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ANALYSIS OF AN INJECTION-LOCKED BISTABLE SEMICONDUCTOR LASER WITH THE FREQUENCY CHIRPING

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Abstract—Modifying the rate equations of an injection-locked semiconductor laser, we have analyzed its optical bistability with the effect of frequency chirping of injected light. Comparison between the bistable steady-state characteristics of the laser in two cases: With and without frequency chirping is done by studying the effect of parameters such as frequency detuning, carrier injection rate, and cavity length. Then we have made a comparison between the bistable dynamic characteristics of the laser for these two cases. The results of the analysis show that the effect of frequency chirping on the bistability behavior is negligible.

1. INTRODUCTION

Optically bistable semiconductor devices have attracted much attention in optical communication systems and information processing. This is because of their useful features of inherent optical gain, low switching power ($\sim \mu$W), and high switching speeds ($\sim$ ns). Researchers have widely utilized optical bistability in optical logic gates [1], optical flip-flops [2, 3], and optical signal regeneration [4].

If a semiconductor laser diode is biased below its oscillation threshold, then it acts as a semiconductor laser amplifier (SLA) so that the externally injected optical signal can be amplified in the active region of the SLA. On the other hand, the intensity dependent refractive index and the nonlinear gain saturation make the resonant frequency of the SLA to be dependent on the power of input optical

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signal. Dispersive optical bistability (OB) is observed, when the input optical signal is strong enough.

OB is also observed in an optically injection-locked semiconductor laser. In this case, the laser diode is biased above its oscillation threshold, the stimulated optical field of the laser diode is locked to the incoming optical signal. The OB behavior can be observed due to two different operating modes of the device, i.e., "locked mode" and "free-oscillation mode". In the former mode, the laser acts at the same frequency as that of the injecting light, ω_1. It occurs when the power ratio between the injected optical field and the emitted optical field from the laser diode is sufficiently high. In the free oscillation mode the laser operates at the wavelength of its original longitudinal-mode, ω_0 (in absence of any injected light).

The injection-locked properties of a laser diode were predicted by Lang [5]. His work was based on the existence of an asymmetry between locked longitudinal-mode intensity and frequency detuning, $\Delta\omega = \omega_1 - \omega_0$. This asymmetric characteristic was first observed experimentally by Kobayashi et al. [6]. Kawaguchi et al. demonstrated, for the first time, bistability in the locking curve using single longitudinal-mode lasers [7]. Behavior of injection-locked bistable semiconductor lasers, using a multi-detuned light injection, has also been studied by Okada et al. [8, 9]. Sivaprakasam et al. reported for the first time, experimental observation of negative hysteresis in an injection-locked diode laser [10]. Recently, Y. H. Won et al. demonstrated an all-optical flip-flop using optical bistability of injection-locked Fabry-Perot laser diodes [11].

On the other hand, the bistable action of an injection-locked semiconductor laser is achieved by injecting the optical signal at the frequency detuning within the stable locking band. Such an injection can be provided by the design illustrated in Fig. 1. The first laser (LD1), biased above the threshold, is used to generate the signal light and its output is injected into the second laser (BLD2) which works as an OB device. Each laser is supposed to be isolated from external reflection. In order to sweep different values of the optical output power of the laser source (LD1), the current of this laser can be modulated. This causes its frequency of emitted light varies with the optical output power; i.e., its output light frequency chirps. Therefore, for this injection scheme, the effect of frequency chirping on the bistability behavior becomes an important issue and is not considered so far.

To take the role of the injection light frequency chirping into account, in the design scheme of Fig. 1, first we have modified the rate equations in Section 2. Then in Section 3, we have studied the steady-state optical output versus optical input power in two cases:

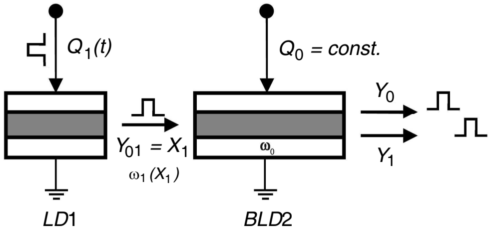

Figure 1. A layout proposed to study the behavior of an injection-locked bistable semiconductor laser with one laser source.

