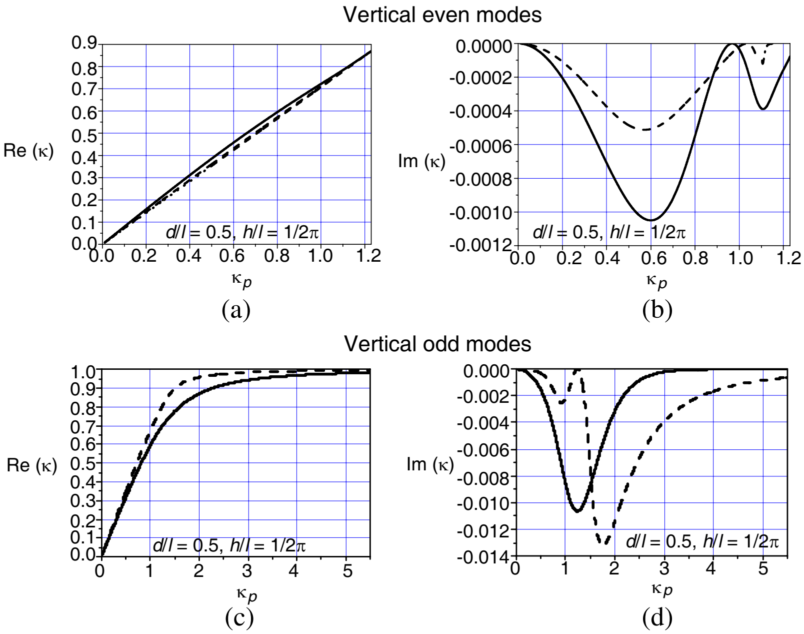


Figure 6. The behavior of families of eigen frequencies κ_n, solid lines for $n = 1$, dashed — $n = 2$, when κ_p is varying: $\mathrm{Re}\,(\kappa)$ are presented in (a) and (c); $\mathrm{Im}\,(\kappa)$ — in (b) and (d).

normalized width of the layer of metamaterial h/l is varying. Suppose that the width of grating slots and characteristic frequency are fixed: $d/l = 0.5$, $\kappa_p = 0.5$. In Fig. 6 the curves $\mathrm{Re}\,(\kappa)$ and $\mathrm{Im}\,(\kappa)$ when h/l is varying are presented for several eigen oscillations (solid and dashed lines correspond to the symmetric with respect to the plane $y = 0$ oscillations, and dotted-dashed line presents the antisymmetric oscillations).

Consider first the case when $d/l \approx 1$, in this situation the real parts of eigen frequencies can be approximated by the roots of (12). For sufficiently small width of layer ($\bar{h} = \frac{2\pi h}{l} \ll 1$) and for $\kappa_p < 1$, we can obtain the following approximate formulas for eigen frequencies

$$\kappa_n^+ \approx \kappa_p \sqrt{\frac{n\bar{h}}{2}}, \quad \kappa_n^- \approx \kappa_p \left(1 - \frac{n^2 \bar{h}}{2\sqrt{n^2 - \kappa_p^2}} \right) \tag{13}$$

Here κ_n^+ and κ_n^- are the roots of (12) with sign "+" and "−" correspondently. As it follows from (13) these roots have following

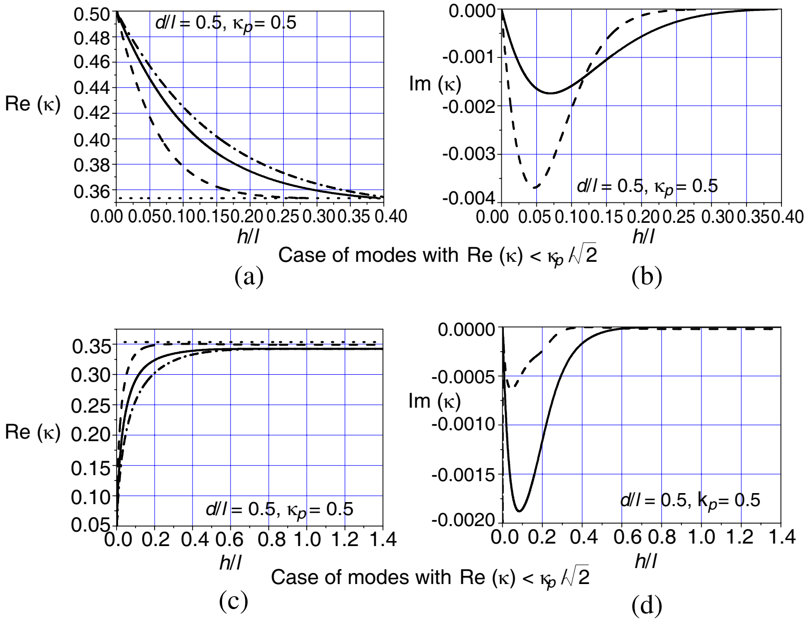


Figure 7. The behavior of eigen frequencies when h/l is varying: Family of curves $\mathrm{Re}\,(\kappa)$ are presented in (a) and (c); $\mathrm{Im}\,(\kappa)$ — in (b) and (d). Solid and dashed lines correspond to the horizontal even modes (symmetric oscillations with respect to the plane $y = 0$); dotted-dashed lines present the horizontal odd modes (anti-symmetric oscillations), they have $\mathrm{Im}\,(\kappa) = 0$.

