Progress In Electromagnetics Research B, Vol. 15, 133–150, 2009

SCATTERING BY MULTILAYERED STRUCTURES USING THE EXTENDED METHOD OF AUXILIARY SOURCES EMAS

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Abstract—In this paper, an electromagnetic model based upon the method of auxiliary sources is developed around multilayered structures without any restriction of physical and geometrical macroscopic parameters. Thus, the multilayered structure is considered as a superposition of a finite number of strongly coupled and recovered layers. The extended method of auxiliary sources EMAS is tested for a dielectric shell, a multilayered dielectric cylinder for different medium conductivities and a conducting cylinder coated with a dielectric or a metamaterial. Furthermore, we validate that the coupling between far layers can be neglected for lossy mediums. Numerical results computed in this paper reveal the validity and the accuracy of the aforementioned model in comparison with moment and hybrid methods.

1. INTRODUCTION

Numerical methods are involved to act where the application of applied physics cannot be purchased analytically and are widely used to solve applied electrodynamics problems with differential equations and at least one boundary condition [1, 2]. The method of auxiliary sources MAS is meshless, not requiring an elaborate discretisation of the domain, no integration over the boundary, easy to implement [3]... and widely used to model scattering problems (photonics, metamaterials, arrays...) [4, 5].

The characteristic feature of the auxiliary sources method MAS compared to mesh ones (MoM, FEM, FDTD...) is to interchange

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the differential equation and boundary conditions [6], inducing the singularity elimination in the singular integral equation by shifting the auxiliary sources contour relative to integration one [7]. Proceeding from this, the electromagnetic fields are directly estimated by expanding them over the basis generated by the particular solutions of Helmoltz equation [8].

In this paper, we extensively investigate the usefulness and spread of the MAS to multilayered structures used in RCS minimization, radar furtivity, cloaking, directive sources, coated micro lenses and so on [9].

According to the method of auxiliary sources and for a dielectric cylinder without coating, we have to introduce two bases of auxiliary sources around the boundary. The interior one acts inside the cylinder and the exterior one outside [11], knowing that the auxiliary sources are the particular solutions of Helmoltz equation [12].

When multilayered structures are modeled by the method of moments (MM), multilayered Green's functions are introduced and subject to critical calculations inducing a time consuming and slowly converging [10].

The extension of the MAS technique to multilayered cylinder implies the set of two AS bases around every boundary. The continuity of the tangential electric and magnetic fields [13] is enforced just on the collocation points distributed on the boundaries.

The main idea here is to subdivide the inhomogeneous structure in finite, homogeneous, recovered and strongly coupled mediums [14–16]. For each boundary, the sum of the radiated electromagnetic fields by the overall upper different auxiliary sources bases and the lower ones satisfies the tangential continuity condition leading to a linear system. The method established here can deal with sets of objects having different electromagnetic parameters and shapes. However, we will limit our theoretical study to circular cylindrical shapes.

The electromagnetic model is formulated for multilayered cylindrical structures. The EMAS is alleviated when we neglect the electromagnetic fields for further AS bases from the considered boundary, leading the matrix simplification by increasing the zeros [14].

The time factor is assumed to be $e^{j\omega t}$, and suppressed throughout this paper.

2. FORMULATION

Let us consider an infinite z-axis cylindrical structure constituted by several coaxial cylindrical dielectric layers. We assume that the structure is illuminated by a plane wave impinging from a direction with polar angle φ_{inc}. The plane wave polarization is assumed to be transverse magnetic (TM) with respect to the axis z. We denote the permittivity of the jth dielectric layer by ε_j. The geometry is shown in Fig. 1.

The incident transverse magnetic wave has an electric and magnetic fields:

$$E_z^{inc}(x,y) = E_0 \exp[j(k_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))]\hat{z} \tag{1}$$

$$H^{inc}(x,y) = -\frac{E_0}{Z_0}(\hat{x}\sin\varphi_{inc} - \hat{y}\sin\varphi_{inc}) \exp[j(k_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))] \tag{2}$$

Here, k_0 is the vacuum wave number, and z denotes the unit vector in the z direction, since the incident electric field is z directed and independent of z, we deduce that the scattered field is z directed too, reducing the scattering problem to a bidirectional one.

For every boundary, two auxiliary sources are regularly distributed outside (up) and inside (down) the boundary contour on which are positioned the collocation points. (Fig. 2).