With and without frequency chirping, where the frequency detuning, carrier injection rate, and cavity length are used as parameters. In Section 4, by injecting an optical pulse into the bistable laser, the dynamic responses of the laser for the two cases with and without frequency chirping are compared. Finally, we conclude the paper in Section 5.

2. THE RATE EQUATIONS

The rate equations describing the bistable behavior of an injection-locked semiconductor laser are as follows [12–14]:

$$\frac{dn}{dt} = Q - \frac{n}{\tau_s} - (g)(n - n_T)[1 - (s)(N_0 + N_1)](N_0 + N_1) \quad (1a)$$

$$\frac{dN_0}{dt} = -\frac{N_0}{\tau_p} + \Gamma(g)(n - n_T)[1 - (s)(N_0 + N_1)]N_0 + \Gamma\beta\frac{n}{\tau_s} \quad (1b)$$

$$\frac{dE_1}{dt} = -\frac{E_1}{2\tau_p} + \Gamma(g)(n - n_T)[1 - (s)(N_0 + N_1)]\frac{E_1}{2} + i[\Omega(n) - \omega_1]E_1 + \frac{E_{ext}}{2\tau_c} \quad (1c)$$

where

$$N_1 = \frac{\eta}{\hbar\omega_1 c}|E_1|^2 \quad (2)$$

$$\tau_c = nL/cT \quad (3)$$

$$\Omega(n) = \Omega(n_{th}) + \frac{d\Omega}{dn}(n - n_{th}) \quad (4)$$

and

$$n_{th} = n_T + 1/\Gamma g\tau_p \quad (5)$$

Table 1. Description of the laser parameters.

Symbol	Quantity
Q	the injection rate of carrier density that is above its threshold value $Q_{th} = n_{th}/\tau_s$
n	the carrier density at the frequency ω_0
N_0	the photon density at the frequency ω_0
E_1	the locked optical field at the frequency ω_1
N_1	the locked photon density at the frequency ω_1
τ_s	the carrier lifetime
τ_p	the photon lifetime
g	the differential gain coefficient
s	the gain saturation coefficient
n_T	the carrier density at transparency
Γ	the confinement factor of optical power within the active layer
β	the spontaneous emission factor
η	the refractive index of the active layer of the semiconductor laser
c	the velocity of light in vacuum
E_{ext}	the input optical field
τ_c	the rate of the optical injection
L	the cavity length
T	the transmission coefficient of the optical field
$\omega(n)$	the cavity-resonant frequency depending on the carrier density
n_{th}	the carrier density at threshold without optical input
$d\Omega/dn$	the rate of change of the resonance frequency with the carrier density caused by the carrier-induced refractive index change with the optical input

All of the parameters given in the above equations are introduced in Table 1. Moreover, defining the photon density saturation as $N_s = 1/g\tau_s$ and the normalized gain saturation coefficient as $\varepsilon = sN_s$, we introduce a new set of dimensionless parameters Y_0, Y_1, and X_1 as follows:

$$\begin{aligned} Y_0 &\equiv N_0/N_s = N_0 g\tau_s && \text{(6a)}\\ Y_1 &\equiv N_1/N_s = N_1 g\tau_s && \text{(6b)}\\ X_1 &\equiv \sqrt{\frac{\eta}{\hbar\omega_1 c N_s}}E_{ext} = \sqrt{\frac{\eta g\tau_s}{\hbar\omega_1 c}}E_{ext} && \text{(6c)} \end{aligned}$$

Y_0, Y_1, and X_1 are the normalized optical output power in absence of the light injection, the normalized optical output power in presence of the light injection, and the normalized optical input power, respectively.

On the other hand if E_{1R} and E_{1I} are the real and imaginary parts of E_1 and also Y_{1R} and Y_{1I} are the real and imaginary parts of Y_1 then we have [8]:

$$\begin{aligned} Y_{1R} &\equiv \sqrt{\frac{\eta}{\hbar\omega_1 c N_s}}E_{1R} = \sqrt{\frac{\eta g\tau_s}{\hbar\omega_1 c}}E_{1R} && \text{(7a)}\\ Y_{1I} &\equiv \sqrt{\frac{\eta}{\hbar\omega_1 c N_s}}E_{1I} = \sqrt{\frac{\eta g\tau_s}{\hbar\omega_1 c}}E_{1I} && \text{(7b)}\\ E_1 &= \sqrt{E_{IR}^2 + E_{1I}^2} && \text{(7c)}\\ Y_1 &= \sqrt{Y_{1R}^2 + Y_{1I}^2} && \text{(7d)} \end{aligned}$$

