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SCATTERING CROSS SECTION OF A META-SPHERE

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Abstract—The scattering cross section of a meta-sphere is determined and comparison is made to a normal right-handed spherical dielectric scatterer. A meta-sphere is a term used for a sphere embedded inside a medium that gives effective doubly-negative permittivity and permeability. It is found that the scattering resonances can be manipulated via the meta-sphere parameters while the issue of reducing the scattering cross section to zero is examined.

1. INTRODUCTION

There is much interest in the analysis of metamaterials [1–3] in view of the fact that composite media of this kind can be utilized in a number of applications such as the design of small antennas. In the case of the latter, there has been interest in determining the resonant conditions for sub-wavelength patch antennas or overcoming the antenna quality restrictions, the so called Q-limit. Many proposals [4, 5] for electrically small antennas require composite media with resonant inclusions embedded in them that can be made to exhibit superior performance characteristics when compared to some conventional antenna designs. It is shown that an electrically small antenna may be designed via a spherical shell that has negative permittivity and permeability [6–9]. For this reason it is of interest to investigate the dependence of the scattering cross section on the parameters which define the composite configuration, ie, the meta-sphere. The scattering cross section and associated resonances are also important in the design of materials with radar absorbing properties. These and other numerous applications rely on the peculiar behaviour of metamaterial structures that posses negative permittivity and permeability respectively. Furthermore, it

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is well known that the effective permittivity and permeability of such media can be extracted from the scattering characteristics. In what follows, motivated by these applications, we examine the scattering cross section of a meta-sphere embedded in a medium.

2. SCATTERING CROSS SECTION

We consider a meta-sphere as being a composite of two volumes, that is, an inner core of radius a_1 with permittivity ϵ_1 and permeability μ_1 and a mantle layer of radius a_2 with permittivity ϵ_2 and permeability μ_2 respectively. Surrounding the meta-sphere is a medium with permittivity ϵ_0 and permeability μ_0. We are interested in the resonant scattering cross section (SCS) of the meta-sphere and this requires relating the former to the effective polarisation factor α_{eff} [10] of the meta-sphere and its surrounding medium. This concept has been used in the solution of uniform homogeneous (right-handed) dielectric spheres [11, 12]. Based on such work, the total scattering cross section due to N meta-spheres in a medium is defined to be

$$\sigma_{tot} = \sum_{n=1}^{N} \sigma_n = N\sigma \tag{1}$$

where σ corresponds to the scattering cross section of an individual meta-sphere so that [11],

$$\sigma = \frac{k^4}{6\pi} \left|\alpha_{\mathit{eff}}\right|^2 \tag{2}$$

where,

$$\alpha_{\mathit{eff}} = 4\pi a_2^3 \left(\frac{\gamma}{1 - \frac{2}{3} k^3 a_2^3 \gamma i} \right) \tag{3}$$

Substituting (3) into (2) we have the final form for the scattering-cross section:

$$\sigma = \frac{8\pi}{3} k^4 a_2^6 \left(\frac{\gamma^2}{1 + \frac{4}{9} k^6 a_2^6 \gamma^2} \right) \tag{4}$$

with

$$\gamma = \frac{(\gamma_1 - 1)(\gamma_2 + 2) + (\gamma_2 - 1)(2\gamma_1 + 1)\left(a_1/a_2\right)^3}{(\gamma_1 + 2)(\gamma_2 + 2) + 2(\gamma_2 - 1)(\gamma_1 - 1)\left(a_1/a_2\right)^3} \tag{5}$$

being the polarisability of a sphere [10]. Here $k = 2\pi/\lambda$, λ is the wavelength and $\gamma_1 = \epsilon_2/\epsilon_0$, $\gamma_2 = \epsilon_1/\epsilon_2$ for the relative permittivities and similarly $\gamma_1 = \mu_2/\mu_0$ and $\gamma_2 = \mu_1/\mu_2$ for the relative permeabilities of the meta-sphere. From (5) we can obtain the limit for a normal right-handed homogeneous sphere with permittivities $\gamma_1 = \epsilon_1/\epsilon_0$ or permeabilities $\gamma_1 = \mu_1/\mu_0$ since we can set $a_1 = a_2$ and $\gamma_2 = 1$ in (5) which reduces to