The continuity of the tangential total electric field on every

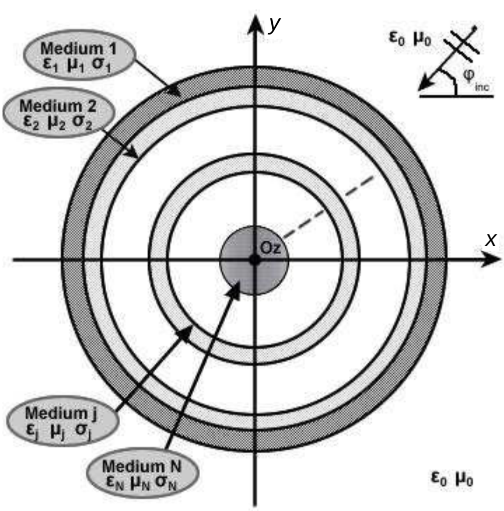

Figure 1. Geometry of the problem.

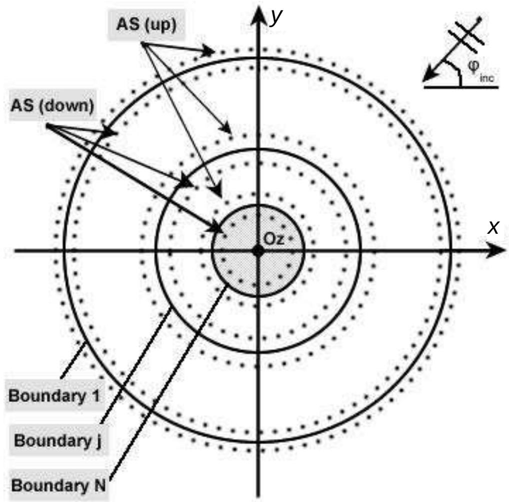

Figure 2. Bases of auxiliary sources repartition around the physical contours.

boundary collocation point leads to:

$$\text{For the boundary 1: } E_{(1)}^{1up} = \sum_{l=1}^{N} E_{(1)}^{ldown} + E_{(1)}^{inc} \tag{3}$$

$$\text{For the boundary 2: } E_{(2)}^{1up} + E_{(2)}^{2up} = \sum_{l=2}^{N} E_{(2)}^{ldown} \tag{4}$$

$$\text{For the boundary 3: } E_{(3)}^{1up} + E_{(3)}^{2up} + E_{(3)}^{3up} = \sum_{l=3}^{N} E_{(3)}^{ldown} \tag{5}$$

$$\text{For the boundary } k\text{: } \sum_{l=1}^{k} E_{(k)}^{lup} = \sum_{l=k}^{N} E_{(k)}^{ldown} \tag{6}$$

$$\text{For the boundary } N\text{: } \sum_{l=1}^{N} E_{(N)}^{lup} = E_{(N)}^{Ndown} \tag{7}$$

$E_{(k)}^{lup}$ is the total electric field radiated by all the upper auxiliary sources of the boundary l and acting on the boundary k.

$E_{(k)}^{ldown}$ is the total electric field radiated by all the down auxiliary sources of the boundary l and acting on the boundary k.

We suppose that the number M of auxiliary sources per basis is equal to the number of collocation points for each boundary. The above $M \cdot N$ equations have $2M \cdot N$ unknowns.

The continuity of the tangential total magnetic field on every boundary collocation point leads to:

$$\text{For the boundary 1: } H_{t(1)}^{1up} = \sum_{l=1}^{N} H_{t(1)}^{ldown} + H_{t(1)}^{inc} \tag{8}$$

$$\text{For the boundary 2: } H_{t(2)}^{1up} + H_{t(2)}^{2up} = \sum_{l=2}^{N} H_{t(2)}^{ldown} \tag{9}$$

$$\text{For the boundary 3: } H_{t(3)}^{1up} + H_{t(3)}^{2up} + H_{t(3)}^{3up} = \sum_{l=3}^{N} H_{t(3)}^{ldown} \tag{10}$$

$$\text{For the boundary } k\text{: } \sum_{l=1}^{k} H_{t(k)}^{lup} = \sum_{l=k}^{N} H_{t(k)}^{ldown} \tag{11}$$

$$\text{For the boundary } N\text{: } \sum_{l=1}^{N} H_{t(N)}^{lup} = H_{t(N)}^{Ndown} \tag{12}$$

The index t denotes the tangential component of the magnetic field. We obtain a linear system with $2M \cdot N$ equations and unknowns. In the purpose to develop the system coefficients, we must clarify some details in the future calculus.

If we consider the boundary 3 for example, these upper auxiliary sources will radiate electromagnetic fields in the interior region bounded by this contour and filled with the medium 3. On the other hand, the auxiliary sources placed down the boundary 3 will radiate outside the region filled with the medium 2.