Using these new dimensionless parameters, we can write the following rate equations [8]:

$$\begin{aligned} \frac{dn}{dt} &= Q - \frac{n}{\tau_s} - \left(\frac{n-n_T}{\tau_s}\right)\left[1-(\varepsilon)(Y_0+Y_1)\right](Y_0+Y_1) && \text{(8a)}\\ \frac{dY_0}{dt} &= -\frac{Y_0}{\tau_p} + (\Gamma g)(n-n_T)\left[1-(\varepsilon)(Y_0+Y_1)\right]Y_0 + \Gamma\beta g n && \text{(8b)}\\ \frac{dY_{1R}}{dt} &= \left\{-\frac{1}{\tau_p} + (\Gamma g)(n-n_T)\left[1-(\varepsilon)(Y_0+Y_1)\right]\right\}\frac{Y_{1R}}{2}\\ &\quad + \left[\left(\frac{g\alpha}{2}\right)(n-n_{th}) + \Delta\omega_1\right]Y_{1I} + \frac{X_1^{\frac{1}{2}}}{2\tau_c} && \text{(8c)}\\ \frac{dY_{1I}}{dt} &= \left\{-\frac{1}{\tau_p} + (\Gamma g)(n-n_T)\left[1-(\varepsilon)(Y_0+Y_1)\right]\right\}\frac{Y_{1I}}{2}\\ &\quad - \left[\left(\frac{g\alpha}{2}\right)(n-n_{th}) + \Delta\omega_1\right]Y_{1R} && \text{(8d)} \end{aligned}$$

In the above equations $\Delta\omega_1 = \omega_1 - \omega_0$ is frequency detuning for the input light, in which $\omega_0 = \Omega(n_{th})$ is the threshold resonant frequency for the bistable semiconductor laser, in absence of the light injection, and $\alpha = -(2/g)(d\Omega/dn)$ is the line width enhancement factor.

At the case of frequency chirping in the input light, the detuning $\Delta\omega_1$ is no longer remaining constant, but it is depending on the optical input power X_1. In order to obtain relationship between $\Delta\omega_1$ and X_1, we use the photon density rate equation for the laser source (LD1) as follows [12–14]:

$$\frac{dX_1}{dt} = -\frac{X_1}{\tau_{p1}} + (\Gamma_1 g_1)(n_1 - n_{T1})\left[1 - \varepsilon_1 X_1\right] X_1 + \Gamma_1 \beta_1 g_1 n_1 \tag{9}$$

Also if the cavity-resonant frequency of the laser source which is dependent on the carrier density is denoted by $\Omega(n_1)$, then we write:

$$\Omega\left(n_1\right) = \Omega\left(n_{th1}\right) + \frac{d\Omega}{dn}\left(n_1 - n_{th1}\right) \tag{10}$$

The parameters: n_1, τ_{p1}, g_1, ε_1, n_{T1}, Γ_1, β_1, and n_{th1} for the laser source are defined similarly to those which are defined for the bistable laser.

Now, using the definition of the linewidth enhancement factor and Eq. (10) we obtain the following equation:

$$\omega_1 - \omega_{th} = -\left(\alpha_1 g_1/2\right)\left(n_1 - n_{th1}\right) \tag{11}$$

in which $\omega_{th} = \Omega(n_{th1})$ is the threshold resonant frequency for LD1. By solving Eq. (9) for the steady state condition, the dependency of ω_1 on X_1, i.e., the case of frequency chirping, is calculated and hence we obtain the frequency detuning $\Delta\omega_1$ as:

$$\begin{aligned} \Delta\omega_1\left(X_1\right) &= -(\alpha_1 g_1/2)\left[\frac{(n_{th1} - n_{T1})\varepsilon_1 X_1^2 - \beta_1 n_{th1}}{\beta_1 + \left(1 - \varepsilon_1 X_1\right) X_1}\right] + \Delta\omega_{th} \\ &= -(\alpha_1/2\Gamma_1)\left[\frac{\varepsilon_1 X_1^2 - \frac{\beta_1 n_{th1}}{(n_{th1} - n_{T1})}}{\beta_1 + \left(1 - \varepsilon_1 X_1\right) X_1}\right]\frac{1}{\tau_{p1}} + \Delta\omega_{th}, \end{aligned} \tag{12}$$