$$\gamma = \frac{(\gamma_1 - 1)}{(\gamma_1 + 2)} \tag{6}$$

In this limit, (6) is proportional to the polarisability factor in the solution for the right-handed dielectric sphere mentioned above. Suppose that the outer layer of the meta-sphere has a frequency dependent permittivity according to the Drude model. Then we have,

$$\varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2 + \beta_e \omega i} \tag{7}$$

where ω_p is the plasma frequency and β_e is the damping constant. At the same time we let the inner-layer be frequency dependent as well, that is, we have a frequency dependent magnetic core [13]:

$$\mu_1 = 1 - \frac{\omega^2}{\omega^2 - \omega_m^2 + \beta_m \omega i} \tag{8}$$

where ω_m is the magnetic resonance and β_m is the damping coefficient. Such a configuration can be used to obtain doubly negative effective permittivity and permeability for inclusions embedded inside a medium with permittivity ϵ_0 and permeability μ_0 according to mean field theory. The scattering characteristics of the meta-sphere are shown in Fig. 1. The dashed curve corresponds to the limit for γ as given by (6). This result is the same as that predicted for a normal right-handed dielectric sphere with resonance at $\omega_0 = \omega_p/\sqrt{3} = 2.15/\sqrt{3} = 1.2413\,\text{GHz}$. The solid curves are the shifted cross section scattering resonances with the same parameters as for the dielectric sphere case except that the inner core has permittivity $\epsilon_1 = 0.1$ for the light curve and $\epsilon_1 = 2.0$ for the dark curve showing, in the case of the latter, two resonances where $\sigma \approx -70dB\lambda^2$. Hence with a meta-sphere, it is possible to obtain multiple resonances where $\sigma \to 0$ using only one set of parameters or material attributes as opposed to a regular dielectric sphere. Parameters, common for all cases, are $\epsilon_1 = 1.0$, $\omega_p = 2.15$, $\beta_e = 0.005$, $a = a_2 = 0.01$, $a_1 = 0.95a_2$ and $\lambda = 0.1$.

In Fig. 2, the solid curve shows the scattering response due to the frequency dependent outer layer of the meta-sphere while the dashed

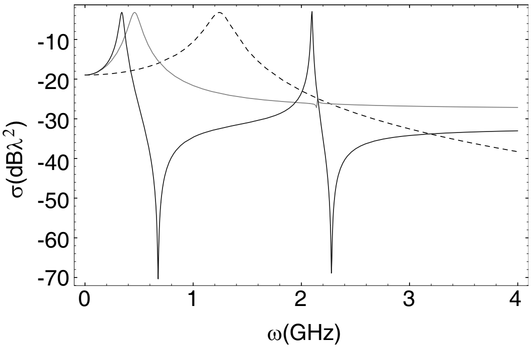

Figure 1. Plots showing appearance of two scattering cross section resonances using the same material parameters in both cases. In particular, the dashed curve corresponds to that predicted for a normal right-handed dielectric sphere which is shown for comparison.

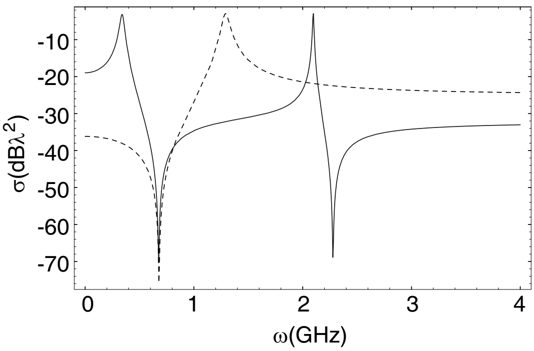

Figure 2. Solid curve shows the scattering response due to the frequency dependent outer layer of the meta-sphere while the dashed curve represents the frequency response due to the magnetic core. Both the magnetic core and metallic layer can be made to resonate at the same frequency.