Following this reasoning and considering the boundary k, the upper and down regions fields can be written as:

$$E_{(k)}^{lup} = \sum_{i=1}^{i=M} a_{(l)}^{up} H_0^{(2)} \left[K_{(l)} \left| r_{Cm}^{n} - r_{Si}^{lup} \right| \right] \tag{13}$$

$$E_{(k)}^{ldown} = \sum_{i=1}^{i=M} a_{(l)}^{down} H_0^{(2)} \left[K_{(l-1)} \left| r_{Cm}^{n} - r_{Si}^{ldown} \right| \right] \tag{14}$$

where, $H_0^{(2)}$ is the Hankel function of the second kind of zero order, $K_{(l-1)}$ the wavenumber in the medium $l-1$, r_{Cm}^{n} the space vector of the collocation point m on the nth boundary, r_{Si}^{lup} the space vector of the upper auxiliary source i on the boundary l and $a_{(l)}^{up}$ the unknown complex currents.

We define the function: $\delta(i-j) = 1$ if $i = j; 0$ elsewhere.

Then, the electric field continuity condition for the nth boundary will be expressed as:

$$\sum_{l=1}^{n} E_{(n)}^{lup} - \sum_{l=n}^{N} E_{(n)}^{ldown} = \delta(n-1)E_{(n)}^{inc} \tag{15}$$

After replacing the electric fields by their expressions:

$$\sum_{l=1}^{l=n}\left(\sum_{i=1}^{i=M} a_{(l)i}^{up} H_0^{(2)}\left[k_{(l)}\left|r_{Cm}^{n} - r_{Si}^{lup}\right|\right]\right) - \sum_{l=n}^{l=N}\left(\sum_{i=1}^{i=M} a_{(l)i}^{down} H_0^{(2)}\left[k_{(l-1)}\left|r_{Cm}^{n} - r_{Si}^{ldown}\right|\right]\right) = \delta(n-1)E_{(n)}^{inc}(m) \tag{16}$$

While varying $1 \ll m \ll M$ and $1 \ll n \ll N$ we obtain $M \cdot N$ equations.

The continuity of the total tangential magnetic field on nth boundary permit:

$$\sum_{l=1}^{n} H_{t(n)}^{lup} - \sum_{l=n}^{N} H_{t(n)}^{ldown} = \delta(n-1)H_{t(n)}^{inc} \tag{17}$$

$$\sum_{l=1}^{n} n_{(n)m} \wedge H_{(n)m}^{lup} - \sum_{l=n}^{N} n_{(n)m} \wedge H_{(n)m}^{ldown} = \delta(n-1)n_{(n)m} \wedge H_{(n)m}^{inc} \tag{18}$$

where, $n_{(n)m}$ is the unit vector perpendicular to the nth boundary just on the collocation point m.

According to the plane wave scattering by a dielectric cylinder, we have:

$$a_{(l)}^{up} = -\left(k_{(l)}Z_{(l)}/4\right) I_{(l)}^{up} \tag{19}$$

$$a_{(l)}^{down} = -\left(k_{(l-1)}Z_{(l-1)}/4\right) I_{(l)}^{down} \tag{20}$$

where, $Z_{(l)}$ is the medium impedance and $I_{(l)}^{up}$ the upper auxiliary source complex current.

Therefore the magnetic field components for the nth boundary will be expressed as:

$$H_{y(n)}^{lup} = \sum_{i=1}^{i=M} \left(k_{(l)} \left(x - x_{(n)i}^{up}\right) / 4jR_{(n)i}^{up}\right) H_1^{(2)} \left[k_{(l)} R_{(n)i}^{up}\right] I_{(l)i}^{up} \quad (21)$$

$$H_{x(n)}^{lup} = \sum_{i=1}^{i=M} \left(k_{(l)} \left(y_{(n)i}^{up} - y\right) / 4jR_{(n)i}^{up}\right) H_1^{(2)} \left[k_{(l)} R_{(n)i}^{up}\right] I_{(l)i}^{up} \quad (22)$$

$$H_{y(n)}^{ldown} = \sum_{i=1}^{i=M} \left(k_{(l-1)} \left(x - x_{(n)i}^{down}\right) / 4jR_{(n)i}^{down}\right) H_1^{(2)} \left[k_{(l-1)} R_{(n)i}^{down}\right] I_{(l)i}^{down} \quad (23)$$

$$H_{x(n)}^{ldown} = \sum_{i=1}^{i=M} \left(k_{(l-1)} \left(y_{(n)i}^{down} - y\right) / 4jR_{(n)i}^{down}\right) H_1^{(2)} \left[k_{(l-1)} R_{(n)i}^{down}\right] I_{(l)i}^{down} \quad (24)$$

Here, $x_{(n)i}^{up}$ denote the abscise of the upper auxiliary source i on the nth boundary and $R_{(n)i}^{up}$ the distance between the auxiliary source $n^\circ i$ and the point with (x,y) coordinates.

