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SYNTHESIS OF SPARSE CYLINDRICAL ARRAYS USING SIMULATED ANNEALING ALGORITHM

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Abstract—In this paper, simulated annealing algorithm (SA) is applied to the synthesis of cylindrical conformal arrays in order to suppress the peaks of side lobes by acting on the elements' positions. There are multiple optimization constraints including the number of elements, aperture and minimum element spacing. A constraint matrix is designed to make the solution meet the restriction on the minima distance between elements, and the individual matrix which forms on the basis of constraint matrix is used to express array configurations. The SA does not act on the elements' positions directly but on the variables in a smaller solution space, this indirect description method makes the SA more computationally efficient. The simulation results confirm the great efficiency and robustness of the proposed method.

1. INTRODUCTION

In recent years, the sparse array antenna has received more and more attention because of its advantages, such as narrow scanning beams, weak cross coupling between elements, low cost of antenna system, etc., but it also has shortcomings. For example, it has higher peak side lobe level [1, 3–5]. So how to suppress the peak side lobe level is an important research problem in sparse array design. Sparse arrays can be classified into two types: thinning arrays and sparse arrays. Thinning arrays is formed by removing some elements from uniform arrays, and the element spacing is integral times of original uniform arrays. The elements of sparse arrays are randomly distributed on the array surface, and the element spacing is indivisible [1, 4]. By contrast, sparse arrays have more freedom than thinning arrays. Sparse

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arrays are facing nonlinear and complicate problems, and traditional numerical computation methods can not get the desirable solution. So more and more artificial intelligence methods are introduced in the field, such as dynamic programming algorithm [9], simulated annealing algorithm [2, 6–8, 10], particle swarm algorithm [5], genetic algorithm [1, 3, 4, 11], etc. Compared with linear and planar arrays, conformal cylindrical arrays have curved structure and are conformal to the carrier, so the synthesis problem of this type of antenna array is multidimensional and nonlinear problem, which is more difficult than the synthesis of linear and planar array [5]. The conformal cylindrical arrays which include the restriction on the number of elements, array aperture and minimum element spacing, which will be discussed in this paper. The peak side lobe level of cylindrical conformal arrays is suppressed by acting on the elements' position with the SA.

2. OPTIMIZATION MODEL

We assume an array of N elements located over a cylindrical surface of radius R, as shown in Fig. 1. The angular region of the cylindrical array extends from $-\theta$ to θ; the height region extends from $-H$ to H; and there are always elements on the four vertices to keep the aperture constant. The optimization task is to search the optimal solution of position vector $\vec{D} = [d_1, d_2, \ldots, d_N]^r$ to suppress the peak side lobe level of cylindrical arrays, subject to the constraint of $|d_k - d_l| \geq d_c > 0$, where d_k is the location coordinates of the Kth element, and d_c is the design constraint of minimum element spacing. The optimization

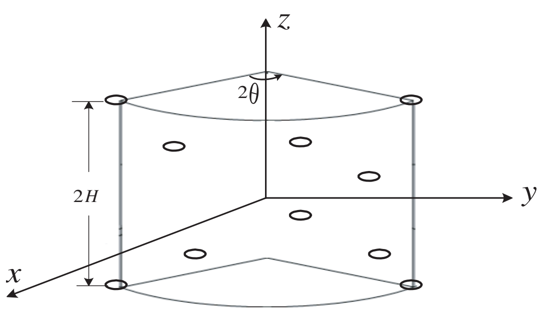

Figure 1. Diagram of a cylindrical array.

model can be described as follow:

$$\begin{cases} M \text{ in } \{\text{SLL}\} = f(D) \\ \text{s.t. } -\theta \leq \mathrm{Re}(d_i) \leq \theta \\ -H \leq \mathrm{Im}(d_i) \leq H \\ |d_k - d_l| \geq d_c > 0 \\ 1 \leq k, \quad l \leq N \end{cases} \tag{1}$$

In order to facilitate the operation of variables, let a matrix $\mathbf{F}$ with Q rows and P column expresse the position vector $\vec{D} = [d_1, d_2, \ldots, d_N]^r$. Because the elements are located over the surface of cylindrical array, the location coordinates can be determined by fixing the height and azimuth of elements, namely d_i can be expressed by $\theta_i + jh_i$. The real and image parts denote the azimuth angle and the height respectively. If $N = P \times Q$, individual matrix $\mathbf{F}$ is a full matrix. On the other hand, $N < P \times Q$, F is a sparse matrix, and there are $Q \times P - N$ elements to be thinned.