curve represents the frequency response due to the magnetic core. In this example, the two are made to resonate at the same frequency which is around $\omega \approx 0.685\,\text{GHz}$. Parameters are the same as in Fig. 1 for the permittivities and here we take $\mu_0 = 1.0$, $\mu_2 = 0.1$, $\omega_m = 0.92$ and $\beta_m = 0.009$ for the permeabilities. When the meta-sphere in the medium is perfect-conducting, (5) reduces to $\gamma = 1$ so that σ becomes,

$$\sigma = \frac{8\pi}{3} k^4 a_2^6 \left(\frac{1}{1 + \frac{4}{9} k^6 a_2^6} \right) \approx \frac{8\pi}{3} k^4 a_2^6 \tag{9}$$

where the approximation holds if the term $4/9k^6 a_2^6$, which is due to the radiated reaction field, is negligible.

3. ZERO SCATTERING CROSS SECTION

Figures 1 and 2 show that σ can be manipulated dramatically for particular frequencies using the same intrinsic physical characteristics of the meta-sphere. Ideally we would like to reduce σ to zero or close to zero as much as possible. In order to reduce the scattering cross section to zero we require that the polarisability of the meta-sphere is zero or more precisely $\gamma = 0$. By solving for γ_2 (or γ_1) we have

$$\gamma_2 = \frac{a_1^3(2\gamma_1 + 1) - 2a_2^3(\gamma_1 - 1)}{a_1^3(2\gamma_1 + 1) + a_2^3(\gamma_1 - 1)} \tag{10}$$

From the definitions of γ_1 and γ_2 the permittivity response of the meta-sphere becomes

$$\varepsilon_1 = \varepsilon_2 \left[\frac{a_1^3(2\varepsilon_2 + \varepsilon_0) - 2a_2^3(\varepsilon_2 - \varepsilon_0)}{a_1^3(2\varepsilon_2 + \varepsilon_0) + a_2^3(\varepsilon_2 - \varepsilon_0)} \right] \tag{11}$$

and similarly we obtain

$$\mu_1 = \mu_2 \left[\frac{a_1^3(2\mu_2 + \mu_0) - 2a_2^3(\mu_2 - \mu_0)}{a_1^3(2\mu_2 + \mu_0) + a_2^3(\mu_2 - \mu_0)} \right] \tag{12}$$

Table 1. For the fixed permittivities and permeabilities shown, (11) and (12) are used in order to obtain the values for the cross section of the meta-sphere. Here we define $a_1/a_2 = 0.95$.

ϵ_0	ϵ_2	ϵ_1 from (11)	σ
4.0	0.5	7.17196	0
1.5	-6.0	2.18061	0
μ_0	μ_2	μ_1 from (12)	σ
3.0	4.0	2.8496	0
2.0	-5.0	2.5763	0

for the permeability response. Thus, given the radius a_1 representing the meta-sphere inner volume and a_2 for the outer layer volume as well as the outer layer permittivity and permeability ϵ_2 and μ_2 respectively, the permittivity ϵ_1 and permeability μ_1 of the inner volume of the meta-sphere are determined that make the scattering cross section zero ($\sigma = 0$). Table 1 shows typical values for ϵ_1 and μ_1 that together with the fixed parameters shown reduce σ to zero.

4. CONCLUSION

It has been shown that a meta-sphere embedded in a medium exhibits interesting scattering cross section behaviour, including multiple resonances. We have derived expressions which enable these resonant structures to be computed and shown how they can be controlled to yield a vanishingly small scattering cross section. These results can be exploited in the design of small antennas where they afford a means of impedance matching and also in applications where the objective is to minimize the scattering cross section.

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