The final linear system will have this expression:

$$\begin{aligned} &\sum_{l=1}^{l=n} \sum_{i=1}^{i=M} - \left(k_{(l)} Z_{(l)}/4\right) H_0^{(2)} \left[k_{(l)} \left|r_{Cm}^n - r_{Si}^{lup}\right|\right] I_{(l)i}^{up} \\ &- \sum_{l=n}^{l=N+1} \sum_{i=1}^{i=M} - \left(k_{(l-1)} Z_{(l-1)}/4\right) H_0^{(2)} \left[k_{(l-1)} \left|r_{Cm}^n - r_{Si}^{ldown}\right|\right] I_{(l)i}^{down} \\ &= \delta(n-1) E_{(n)m}^{inc} \end{aligned} \quad (25)$$

$$\begin{aligned} &\sum_{l=1}^{l=n} \sum_{i=1}^{i=M} \left(k_{(l)}/4jR_{(n)i}^{up}\right) \left(n_{x(n)m} \left(x_m - x_{(n)i}^{up}\right)\right. \\ &\left.-n_{y(n)m} \left(y_{(n)i}^{up} - y_m\right)\right) H_1^{(2)} \left[k_{(l)} R_{(n)i}^{up}\right] I_{(l)i}^{up} \\ &- \sum_{l=n}^{l=N} \sum_{i=1}^{i=M} \left(k_{(l-1)}/4jR_{(n)i}^{down}\right) \left(n_{x(n)m} \left(x_m - x_{(n)i}^{down}\right)\right. \\ &\left.-n_{y(n)m} \left(y_{(n)i}^{down} - y_m\right)\right) H_1^{(2)} \left[k_{(l-1)} R_{(n)i}^{down}\right] I_{(l)i}^{down} \\ &= \delta(n-1) \left(n_{x(n)m} H_{y(n)m}^{inc} - n_{y(n)m} H_{x(n)m}^{inc}\right) \end{aligned} \quad (26)$$

If we write the system under the form: $\mathbf{Z} \cdot I = V$, where $\mathbf{Z}$ the square matrix representing physically the total mutual coupling between the different mediums.

$$\mathbf{Z} = \begin{vmatrix} \begin{vmatrix} Z_{11}^{e,up} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ Z_{N1}^{e,up} & \cdots & \cdots & Z_{NN}^{e,up} \end{vmatrix} & \begin{vmatrix} Z_{11}^{e,down} & \cdots & \cdots & Z_{1N}^{e,down} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & Z_{NN}^{e,up} \end{vmatrix} \\ \begin{vmatrix} Z_{11}^{h,up} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ Z_{N1}^{h,up} & \cdots & \cdots & Z_{NN}^{h,up} \end{vmatrix} & \begin{vmatrix} Z_{11}^{h,down} & \cdots & \cdots & Z_{1N}^{h,down} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & Z_{NN}^{h,down} \end{vmatrix} \end{vmatrix}$$

Every element of $\mathbf{Z}$ is a $M \cdot N$ square matrix; $Z_{N1}^{h,up}$ represent the magnetic effect of the upper auxiliary sources of boundary 1 on the boundary N over the collocation points.

3. NUMERICAL RESULTS

In this section are reported some numerical results obtained by computer EMAS implementation code to verify the validity and accuracy of the aforementioned numerical model. In the following examples the permeability is assumed to be that of free space everywhere except for the PEC coated with a metamaterial.

The spatial distribution of scattered power is characterized by a cross section. This fictitious area is the radar cross section and for 2D structures is the scattering width defined as:

$$SW = \lim_{\rho \to \infty} \left[2\pi\rho \frac{\left|E^{sc}\right|^2}{\left|E^{inc}\right|^2} \right] \tag{27}$$

For given auxiliary surfaces, the convergence rate and the accuracy of the method are only dependent on the number of auxiliary sources M. According to MAS, the approximate solution of the boundary problem will tend to exact solution as $M \to \infty$. Therefore, the convergence is ensured [20].

The boundary condition error for the boundary 1 is defined by the ratio of the absolute difference between the tangential electric fields intensity around the considered boundary to the maximum magnitude of the corresponding incident field:

$$\Delta E_{bc} = \frac{\left\| \sum_{l=1}^{N} E_{(1)}^{ldown} + E_{(1)}^{inc} - E_{(1)}^{1up} \right\|}{\max \left\| E_{(1)}^{inc} \right\|} * 100 \tag{28}$$

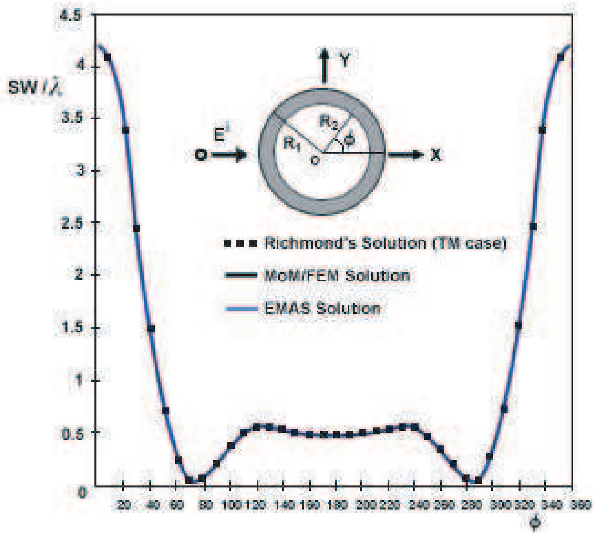

Figure 3. Normalized bistatic echo width of a dielectric cylindrical shell.