Individual matrix $\mathbf{F}$ can be expressed as follow:

$$\mathbf{F} = \begin{bmatrix} -\theta + jH & \theta_{12} + jh_{12} & \cdots & \theta_{1P} + jH \\ \theta_{21} + jh_{21} & \theta_{22} + jh_{22} & \cdots & \theta_{2P} + jh_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ -\theta - jH & \theta_{Q2} + jh_{Q2} & \cdots & \theta - jH \end{bmatrix} \tag{2}$$

The minimum element spacing is d_c both in ϕ- and Z-directions. It is easy to prove that the solution would meet the constraint of the minimum element spacing. According to $l = \theta \times R$, the restriction on radian in ϕ-direction can be changed into restriction on azimuth angle, namely $|\theta_i - \theta_j| \geq d_0 > 0$. Taking the real and image parts of individual matrix $\mathbf{F}$ apart, make them meet the constraint of the minimum element spacing respectively. Let the vector $\vec{\theta}$ denote the elements of the ith row of the individual matrix, assuming that $\theta_{ij} = x_j + (j-1)d_0$, then:

$$\vec{\theta}^r = \begin{bmatrix} x_1 \\ x_2 + d_0 \\ x_3 + 2d_0 \\ \vdots \\ x_P + (P-1)d_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_P \end{bmatrix} + \begin{bmatrix} 0^\circ \\ d_0 \\ 2d_0 \\ \vdots \\ (P-1)d_0 \end{bmatrix} = \vec{\mathbf{x}} + \begin{bmatrix} 0^\circ \\ d_0 \\ 2d_0 \\ \vdots \\ (P-1)d_0 \end{bmatrix} \tag{3}$$

If $x_1 \leq x_2 \leq \cdots \leq x_P$, the transformation can meet the constraint of minimum element spacing in ϕ-direction. The constraint matrix about azimuth angle can be expressed as follow:

$$C_{\mathrm{Re}} = \begin{bmatrix} 0 & d_0 & \cdots & (P-1)d_0 \\ 0 & d_0 & \cdots & (P-1)d_0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & d_0 & \cdots & (P-1)d_0 \end{bmatrix} \tag{4}$$

According to $\theta_P = x_P + (P-1)d_0 \leq \theta$, we can obtain $x_P \leq \theta - (P-1) \cdot d_0$, so the solution space is downsized from original $[-\theta, \theta]$ to $[-\theta, \theta - (P-1) \cdot d_0]$. This indirect manner of individual description allows the SA to search a smaller space.

We can obtain the constraint matrix about the image part of individual matrix in a similar way:

$$C_{\mathrm{Im}} = \begin{bmatrix} (Q-1)d_c & (Q-1)d_c & \cdots & (Q-1)d_c \\ \vdots & \vdots & \vdots & \vdots \\ d_c & d_c & d_c & d_c \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

The solution space is downsized from the original $[-H, H]$ to $[-H, H-(Q-1) \cdot d_c]$, and it is highly advantageous for SA to find an excellent solution. The constraint matrix about the individual matrix can be expressed as follow:

$$\begin{aligned} C &= C_{re} + C_{im} \\ &= \begin{bmatrix} 0+j(Q-1)d_c & d_0+j(Q-1)d_c & \cdots & (P-1)d_0+j(Q-1)d_c \\ 0+j(Q-2)d_c & d_0+j(Q-2)d_c & \cdots & (P-1)d_0+j(Q-1)d_c \\ \vdots & \vdots & \vdots & \vdots \\ 0 & d_0 & \cdots & (P-1)d_0 \end{bmatrix} \end{aligned} \tag{6}$$

