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PEMC PARABOLOIDAL REFLECTOR IN CHIRAL MEDIUM SUPPORTING POSITIVE PHASE VELOCITY AND NEGATIVE PHASE VELOCITY SIMULTANEOUSLY

T. Rahim and M. J. Mughal

Faculty of Electronic Engineering
GIK Institute of Engineering Sciences and Technology
Topi, Swabi 23640, N.W.F.P., Pakistan

Q. A. Naqvi

Electronics Department
Quaid-i-Azam University
Islamabad 45320, Pakistan

Abstract—Focal region fields of a perfect electromagnetic conductor (PEMC) paraboloidal reflector placed in homogenous and reciprocal chiral medium are analyzed under the effect of high chirality parameter. The chiral medium supports both negative phase velocity (NPV) and positive phase velocity (PPV) modes simultaneously under strong electromagnetic coupling. The geometrical optics (GO) fields have singularities at the focal region. Therefore, to avoid these singularities Maslov's method is used for the focal region fields. The results obtained using this method are solved numerically, and line plots for the reflected field of the paraboloidal PEMC reflector are obtained for different values of admittance of the PEMC paraboloidal reflector and chirality parameter of the medium.

1. INTRODUCTION

When the phase velocity vector is directed opposite to the direction of poynting vector the wave is said to be traveling with negative phase velocity (NPV). This idea initially introduced by Veselago [1] was confirmed later on by many experiments [2]. Initially, NPV was realized by imposing conditions on the permittivity ε and permeability

Corresponding author: T. Rahim (rahim372@gmail.com.).

μ of the mediums [5, 6]. Recently, conditions for the chiral medium to support NPV propagation have been derived [7]. The chirality parameter of the chiral medium is increased such that the chiral medium supports positive phase velocity (PPV) and negative phase velocity (NPV) simultaneously. The paraboloidal reflector in this case is made of PEMC. The PEMC is a non-reciprocal generalization of both perfect electric conductor (PEC) and perfect magnetic conductor (PMC). It was introduced by Lindell and Sihvola [3, 4]. This medium is defined by a scalar parameter M, which is admittance of the surface interface.

Geometrical optics (GO) approximation or asymptotic ray theory (ART) is well known in electromagnetics. However, GO shows singularities at the focal region. Therefore, Maslov's method is used to find the field expressions around the focal region [9–19]. Maslov's method combines the simplicity of ray theory and the generality of Fourier transform to avoid the singularity at the caustics. In the present work, our interest is to apply Maslov's method to finding the fields around the focal region of a PEMC paraboloidal reflector placed in isotropic and homogeneous chiral medium. This work is an extension of the previous work, in which field around the focal region of PEC paraboloidal reflector supporting PPV and NPV simultaneously were studied [10], to the case in which the paraboloidal reflector is made of PEMC. We suggest that this paper should be read along with [9–11].

In Section 2, an introduction to chiral medium supporting both negative phase velocity is given. In Section 3, reflection of plane waves from PEMC plane boundary is studied. In Section 4, we have discussed the geometrical optics field of PEMC paraboloidal reflector placed in chiral medium and plots around focal point are discussed. Concluding remarks are presented in Section 5.

2. CHIRAL MEDIUM AND NEGATIVE PHASE VELOCITY

Chiral medium is a microscopically continuous medium composed of chiral objects, uniformly distributed and randomly oriented. A chiral object is, by definition, an object that lacks bilateral symmetry. It cannot be superimposed on its mirror image either by translation or by rotation. This is also known as handedness. Chiral medium supports left circularly polarized (LCP) and right circularly polarized (RCP) modes. Many constitutive relations can be used to define a chiral medium. In this paper, we will use Drude-Born-Fadorov (DBF) constitutive relations as follows [20]

$$\mathbf{D} = \epsilon_0(\mathbf{E} + \beta\nabla \times \mathbf{E}), \quad \mathbf{B} = \mu_0(\mathbf{H} + \beta\nabla \times \mathbf{H})$$

where, ϵ, μ and β are permittivity, permeability and chirality parameters respectively. ϵ, μ have usual dimensions, and β has the dimension of length. Using above constitutive relations, Maxwell's equations result in coupled differential equations. Uncoupled differential equations for $\mathbf{E}$ and $\mathbf{H}$ are obtained by using the following transformation [20]

$$\mathbf{E} = \mathbf{Q}_L - j\sqrt{\frac{\mu}{\epsilon}}\mathbf{Q}_R, \quad \mathbf{H} = \mathbf{Q}_R - j\sqrt{\frac{\epsilon}{\mu}}\mathbf{Q}_L$$