3.1. Dielectric Cylindrical Shell

Figure 3 shows the RCS of a dielectric circular cylindrical shell under TM plane wave incidence, all the parameters are identical with those in [17]. The incident wave frequency is set to 300 MHz. The number of auxiliary sources $M = 100$ and $d_{aux} = 0.0125\lambda$, where d_{aux} is the distance between the auxiliary contour and the boundary. The EMAS results are compared with those given by the moment method solution. The agreement between the results is obvious.

3.2. Four Layered Circular Cylinder

Figure 4 shows the RCS of a four-layered circular homogeneous cylinder under TM plane wave incidence with $M = 150$ and $d_{aux} = 0.0075\lambda$. All dielectric layers are assumed to have the same conductivity. EMAS and MM/FEM solutions agree very well except there are some discrepancies in the backscattering region when the layers are lossless. The boundary condition error is plotted in Fig. 5; the maximum error along the boundary1 predicted by the EMAS is 0.28%. On the other hand, the minimum error induced by MM/FEM method [17] just on the

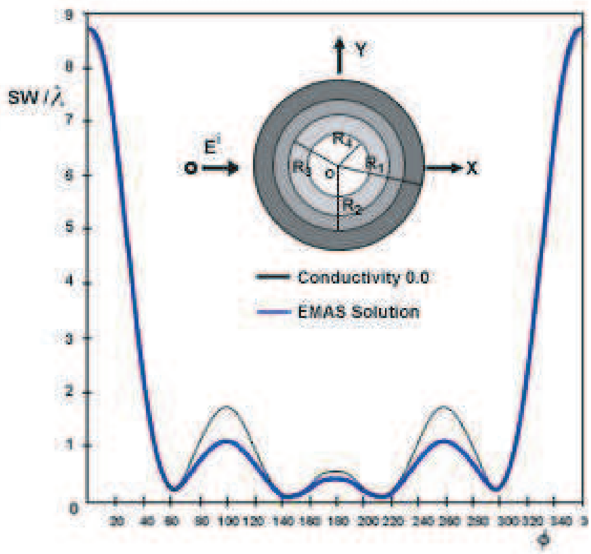

Figure 4. Normalized bistatic echo width of a multilayered circular cylinder, lossless mediums.

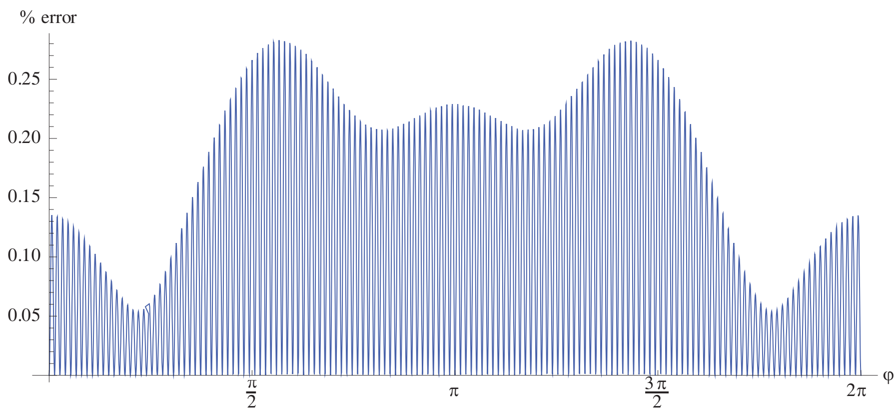

Figure 5. Boundary condition error for multilayered circular cylinder, lossless mediums.

boundary and for only one dielectric cylinder is 1.415%. Therefore, we can deduce that EMAS solution is more accurate than MM/FEM one.