An individual matrix can be obtained by: we get a $Q \times P$ stochastic matrix whose elements are among the range of $[-\theta, \theta-(P-1) \cdot d_0]$ and reset the elements' order of each row vector. Let the right number be the biggest and gradually decrease toward the left, and then the stochastic matrix is expressed as matrix notation R. Again get a $Q \times P$ stochastic matrix whose elements are among the range of $[-H, H-(Q-1) \cdot d_c]$. Let the bottom number be the biggest and gradually decrease toward the top, and the stochastic matrix can be expressed as matrix notation $\mathbf{I}$, then a template of individual matrix can be obtained as follows:

$$\mathbf{F} = R + jI + C \tag{7}$$

We need to make some definitions during the SA process:

Definition 1: For a sparse matrix $\mathbf{F}$, if the element of the pth row qth column is thinned, the element of corresponding index matrix $\mathbf{S}$ is settled as "0", otherwise, "1".

$$\mathbf{F} = \begin{bmatrix} -\theta + jH & 0 & \cdots & \theta + jH \\ \theta_{21} + jh_{21} & \theta_{22} + jh_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\theta - jH & \theta_{Q2} + jh_{Q2} & \cdots & \theta - jH \end{bmatrix} \tag{8}$$

The index matrix $\mathbf{S}$ should be:

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & \cdots & 1 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{9}$$

Definition 2: In the optimization process, if the individual matrix is a full matrix, the constraint matrix equals C; if the individual matrix is a sparse matrix, the corresponding elements of C are settled as "0" according to the index matrix.

We choose isotropic elements in this paper, namely $f_{mn}(\phi, \theta) = 1$, $A_{mn} = 1$, and the main lobe direction is chosen as (ϕ_0, θ_0). The array factor for the cylindrical array is given by [5, 8, 13]:

$$\begin{aligned} F(\phi, \theta; \phi_0, \theta_o) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} & f_{mn}(\phi, \theta) A_{mn} \exp\{jk[R(\sin\theta_0 \cos(\phi_0 - \phi_{mn}) \\ & - \sin\theta \cos(\phi - \phi_{mn})) + h(\cos\theta_0 - \cos\theta)]\} \end{aligned} \tag{10}$$

where k is the wave number, and ϕ_{mn} is the azimuth angle.

Assuming that $u = \sin\theta\cos\phi$, $v = \sin\theta\sin\phi$, $u_0 = \sin\theta_0\cos\phi_0$, $v_0 = \sin\theta_0\sin\phi_0$, the array factor can be transformed as follows:

$$\begin{aligned} F(\phi, \theta; \phi_0, \theta_o) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} & \exp\{jk[R((u_0 - u)\cos\phi_{mn} + (v_0 - v)\sin\phi_{mn}) \\ & + h\left(\sqrt{1 - u_0^2 - v_0^2} - \sqrt{1 - u^2 - v^2}\right)\Big]\Big\} \end{aligned} \tag{11}$$

We define the fitness function as the maximum SLL in the entire ϕ plane

$$\text{fitness } (D) = \max\left\{\left|\frac{F(\phi, \theta, \phi_0, \theta_0)}{FF_{\max}}\right|\right\} \tag{12}$$

where $FF_{\max}$ is the peak of main beam, the region of ϕ and θ is valid excluding the main beam.

The objective function is defined as:

$$f(D) = \min\{\text{fitness } (D)\} \tag{13}$$

3. SIMULATED ANNEALING ALGORITHM

The simulated annealing is a probabilistic method of optimization. Initially, it aims to simulate the behavior of the molecules of a pure substance during the slow cooling that results in the formation of a perfect crystal [10]. This technique aims to solve the problems similar to physical annealing process. It only needs the fitness function's information and adapts to poly-dimensional problems which are discontinuous and have many locals.

3.1. The Advantages of the SA

1) Be able to solve the approximate solution of energy function which is nonlinear and complicated;
2) Be able to overcome the dependence on initial value;
3) Be able to escape from local minima and find global minima;

3.2. The Steps of the SA

1) Set the initial parameters and construct the constraint matrix;
2) Construct an initial individual matrix **F** according to (7), and its fitness function $f(F_{\text{old}})$ is evaluated.
3) Perform perturbation on the real part R and the image part I of the individual matrix respectively, the iteration is given as:

$$\begin{aligned} R_{\text{new}} &= R_{\text{old}} + 0.01P_{\text{rand}} \\ I_{\text{new}} &= I_{\text{old}} + 0.01P_{\text{rand}} \end{aligned} \tag{14}$$

   where P_{rand} is a random matrix subjected to the normal distribution. In this process, if its element is out of the feasible region, a random number in the feasible region substitutes for it. A new individual matrix F_{new} can be obtained according to (7). A new fitness function value $f(F_{\text{new}})$ is then evaluated.
4) Judge the new individual matrix to be accepted or not according to the Metropolise sampling principle: if $f(F_{\text{new}}) \leq f(F_{\text{old}})$ the new individual matrix is accepted and F_{old} is replaced by F_{new}.