in which $\Delta\omega_{th} = \omega_{th} - \omega_0$ is the threshold detuning frequency which is equal to $\Delta\omega_1$ in the case of without frequency chirping. The frequency detuning $\Delta\omega_1$ is no longer remaining constant in presence of frequency chirping within the BLD. Then in this case, we can take the threshold detuning $\Delta\omega_{th}$ as a fixed parameter in analysis of optical bistability.

3. STEADY-STATE CHARACTERISTICS

The steady-state characteristics of optical output versus optical input are obtained by setting the left-hand side of Eqs. (8a)–(8d) equal to zero. Using a laser source, we feed the active region of the bistable semiconductor laser by an optical beam of power X_1 at frequency ω_1. For the sake of comparison, in our analysis, we set the original state of $Y_0 = 0.5$, the same as that is used in [8], and analyzed the bistable semiconductor laser without frequency chirping. The results similar to those reported in [8] are obtained.

In our analysis, the values of the parameters for the source and bistable lasers are set equal and given in Table 2, which are the same as those used in [8].

Figures 2–4 illustrate the dependency of the steady-state characteristics of the bistable semiconductor laser on the frequency detuning, carrier injection rate, and cavity length, respectively. The results labeled by "i" and "k" show the case "in absence of the frequency chirping", while those labeled by "j" and "l" show the case "in presence of the frequency chirping". The OB behavior of the injection-locked semiconductor laser (BLD2) are observed through operating locked mode or free-oscillation mode if either Y_1 or Y_0 is at a high level, respectively. We should note that the $Y_0 - X_1$ characteristics are illustrated in log-log scales and the $Y_1 - X_1$ characteristics are illustrated in linear-log scales.

As frequency chirping is concerned, a careful examination of Figs. 2–4 (i.e., the curves "j" in comparison with "i" and "l" in comparison with "k") shows that the frequency chirping causes a small upward jump in the upper state of Y_1, whereas it leads to a small down

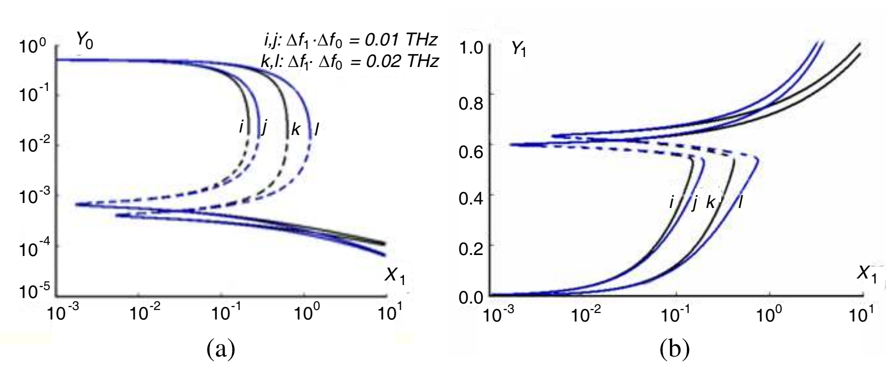

Figure 2. Steady-state output characteristics of a bistable semiconductor laser versus its optical input power for two different frequency detuning ($L = 200\,\mu$m, $Q = 1.31\,Q_{th}$) without frequency chirping ("i" and "k") and with frequency chirping ("j" and "l"): (a) Free oscillation mode; (b) locked mode.