asymptotic behavior $\kappa_n^+ \to 0$, $\kappa_n^- \to \kappa_p$ when $\bar{h} \to 0$. Numerical modeling proved, that the same behavior of eigen frequencies are characteristic for $\frac{d}{l} \neq 1$ (see Figs. 7(a),(c)). The real parts of eigen frequencies meeting condition $\mathrm{Re}\kappa > \kappa_p/\sqrt{2}$ tend asymptotically to κ_p with layer width decreasing; when layer width is increasing eigen frequencies tend to $\kappa = \kappa_p/\sqrt{2}$, see Fig. 7(a). As we have mentioned above, the value $\kappa = \kappa_p/\sqrt{2}$ is the accumulation point. If for the fixed values of κ_p and h/l eigen frequencies meet the condition $\mathrm{Re}\kappa < \kappa_p/\sqrt{2}$, then with $h/l \to 0$ they tend to zero (see Figs. 6(b), (d)); in opposite situation, when $h/l \to \infty$, $\mathrm{Re}\kappa_n$ tend to corresponding eigen frequencies of grating placed on the half space of metamaterial [15] (see Fig. 7(c)). For imaginary parts the following regularities take place. For $h/l \to 0$ imaginary parts of all types of oscillations ($\mathrm{Re}\kappa > \kappa_p/\sqrt{2}$ and $\mathrm{Re}\kappa < \kappa_p/\sqrt{2}$) tend to zero (see Figs. 7(b), (d)). With layer width increasing imaginary parts of eigen frequencies with $\mathrm{Re}\kappa < \kappa_p/\sqrt{2}$ tend to imaginary parts of corresponding eigen frequencies of grating placed on the half space of metamaterial [15] (see Figs. 6(d), 7(d)).

Numerical experiments showed that such behavior of eigen frequencies is also typical for another values of characteristic frequency κ_p and other magnitude of slot width.

Consider now the behavior of eigen frequencies with a change of characteristic frequency of metamaterial. When $\kappa_p \to 0$ the eigen frequencies of all types of oscillations tend to zero (see Fig. 7). This is the consequence of the fact that stripe grating in free space has no eigen frequencies (when $\kappa_p = 0$ effective permittivity of metamaterial $\varepsilon = 1$) [9, 13]. The increase of $\kappa_p \to \infty$ influence the behavior of eigen frequencies of the oscillations of different types of symmetry in different way. Thus, the real parts of eigen frequencies, corresponding to the vertical add modes (respective to plane $z = -h/2$ oscillations tend asymptotically to branching point $\kappa = 1$ (see Fig. 7(c)). The real parts of eigen frequencies of vertical even modes (relative to the plane $z = -h/2$ oscillations tend asymptotically to the value $\mathrm{Re}\kappa = \kappa_p/\sqrt{2}$ (see Fig. 7(a). The imaginary parts of oscillations of both type of symmetry vanish when $\kappa_p \to \infty$ (see Figs. 7(b), (d)). Such behavior of eigen frequencies also shows up for other geometrical parameters of stripe grating backed with the layer of metamaterial.

3.2. Diffraction Problem

Consider now the numerical results of the solution to diffraction problem in the case of H-polarization, beginning with the situation when $\kappa < \kappa_p$. As we have mentioned above in this frequency range inequality for effective permittivity $\mathrm{Re}\varepsilon < 0$ holds and real parts of eigen frequencies are also located in this domain of κ. Presume now that $\kappa_p/\sqrt{2} \leq 1$. It can be always arranged by the choice of the dimension of the period of grating ($\kappa_p = \omega_p l/2\pi c$. In this case, as follows from (3), the reflected ($z > 0$) and transmitted ($z < -h$) diffraction fields are the superposition of propagation 0-th order harmonic and infinite number of evanescent surface harmonics decaying exponentially when $z \to \pm\infty$. Thus at large distances from grating in zone of reflection ($z > 0$) the field if formed by the only propagating 0-th harmonic with amplitude R_0 (reflection coefficient), and T_0 (transmitting coefficient). In Fig. 8(a), we present the behavior of $|T_0|(\kappa)$ for normal incidence for various parameters of the problem: Solid line corresponds to the grating backed with layer of metamaterial with parameters $\kappa_p = 0.5$, $\nu = 10^{-5}$, $d/l = 0.5$, $h/l = 1/2\pi$. Dashed line corresponds to the layer of metamaterial with parameters $\kappa_p = 0.5$,

$\nu = 10^{-5}$, $d/l = 1.0$, $h/l = 1/2\pi$; dotted line — to the grating in vacuum with parameters $\kappa_p = 0$, $d/l = 0.5$.