In the above equations $\mathbf{Q}_L$, $\mathbf{Q}_R$ are LCP and RCP waves respectively and satisfy the following equations

$$\left(\nabla^2 + n_1^2k^2\right)\mathbf{Q}_L = 0, \quad \left(\nabla^2 + n_2^2k^2\right)\mathbf{Q}_R = 0$$

where, $n_1 = 1/(1 - k\beta)$ and $n_2 = 1/(1 + k\beta)$ are equivalent refractive indices of the medium seen by LCP and RCP waves respectively, and $k = \omega\sqrt{\epsilon\mu}$. For $|k\beta| < 1$, both refractive indices remain positive and we have PPV propagation for both modes [11]. For $k\beta > 1$, $n_1 < 0$ and $n_2 > 0$, LCP wave travels with NPV and RCP wave with PPV. For $k\beta < -1$, RCP wave travels with NPV and LCP wave with PPV. It may be noted that for $|k\beta| < 1$ and $\epsilon < 0$, both RCP and LCP waves travel with NPV, simultaneously. Recently, it has been introduced that under certain circumstances NPV can arise even when the reflection is positive [21]. We have considered the situation when one wave is negatively reflected and also travels with NPV while the other wave travel with PPV. We have examined the case of $k\beta > 1$ only, because for $k\beta < -1$; we can get the solutions from $k\beta > 1$ by interchanging the role of LCP and RCP waves.

3. PLANE WAVE REFLECTION FROM PEMC PLANE

Perfect electromagnetic conductor (PEMC) is a non-reciprocal generalization of both perfect electric conductor (PEC) and perfect magnetic conductor (PMC). PEMC boundary can be realized in terms of a layer of certain non reciprocal materials resting on a PEC plane [3, 4]. Parameters of a bi-isotropic medium can be chosen so that the interface of the layer acts as a PEMC boundary. The boundary conditions for PEC and PMC are given as

$$\mathbf{n}\times\mathbf{E} = 0, \quad \mathbf{n}\cdot\mathbf{B} = 0 \quad (\text{PEC}) \qquad \mathbf{n}\times\mathbf{H} = 0, \quad \mathbf{n}\cdot\mathbf{D} = 0 \quad (\text{PMC})$$

Boundary conditions for PEMC are more general and are given as

$$\mathbf{n}\times(\mathbf{H} + M\mathbf{E}) = 0, \qquad \mathbf{n}\cdot(\mathbf{D} - M\mathbf{B}) = 0$$

where, **n** is the unit normal to the surface, and M is the admittance of the PEMC boundary. PEMC reduces to PMC when $M = 0$, and PEC when $M \longrightarrow \pm\infty$.

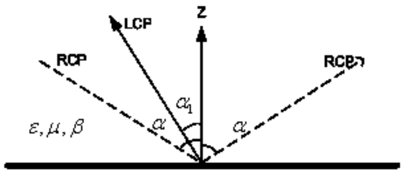

Figure 1. Reflection of RCP wave from PEMC plane.

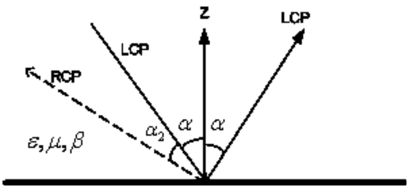

Figure 2. Reflection of LCP wave from PEMC plane.

To study the reflection of RCP wave from PEMC we refer to Figure 1. A RCP wave with unit amplitude, phase velocity ω/kn_2 and making angle α with normal (z-axis), strikes a PEMC plane. Two waves are reflected, a RCP wave with amplitude $(\cos\alpha - \cos\alpha_1)/(\cos\alpha + \cos\alpha_1)$, traveling with phase velocity ω/kn_2 and making an angle α with z-axis and a LCP wave with amplitude

$$\left(\frac{M\eta - j}{M\eta + j}\right)\left(\frac{2\cos\alpha}{\cos\alpha + \cos\alpha_1}\right)$$

and makes an angle $\alpha_1 = \sin^{-1}\{(n_2/n_1)\sin\alpha\}$ with the z-axis. If $k\beta > 1$, α_1 and n_1 becomes negative. Therefore, the LCP wave will be reflected in the opposite direction as shown in Figure 1, it will have a negative phase velocity given by ω/kn_1. Similarly in Figure 2, a LCP wave with unit amplitude, and making an angle α with the normal (z-axis), is incident on a PEMC plane. In this case we also get two reflected waves, a LCP wave with amplitude $(\cos\alpha - \cos\alpha_2)/(\cos\alpha + \cos\alpha_2)$ traveling with phase velocity ω/kn_1 and making an angle α with z-axis and a RCP wave with amplitude

$$\left\{-\left(\frac{1 - jM\eta}{1 + jM\eta}\right)\left(\frac{2\cos\alpha}{\cos\alpha + \cos\alpha_2}\right)\right\}$$

traveling with phase velocity ω/kn_2 and making an angle $\alpha_2 = \sin^{-1}\{(n_1/n_2)\sin\alpha\}$ with z-axis. In this case negative reflection take place for RCP wave.