Figures 6 and 7 show the RCS of a four-layered circular homogeneous cylinder under TM plane wave incidence for different values of conductivity ($\sigma = 0.25$; $\sigma = 0.5$) with $M = 150$ and

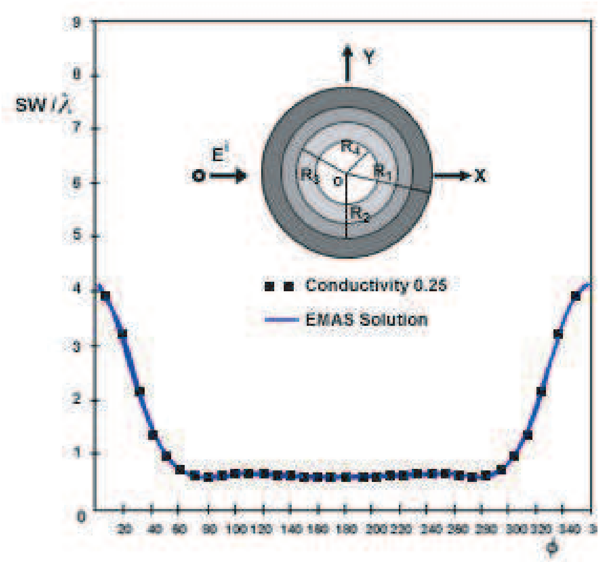

Figure 6. Normalized bistatic echo width of a four-layered circular cylinder, lossy mediums: $\sigma = 0.25$.

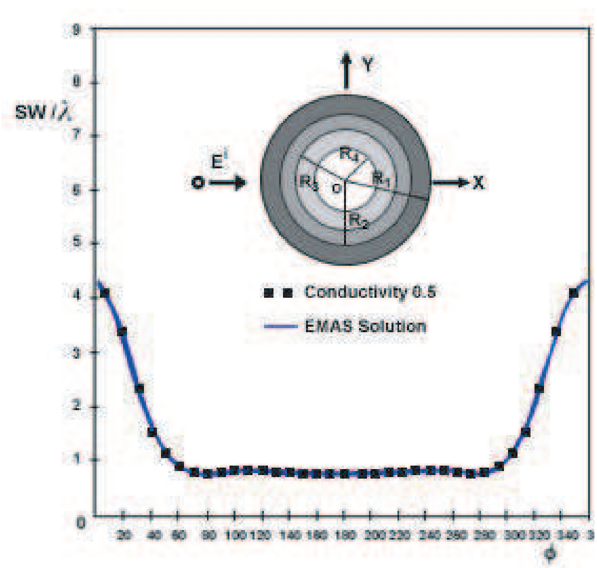

Figure 7. normalized bistatic echo width of a four-layered circular cylinder, lossy mediums: $\sigma = 0.5$.

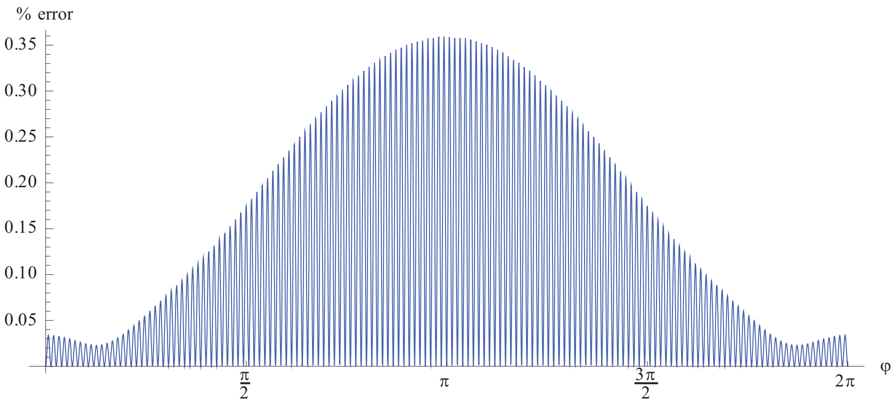

Figure 8. Boundary condition error for multilayered circular cylinder, lossy mediums.

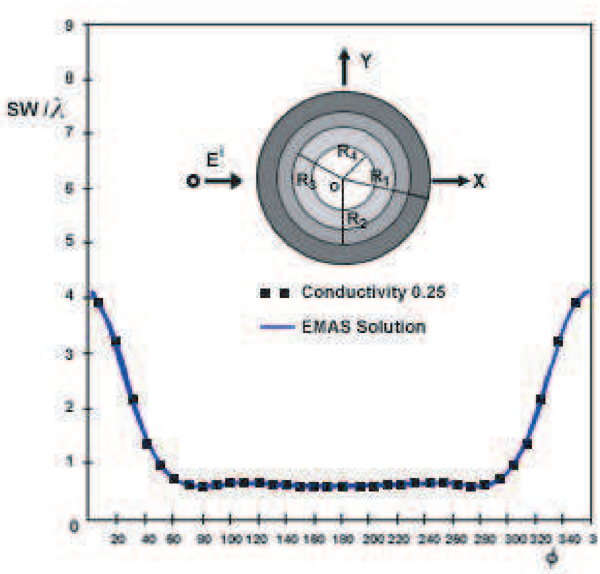

Figure 9. Normalized bistatic echo width of a four-layered circular cylinder, lossy mediums ($\sigma = 0.25$). The coupling between the first and the fourth layer is neglected.