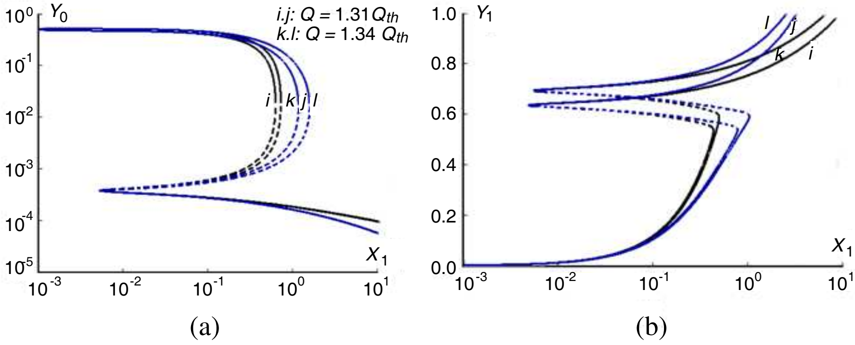

Figure 3. Steady-state output characteristics of a bistable semiconductor laser versus its optical input power for two different carrier injection rates ($L = 200\,\mu\text{m}$) without frequency chirping ("i" and "k", $\Delta f_1 = 0.02\,\text{THz}$) and with frequency chirping ("j" and "l", $\Delta f_{th} = 0.02\,\text{THz}$): (a) Free oscillation mode; (b) locked mode.

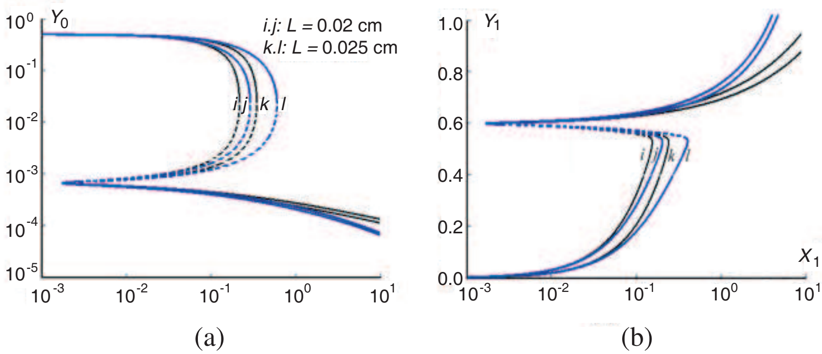

Figure 4. Steady-state output characteristics of a bistable semiconductor laser versus its optical input power for two different cavity lengths ($Q = 1.31Q_{th}$), without frequency chirping ("i" and "k", $\Delta f_1 = 0.02\,\text{THz}$) and with frequency chirping ("j" and "l", $\Delta f_{th} = 0.02\,\text{THz}$): (a) Free oscillation mode; (b) locked mode.

fall in the lower state of Y_0. This result is obtained when we follow the effect of the parameters such as: The frequency detuning, carrier injection rate, or cavity length on the bistable characteristics. This is because, in presence of the frequency chirping, the power dependent frequency detuning $\Delta\omega_1(X_1)$ is a bit larger than the constant frequency detuning $\Delta\omega_1$, in absence of the frequency chirping. This small

Table 2. Values of the bistable laser parameters used for calculations.

Symbol	Value
c	3×10^8 m/s
L	200–300 μm
g	8.56×10^{-5} cm^3/s
n_T	1×10^{18} cm^{-3}
T	0.82
Δf_{th}	0.01–0.03 THz
Q	1.31–1.37 Q_{th}
β	10^{-5}
ε	0.02
Γ	0.46
η	3.8–4.4
$d\Omega/dn$	2.14×10^{-6} cm^3/s
τ_p	1.65 ps
τ_s	2 ns

difference makes the bistability region to shift towards a larger optical input range in the case of the frequency chirping, as we expected. In fact, an increase in the frequency detuning makes the optical input less effective, and hence, a larger optical input power is required for the bistable laser to switch from the free-oscillation mode to the locked mode; i.e., Y_0 falls to a lower level, whereas Y_1 jumps to a higher level. However, as mentioned above, the difference between the power dependent frequency detuning $\Delta\omega_1(X_1)$ and the constant frequency detuning $\Delta\omega_1$ is negligible. In fact, this negligible difference is analytically given by the first term on the right hand side of Eq. (12). Using the values of the parameters given in Table 2, the numerical value of this term, for the normalized input power of $X_1 = 1$, is smaller than 10^{-4} THz; i.e., less than 1% of the minimum value used for the threshold detuning frequency. When the normalized input power reaches $X_1 = 10$, this difference approaches 10^{-3} THz; i.e., 10% of the minimum value used for our numerical analysis. This shows that the effect of frequency chirping on the steady-state bistable performance of an injection-locked semiconductor laser can be neglected.