4. FOCAL REGION FIELD OF PEMC PARABOLOIDAL REFLECTOR USING GO AND MASLOV'S METHOD

Consider the reflection of a plane wave traveling along positive z-axis from a PEMC paraboloidal reflector with axis parallel to the z-axis as shown in Figure 3. The equation for the paraboloidal surface is given by

$$\zeta = g(\xi, \eta) = f - \frac{\rho^2}{4f} = f - \frac{\xi^2 + \eta^2}{4f} \tag{1}$$

where, (ξ, η, ζ) are the initial values of the Cartesian coordinates (x, y, z), f is the focal length of the paraboloidal reflector and $\rho^2 = \xi^2 + \eta^2$. The reflector is placed in homogenous and reciprocal chiral medium defined by constitutive relations as given in Section 2. Let there be two incident plane waves of opposite handedness traveling in chiral medium along positive z-axis, which satisfy the general wave equation and are given as

$$\mathbf{Q}_L = (\mathbf{a}_x + j\mathbf{a}_y) \exp(-jkn_1 z) \tag{2}$$

$$\mathbf{Q}_R = (\mathbf{a}_x - j\mathbf{a}_y) \exp(-jkn_2 z) \tag{3}$$

where, $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vector along x-axis and y-axis respectively. By ignoring the polarization and taking the incident field of unit amplitude we get

$$Q_L = \exp(-jkn_1 z) \tag{4}$$

$$Q_R = \exp(-jkn_2 z) \tag{5}$$

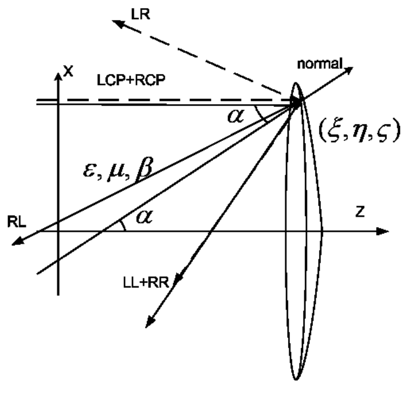

Figure 3. PEMC paraboloidal reflector placed in chiral medium.

These waves makes an angle α with the normal to the surface of a paraboloidal reflector. The unit normal vector to the surface can be written as

$$\mathbf{a}_n = \sin\alpha\cos\gamma\mathbf{a}_x + \sin\alpha\sin\gamma\mathbf{a}_y + \cos\alpha\mathbf{a}_z \tag{6}$$

where, $\mathbf{a}_z$ is the unit vector along z-axis and α, γ are given as

$$\sin\alpha = \frac{\rho}{\sqrt{\rho^2+4f^2}}, \quad \cos\alpha = \frac{2f}{\sqrt{\rho^2+4f^2}}, \quad \tan\gamma = \frac{\eta}{\xi}$$

Four waves are reflected when both LCP and RCP waves hit PEMC paraboloidal reflector. These waves are represented by RR, RL, LL and LR. RR and RL are RCP and LCP reflected waves respectively, when RCP is incident. LL and LR are LCP and RCP reflected waves respectively, when LCP wave is incident. The expressions for the fields around the focal region has been calculated for PEC paraboloidal reflector in [11]. For the case of PEMC case LL and RR has the same initial amplitudes, while RL wave has different initial amplitude as compered with the corresponding case of PEC case. The angle of reflection for all the reflected waves are same as in case of PEC. The field expressions for the LL and RR rays are the same. The field expressions around the focal points for the PEMC paraboloidal reflector are presented as derived in [10].

$$\begin{aligned} u_{LL}(r) &= \frac{j2kn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left(\frac{\cos\alpha-\cos\alpha_2}{\cos\alpha+\cos\alpha_2}\right)\tan\alpha \\ &\quad \times\exp\{-jkn_1 s_{LL}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{7}$$

$$\begin{aligned} u_{RR}(r) &= \frac{j2kn_2 f}{\pi}\int_0^H\int_0^{2\pi}\left(\frac{\cos\alpha-\cos\alpha_1}{\cos\alpha+\cos\alpha_1}\right)\tan\alpha \\ &\quad \times\exp\{-jkn_1 s_{RR}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{8}$$

$$\begin{aligned} u_{RL}(r) &= \frac{jkn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left\{\left(\frac{M\eta-j}{M\eta+j}\right)\frac{2\cos\alpha}{\cos\alpha+\cos\alpha_1}\right\} \\ &\quad \times\sec^{3/2}\alpha\sqrt{X_1}\{\sin\alpha S_1(\tan\alpha S_1+C_1)\}^{1/2} \\ &\quad \times\exp\{-jkn_1 s_{RL}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{9}$$