$d_{aux} = 0.0075\lambda$. The maximum error estimated by EMAS in Fig. 8 is 0.36%. The results predicted by EMAS and MM/FEM are identical.

Figures 9 and 11 reveal the RCS of a four-layered circular

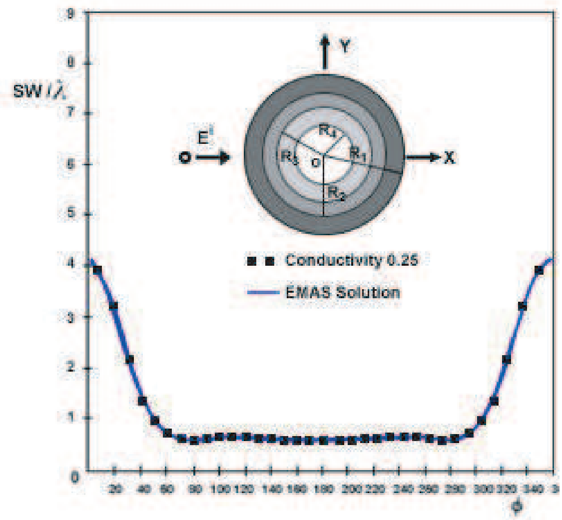

Figure 10. Normalized bistatic echo width of a four-layered circular cylinder, lossy mediums ($\sigma = 0.25$). The coupling between the first and the third, second and fourth layer are neglected.

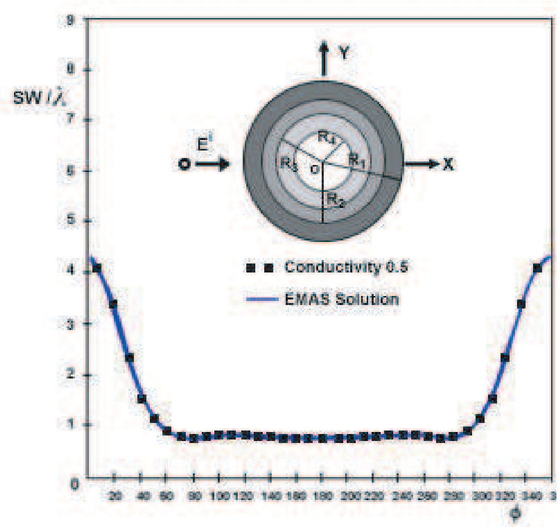

Figure 11. Normalized bistatic echo width of a four-layered circular cylinder, lossy mediums ($\sigma = 0.5$). The coupling between the first and the fourth layer is neglected.

homogeneous cylinder under TM plane wave incidence for different values of conductivity ($\sigma = 0.25$; $\sigma = 0.5$) with $M = 150$ and $d_{aux} = 0.0075\lambda$, when the mutual interaction between the first and the fourth layer is neglected entraining the increase of the zeros in the matrix and consequently the computational cost reduction. The results remain unchanged and agree very well with the reference one.

Figures 10 and 12 illustrate the RCS of a four-layered circular homogeneous cylinder under TM plane wave incidence for respective values of conductivity ($\sigma = 0.25$; $\sigma = 0.5$) with $M = 150$ and $d_{aux} = 0.0075\lambda$, when the mutual interaction between the first and the third layer and the second and fourth layer are neglected: otherwise we suppose that every layer is coupled only with the adjacent ones. In this case, numerical results agree very well with the references. The numerical cost is much more decreased.

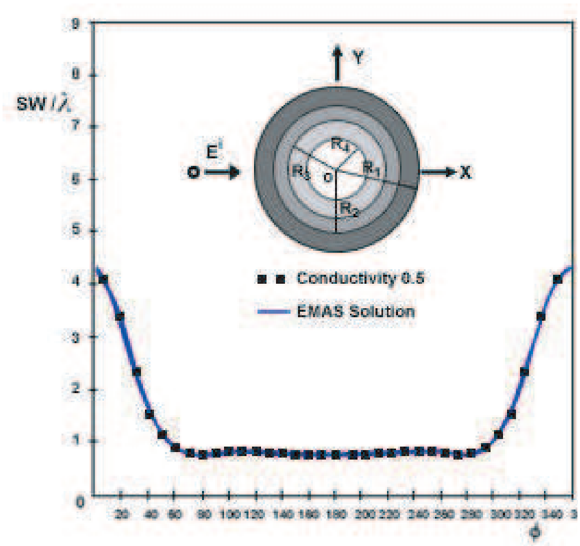

Figure 12. Normalized bistatic echo width of a four-layered circular cylinder, lossy mediums ($\sigma = 0.5$). The coupling between the first and the third, second and fourth layer are neglected.

3.3. Coated Dielectric Circular Cylinder

Figures 13 and 14 illustrate the RCS of a coated circular homogeneous cylinder under TM plane wave incidence with respectively a dielectric and a metamaterial knowing that $M = 120$ and $d_{aux} = 0.01\lambda$.