Another interesting question is: How the parameters such as the frequency detuning, carrier injection rate, and cavity length affect the steady-state characteristics of the bistable semiconductor laser whether the frequency chirping is considered or not (i.e., in Figs. 2–4, the curves "j" in comparison with "l" or "i" in comparison with "k"). Fig. 2 illustrates the dependency of the steady-state characteristics of the bistable semiconductor laser on the frequency detuning, similar to that reported in [8]. The results show that as the frequency detuning increases, the bistability region shifts towards a larger optical input range. This is because an increase in the frequency detuning makes the optical input less effective, and hence, a larger optical input power is required for the bistable laser to switch from the free-oscillation mode to the locked mode; i.e., Y_0 falls to a lower level while Y_1 jumps to a higher level. Fig. 3 illustrates the dependency of the steady-state characteristics of the BLD2 on the carrier injection rate. From this figure, it is evident that the upper state of Y_1 increases with increase in Q. This is due to the fact that an increase in injected carrier density leads to an increase in the laser amplifier gain. This additional gain increases the switching contrast between the upper and lower stable states of Y_1. As we expected, the change in carrier injection rate has no effect on the lower states of Y_0. The effect of variation of the cavity length on the steady-state optical output characteristics of the bistable semiconductor laser is also considered. Fig. 4 illustrates such results for both free oscillation and locked modes. We can see from this figure that an increase in the cavity length leads to some increase in the width of the bistability region. This is because an increase in the cavity length leads to a decrease in the cavity mode spacing, which in turn increases the probability of bistability operation [15, 16].

4. DYNAMIC CHARACTERISTICS

Using the Runge-Kutta method [8], we have solved Eqs. (8a)–(8d) to obtain the dynamic characteristics of bistable semiconductor lasers with and without frequency chirping. In order to study the effect of frequency chirping on the dynamic response, two similar optical pulses of power X_1, one with and the other without frequency chirping, are injected into the bistable laser.

Figure 5 illustrates dynamic responses of switching from locked-mode to free-oscillation mode with and without frequency chirping. From these characteristics, we observe that the speed of switching is almost equal in both cases. In fact, the speed of switching from locked mode to free oscillation mode depends upon the ratio of the upper state to lower state of each optical output power (Y_1 or Y_0) [12]. According to the results of this paper for the steady state condition (Figs. 2–4), this ratio is only a bit larger in presence of the frequency chirping than that in absence of the frequency chirping. Hence, the effect of frequency chirping on the dynamic response is also negligible.

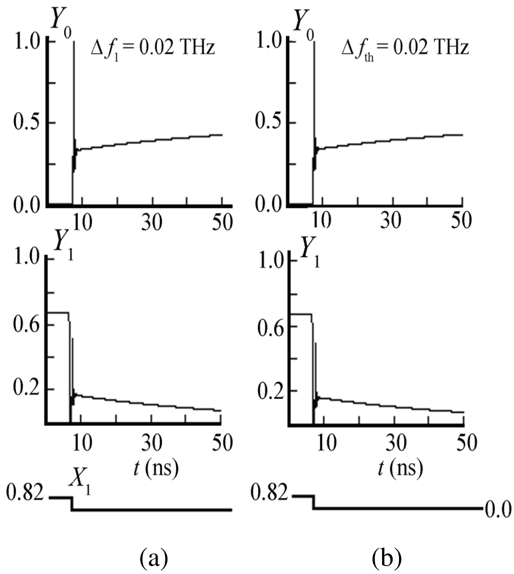

Figure 5. Switching response of a bistable semiconductor laser from locked mode to free-oscillation mode: (a) Without frequency chirping; (b) with frequency chirping.

5. CONCLUSION

In this paper, a special configuration is proposed to study the bistability behavior of an injection-locked semiconductor laser in which only one laser source is used. By modulating the injected current into the laser source, the frequency chirping is observed and the rate equations for the bistable laser are modified. The optical output characteristics of a bistable injection-locked semiconductor laser have been analyzed when an optical input, detuned from the threshold resonant frequency of the bistable semiconductor laser, is injected. The effects of variations in the frequency detuning, carrier injection rate and cavity length on the bistability behavior are also studied. The results of static and dynamic analysis of the bistable injection-locked laser show that the effect of frequency chirping on the bistability behavior is negligible.

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