If $f(F_{\text{new}}) > f(F_{\text{old}})$, the acceptance or rejection about the new individual matrix depends on a probability, and "prob" defined as

$$\text{prob} = e^{\dfrac{f(F_{\text{new}}) - f(F_{\text{old}})}{T}} \tag{15}$$

where T is called "temperature" in the SA. A random number r in the range $[0, 1]$ is generated. If $r <$ prob, the new individual matrix is accepted. Otherwise it is rejected. This procedure is for the purpose of reducing the chance of getting stuck in local solutions [2, 12].

5) To avoid losing the current best solution due to accepting new solution by probability, set a variable to preserve the current best solution.
6) If the acceptance numbers or the rejection numbers exceed the given number, the temperature is lowered. The temperature is chosen to decay exponentially with time: $T_{i+1} = aT_i$ for some $a < 1$ in this paper.
7) When T is less than a appointed number T_{ter}, the algorithm is end.

4. SIMULATION RESULTS

Supposing that the radius of cylindrical array is 5λ, and the height is 2λ, where λ is the working wavelength, $\theta = 30$, the radiation pattern in the u and v-region $(0 \leq u,\ v \leq 1)$ is sampled 50×50 points. Two simulation results are listed as follows.

Table 1. Individual matrix of best array under the condition of $P \times Q = N$.

	1	2	3	4	5	6	7	8	9
1	–30.00	–22.41	–16.40	–10.13	–3.87	2.16	7.94	14.60	30.00
	1.00	0.28	0.350	0.29	0.47	0.47	0.26	0.31	1.00
2	–28.17	–21.51	–15.46	–9.38	–3.62	2.63	8.45	14.22	20.99
	–0.40	–0.43	–0.26	–0.21	–0.05	–0.12	–0.29	–0.23	–0.34
3	–30.00	–21.77	–15.82	–10.06	–3.76	3.04	8.90	15.12	30.00
	–1.00	–0.97	–0.82	–0.77	–0.63	–0.88	–0.89	–0.74	–1.00

1) Let $N = 27$, $P = 9$, $Q = 3$, namely the individual matrix is a full matrix. The minimum elements space is $d_c = 0.5\lambda$, and the basic

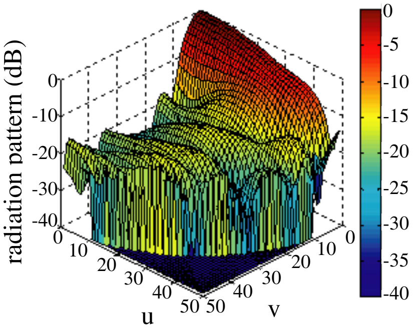

Figure 2. Radiation pattern of best array.

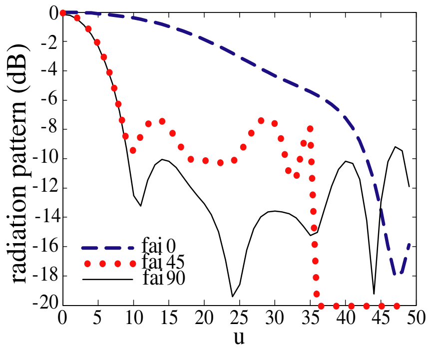

Figure 3. Radiation pattern in 3 tangent planes.