where, $S_1 = \sin(\alpha+\alpha_1)$ and $C_1 = \cos(\alpha+\alpha_1)$. The phase functions in the above equations are given by

$$s_{LL}(p_x,p_y) = n_1\left(2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right) \tag{10}$$

$$s_{RR}(p_x,p_y) = n_2\left(2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right) \tag{11}$$

$$s_{RL}(p_x, p_y) = n_1 \left\{ \frac{n_2}{n_1} f \frac{\cos 2\alpha}{\cos^2 \alpha} - (x\cos\gamma + y\sin\gamma - 2f\tan\alpha) S_1 \right.$$
$$\left. - \left(z - f\frac{\cos 2\alpha}{\cos^2\alpha} \right) C_1 \right\} \tag{12}$$

The term X_1 used in (13) is given by

$$X_1 = \frac{\sqrt{n_1^2 - n_2^2 \sin^2\alpha} + n_2\cos\alpha}{\sqrt{n_1^2 - n_2^2\sin^2\alpha}} \tag{13}$$

The limit of integration is calculated by the formula

$$H = \tan^{-1}(D/2f)$$

where, D is the height of the paraboloidal reflector from the horizontal axis.

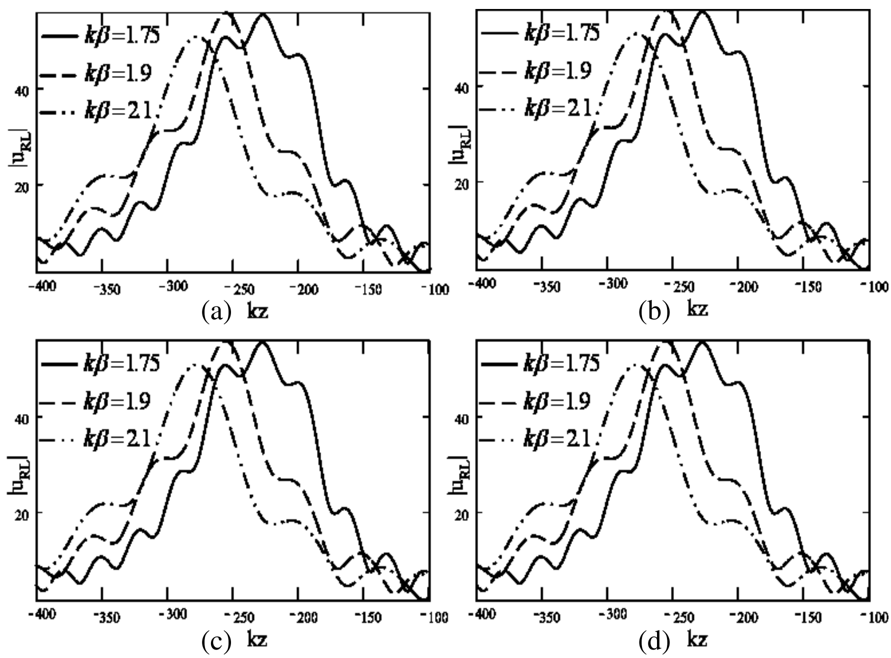

Figure 4. Plots for $|u_{LR}|$ along x-axis, of PEMC paraboloidal reflector for $k\beta = 0$, 0.01, 0.05, 0.1 and (a) $M\eta = 0, \infty$, (b) $M\eta = 0.1$, (c) $M\eta = 1$, (d) $M\eta = 10$.

5. SIMULATION RESULTS

To analyze the reflected field behavior around the PEMC paraboloidal reflector supporting NPV and PPV simultaneously, we have solved Eqs. (7)–(9) numerically, and the results are obtained. The field pattern for LL and RR waves are similar to the case of PEC and are not repeated here [10]. LR wave was not converging to the axis of PEMC paraboloidal surface and does not form a real focus as shown in Figure 3. The RL wave forms real focus and significantly shifted towards left. The plots of $|u_{RL}|$ are given in Figure 4, for $k\beta = 1.75, 1.9, 2.1$ and $M\eta = 0, \infty, 0.1, 1, 10$. The amplitude of the plots remains the same, but the phase term changes with M. This figure shows that the trend of these plots is similar to the case of PEC paraboloidal reflector [10].

6. CONCLUSION

The high frequency field around the focal region of PEMC paraboloidal reflector is analyzed which is placed in reciprocal and homogenous chiral medium having high value of chirality parameter ($k\beta$), using geometrical optics and Maslov's method. It has been seen that fields of LL and RR waves are exactly the same as that of PEC paraboloidal reflector. The field behaviors of LR and RL waves are similar, but not exactly same, to that of PEC paraboloidal reflector case. LR wave diverges and does not form a focus, while focal point of RL wave moves to the left with increase in chirality parameter $k\beta$.

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