In frequency range $0.3 \leq \kappa \leq 0.41$ non of the structures: Grating or layer of metamaterial does not manifest any resonant properties and are practically transparent for electromagnetic wave $|T_0| > 0.9$. In contrary the grating backed with a layer of metamaterial has pronounced resonant character in the same frequency range. There is a discrete set of parameter κ providing the total reflection of the incident field $|T_0| \approx 0$, and the total transparency $|T_0| \approx 1$. The comparison of diffraction and spectral characteristics (see Fig. 3(a) and Fig. 4(a)) results into conclusion that resonances of diffraction

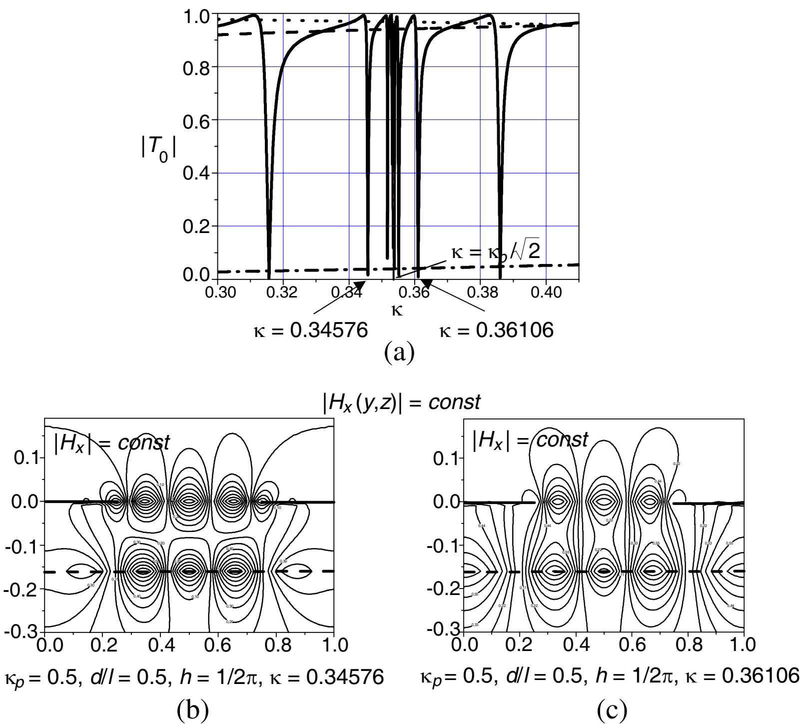


Figure 8. (a) Transmission coefficient $|T_0|$ for normal incidence as a function of κ. Solid line: Grating backed with metamaterial with parameters $\kappa_p = 0.5$, $\nu = 10^{-3}$, $d/l = 1.0$, $h/l = 1/2\pi$; dashed-doted line: The layer of metamaterial $\kappa_p = 0.5$, $\nu = 10^{-5}$, $d/l = 1.0$, $h/l = 1/2\pi$ without grating; dotted line: Grating in vacuum $\kappa_p = 0$, $d/l = 0.5$; (b), (c) field patters, calculated at resonant frequencies. (b) $\kappa = 0.34576$; (c) $\kappa = 0.36106$.

amplitudes occur at frequencies close to eigen ones, when oscillations close to natural ones are excited. The patterns of diffraction fields for couple of resonant values of κ are presented in Fig. 8(b) and Fig. 8(c). We can clearly see that at resonant frequencies for $|R_0|$ the oscillation close to the natural ones of the grating with the layer of metamaterial are excited, compare with Figs. 5(a), (c). As the incident field (normal excitation) is symmetric with respect to the plane$y = 0$, then only the symmetric oscillations are excited.

4. CONCLUSION AND PERSPECTIVES

The systematical extensive analysis of the problem of electromagnetic wave scattering by periodic stripe grating backed with a layer of metamaterial based on rigorous solution to the diffraction and spectral problems is carried out. The characteristic association of micro (metamaterial constructive elements, resulted in negative constitutive parameters) and macro periodicity (metal stripe grating with period of the scale of wavelength of incident wave) implemented in the structure provided it with new type of electromagnetic properties that have been thoroughly studied.