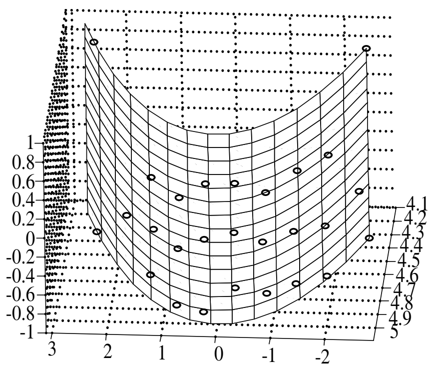

Figure 4. Result array's configuration.

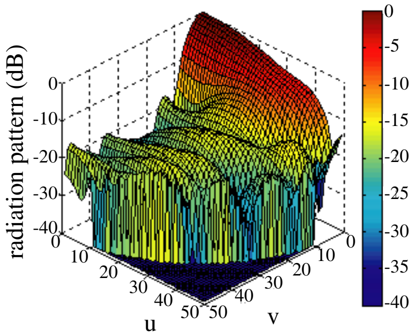

Figure 5. Radiation pattern of best array.

parameters of the SA are chosen as follow: $T = 200, a = 0.95, T_{\text{ter}} = 0.5$. Table 1 shows the individual matrix of best array; Fig. 2 shows the far-field radiation pattern of the best array; and its peak side lobe level is $-13.124\,\text{dB}$ in the entire ϕ plane. Fig. 3 shows the far-field radiation pattern in $\phi = 0$, $\phi = 45$ and $\phi = 90$ planes. The element configuration of the best array is shown in Fig. 4.

2) Let $N = 25, P = 9, Q = 3$. Unlike Simulation 1), two elements of individual matrix are randomly selected to be thinned. The minimum element spacing and the basic parameters of the SA are chosen the same as 1). The optimal solution of this simulation is presented in Table 2 and its peak side lobe in the entire ϕ plane is $-14.147\,\text{dB}$. Figs. 5 and 6 show the radiation pattern of the result array

Table 2. Individual matrix of best array under the condition of $P \times Q < N$.

	1	2	3	4	5	6	7	8	9
1	−30.00 1.00	0	−15.7 0.29	−9.7 0.29	−3.97 0.29	2.84 0.30	0	16.43 0.24	30.00 1.00
2	−29.06 −0.36	−21.59 −0.47	−15.71 −0.35	−9.94 −0.27	−4.12 −0.24	1.67 −0.35	8.39 −0.36	14.70 −0.41	21.59 −0.24
3	−30.00 −1.00	−23.27 −0.98	−16.42 −0.92	−10.67 −0.90	−4.67 −0.77	1.38 −0.91	8.35 −0.86	14.62 −0.94	30.00 −1.00

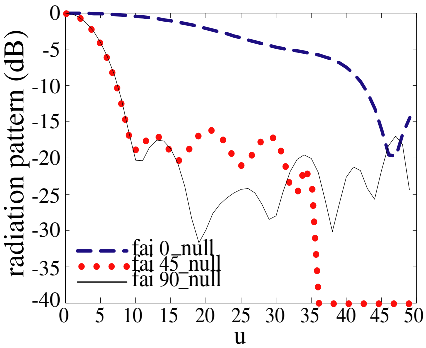

Figure 6. Radiation pattern in 3 tangent planes.

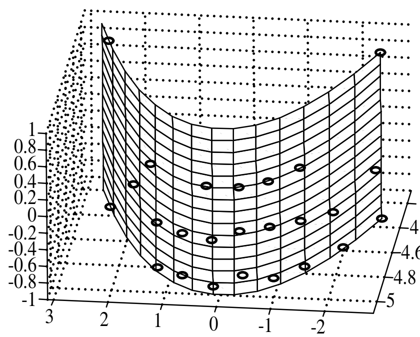

Figure 7. Result array's configuration.

and the radiation pattern in $\phi = 0$, $\phi = 45$, $\phi = 90$ planes respectively. The element configuration of the result array is shown in Fig. 7.

5. CONCLUSION

The synthesis of sparse cylindrical arrays with multiple constraints exhibits some challenges related to the control of the side lobes. In this paper, we let a matrix express a array configuration, and the minimum element spacing is transformed into constraint matrix. This transformation makes the simulation easier and the solution space smaller, so this method makes the SA more computationally efficient. Future work will br mainly aimed at the model of cylindrical conformal arrays. More complicated models including directivity of elements and cross coupling between elements should be considered.

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