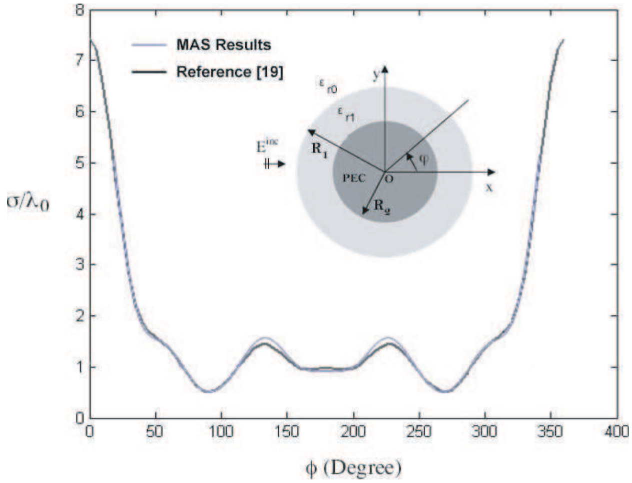

Figure 13. Normalized bistatic echo width of a normal dielectric coated cylinder. ($R_2 = 50\,\text{mm}$, $R_1 = 100\,\text{mm}$, $\varepsilon r1 = 9.8$, $\mu r1 = 1$, $\varphi 0 = 0°$ and $f = 1\,\text{GHz}$).

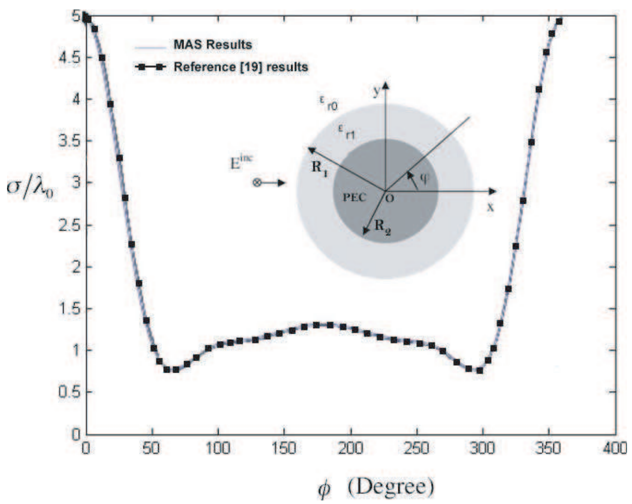

Figure 14. Normalized bistatic echo width of a normal dielectric coated cylinder. ($R_2 = 50\,\text{mm}$, $R_1 = 100\,\text{mm}$, $\varepsilon r1 = -9.8$, $\mu r1 = -1$, $\varphi 0 = 0°$ and $f = 1\,\text{GHz}$).

The EMAS numerical results and the reference ones are identical for matematerial coating and in good agreement for dielectric coating.

4. CONCLUSION

In this paper, we have developed a numerical method EMAS to the scattering problems by dielectric or metamaterial coated cylinders and for multilayered structures. Firstly, the global electromagnetic coupling between different layers was taken into account and the numerical results validate the inner field distributions, secondly, the partial coupling was justified too, when the mutual interaction from far layers was neglected, thus, the problem complexity will be reduced.

The extended method of auxiliary sources permits large perspectives, specially, the modelisation for arrays with multilayered coupled elements.

APPENDIX A.

If one dielectric cylinder illuminated by a TM plane wave. The inner region is denoted by II and the outer one by I. The fields in the region I radiated by one auxiliary source can be written as:

$$E_{iz}^{I} = -\left(k_I Z_I/4\right) H_0^{(2)}\left[k_0 R_i^I\right] I_i^I \tag{A1}$$

$$H_{ix}^{I} = \left(k_I\left(y_i^I - y\right)/4jR_i^I\right) H_1^{(2)}\left[k_0 R_i^I\right] I_i^I \tag{A2}$$

$$H_{iy}^{I} = \left(k_I\left(x - x_i^I\right)/4jR_i^I\right) H_1^{(2)}\left[k_0 R_i^I\right] I_i^I \tag{A3}$$

The fields in the region II radiated by an outside auxiliary source are:

$$E_{iz}^{II} = -\left(kZ/4\right) H_0^{(2)}\left[k_0 R_i^{II}\right] I_i^{II} \tag{A4}$$

$$H_{ix}^{II} = \left(k\left(y_i^{II} - y\right)/4jR_i^{II}\right) H_1^{(2)}\left[k_0 R_i^{II}\right] I_i^{II} \tag{A5}$$

$$H_{iy}^{II} = \left(k\left(x - x_i^{II}\right)/4jR_i^{II}\right) H_1^{(2)}\left[k_0 R_i^{II}\right] I_i^{II} \tag{A6}$$

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