Analytical analysis of the problem and detailed numerical investigation allowed finding out following features and regularities:

- The structure Fig. 1 is an open oscillating system characterized by infinite number of eigen frequencies with finite accumulation points at $\omega = \pm\omega_p/\sqrt{2}$, here ω_p is characteristic frequency of metamaterial, defined by elements it is composed of. Real parts of these eigen frequencies are located within the interval $(-\omega_p, \omega_p)$, imaginary parts are negative (damped resonances, decaying with time).
- In frequency interval $(0, \omega_p)$, where $\mathrm{Re}\varepsilon(\omega) < 0$, there is a discrete set of values of normalized frequency κ, providing both of the resonant regimes: The total reflection of incident field $|T_0| \approx 0$ and total transparency $|T_0| \approx 1$ of the structure.

These phenomena are rather promising for the construction of electrically controlled compact super directive antennas and for synthesis of frequency selective coatings.

We plan to continue to study this type of structures, and as a following step we are going to consider anisotropic metamaterials and after to optimize the geometrical configuration of the stripe grating backed with anisotropic layer of metamaterial.

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REFERENCES

1. Pendry, J. B., "Negative refraction," *Contemporary Physics*, Vol. 45, No. 3, 191–203, 2004.
2. Bilotti, F., A. Alu, N. Engheta, and L. Vegni, "Anomalous properties of scattering from cavities partially loaded with double-negative or single-negative metamaterials," *Progress In Electromagnetics Research*, PIER 51, 49–63, 2005.
3. Lapine, M. and S. Tretyakov, "Contemporary notes on metamaterials," *IET Microwaves, Antennas and Propagation*, Vol. 1, No. 1, 3–11, 2007.
4. Lim, S., C. Caloz, and T. Itoh, "Electronically scanned composite right/left hfnded vicrostrip leaky-wave antenna," *IEEE Microwave and Wireless Components Letters*, Vol. 14, No. 6, 277–279, 2004.
5. Antoniades, M. A. and G. V. Eleftheriades, "Compact linear lead/lag metamaterial phase shifters for broadband applications," *IEEE Antennas and Wireless Propagation Letters*, Vol. 2, 103–106, 2003.
6. Engheta, N., "An idea for thin sub wavelength cavity resonators using metamaterials with negative permittivity and permeability," *IEEE Antennas and Wireless Propagation Letters*, Vol. 1, No. 1, 10–13, 2002.
7. Leskova, T. A., A. A. Maradudin, E. E. Garcia-Guerrero, and E. R. Mendez, "Structured surfaces as optical metamaterials," *Metamaterials*, Vol. 1, No. 1, 19–40, 2007.
8. Pendry, J. B., A. J. Holden, W. J. Stewart, and I. Youhgs, "Extremely low frequency plasmons in metallic structures," *Phys. Rev. Letters*, Vol. 76, 4773–4776, 1996.
9. Shestopalov, V. P. and Y. K. Sirenko, *Dynamic Theory of Gratings*, 214, Kiyev, Naukova Dumka, 1989.
10. Brovenko, A., P. Melezhik, and A. Poyedinchuk, "The method of regularization for one class of dual series equations," *Ukrainskiy Matematichesskiy Zhurnal*, Vol. 53, No. 10, 1320–1327, 2001 (in Russian).
11. Brovenko, A., P. Melezhik, and A. Poyedinchuk, "Diffraction of plane electromagnetic wave by metal grating with magnetoactive

plazma," *Izvestiya VUZov, Radiofizika*, Vol. 47, No. 8, 638–649, 2004 (in Russian).

12. Shestopalov, V. P., *Method of Riemann-Hilbert Problem in Theory of Electromagnetic Wave Diffraction and Propagation*, 400, Kharkov, Publishing House of Kharkov University, 1971 (in Russian).
13. Shestopalov, V. P., Y. A. Tuchkin, A. Y. Poyedinchuk, and Y. K. Sirenko, "Novel methods of solving of direct and inverse problems of diffraction theory," *Analytical Regularization of Boundary Value Problems in Electromagnetics*, 285, Kharkov, Osnova, 1997 (in Russian).
14. Gohberg, I. C. and Y. I. Sigal, "Operator generalization of the theorem about logarithm residue and Rouché's theorem," *Matematicheskiy Sbornik*, Vol. 84, No. 4, 607–629, 1971 (in Russian).
15. Brovenko, A., P. N. Melezhik, A. Y. Poyedinchuk, and N. P. Yashina, "Surface resonances of metal stripe grating on the plane boundary of metamaterial," *Progress In Electromagnetics Research*, PIER 63, 209–222, 2006.
16. Sirenko, Y. K., S. Strom, and N. P. Yashina, "Modeling and analysis of transient progresses in open resonant structures," *New Methods and Techniques*, 353, Springer, 2007.