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SCATTERING FROM PERFECTLY MAGNETIC CONDUCTING SURFACES: THE EXTENDED THEORY OF BOUNDARY DIFFRACTION WAVE APPROACH

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Abstract—In this paper, the uniform scattered fields from a perfectly magnetic conducting (PMC) surface are studied with the *extended* theory of boundary diffraction wave (TBDW). The vector potential is described by considering the *extended* TBDW for the PMC surfaces. The *extended* TBDW is then applied to the problem of scattering from the PMC half plane. The total scattered fields are obtained and compared numerically with the exact solution for the same problem. The numerical results show that the solution of the *extended* TBDW is *very close* to the exact solution.

1. INTRODUCTION

Young was the first person who interpreted the scattered fields as the sum of incident fields and edge diffracted fields [1]. Young could not formulate the scattering phenomena, but afterwards, Sommerfeld obtained the exact solution for the problem of diffraction from a half plane in terms of the Fresnel functions [2]. Later, the analytical definition of Young's ideas was introduced by Maggi-Rubinowicz [3, 4]. They independently showed that Helmholtz-Kirchhoff integral can be converted into a line integral which represents the edge diffracted fields. This definition was known as Maggi-Rubinowicz formulation or the theory of boundary diffraction wave (TBDW). Miyamoto and Wolf generalized the TBDW method for various incident fields [5, 6]. They described a potential function which gives the edge diffracted fields for an opaque surface. The formulation of Miyamoto and Wolf was supported and refined by the studies of Rubinowicz [7, 8].

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The theory of boundary diffraction wave can be easily applied to various diffraction problems. However, the diffracted fields from PEC, PMC, or impedance surfaces can not be solved by using TBDW since the theory is based upon an opaque screen.

TBDW method has been applied to diffraction problems. Otis and Lit were investigated diffraction of a Gaussian laser beam from the edge of an opaque half plane by using the TBDW [9]. The uniform diffracted fields from an opaque half plane were examined for normal and oblique incidence with TBDW approach [10, 11]. The problem of diffraction from an opaque half plane was investigated using the detour parameter with TBDW approach [12]. The potential function of the TBDW was obtained for the impedance surfaces by the asymptotic reduction of the modified theory of physical optics (MTPO) surface integrals in [13]. The uniform line integral representation for TBDW was examined by using the MTPO method [14]. Recent studies in [15–17] have been focused on the applications of the TBDW. The *extended* theory of the boundary diffraction wave that we consider in this paper was first applied to the PEC surfaces by the author [18].

The evaluation of the surface integrals needs *high computation times* for diffraction problems with complex geometries. TBDW method reduces the surface integral to a line integral, resulting in a significant improvement in the computation time. Moreover, the line integral reduction of the surface integrals enables one to evaluate the edge diffracted fields directly, by integrating the reduced integrand along the edge contour.

In this paper, the scattered fields from a perfectly magnetic conducting (PMC) surfaces are examined with the *extended* TBDW approach. The vector potential of the PMC surfaces is obtained and applied to the *extended* theory of boundary diffraction wave. The *extended* TBDW is adapted for PMC surfaces. Verification of the method is performed by applying it to the problem of scattering from the PMC half plane. The uniform scattered fields, which are finite at transition regions, are obtained. Results are compared numerically with the exact solution for the same problem.

A time factor $e^{j\omega t}$ is assumed and suppressed throughout the paper.

2. THE EXTENDED THEORY OF THE BOUNDARY DIFFRACTION WAVE

It is known that the Helmholtz-Kirchhoff integral formula at an aperture in an opaque plane can be expressed with a line integral [3, 4].

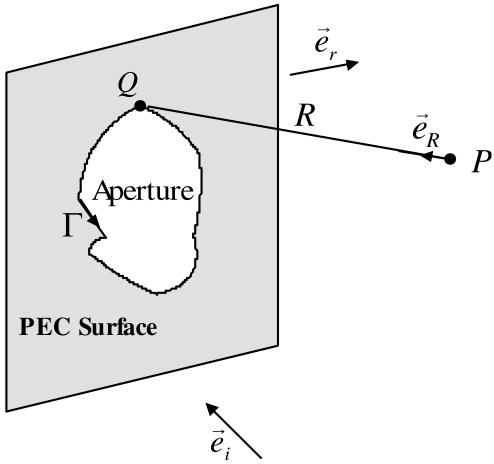

Figure 1. Geometry of the *extended* theory of the Boundary Diffraction Wave.

The scattered fields $U(P)$ at an observation point P can be given as [18]

$$U(P) = U_B(P) + U_{GO}(P). \tag{1}$$

The first term represents the boundary diffraction wave from the boundary line Γ of the aperture surface (see Figure 1). The expression can be given as

$$U_B(P) = \int_\Gamma \vec{W}(Q,P) \cdot d\vec{l}. \tag{2}$$

Where, $\vec{W}(Q,P)$ is the vector potential for the PEC surface. Q is the variable point on the plane surface and $d\vec{l}$ is the line element of the boundary line Γ. The vector potential $\vec{W}(Q,P)$ is symbolically given as

$$\vec{W}(Q,P) = \frac{1}{4\pi}\frac{e^{-jkR}}{R}\left[\vec{e}_R \times \frac{\nabla_Q}{(-jk + \vec{e}_R \cdot \nabla_Q)} U(Q)\right] \tag{3}$$

for PEC surfaces. Here, $\vec{e}_R$ is the unit vector of the vector $\vec{R}$ and R denotes the distance between the observation point P and Q (see Figure 1). As the screen is PEC surface, $U(Q)$ given in Eq. (3) will be the sum of the incident $U_i(Q)$ and reflected $U_r(Q)$ fields at the secondary source point Q [18].

The second term in Eq. (1) represents the contributions of geometrical-optics (GO) fields from the special Q_i points on the PEC surface or aperture. These points are singularities relating to the vector potential $\vec{W}(Q,P)$. Hence, $U_{GO}(P)$ is given symbolically as [18]

$$U_{GO}(P) = \sum_i \lim_{\sigma_i \to 0} \int_{\Gamma_i} \vec{W}(Q_i, P) \cdot d\vec{l} \tag{4}$$

where, $d\vec{l}$ is the line element of the boundary line Γ_i.

3. SCATTERING FROM THE PMC HALF PLANE

The diffraction geometry to be considered in this section is given in Figure 2. In the figure, a homogeneous plane wave is illuminating the PMC half plane. The homogeneous plane wave and the pseudo-reflected wave can be given as

$$U_i(P) = u_i e^{jk\rho\cos(\phi-\phi_0)} \tag{5a}$$

$$U_r(P) = u_i e^{jk\rho\cos(\phi+\phi_0)} \tag{5b}$$

for the PMC half plane problem. Where, k is the wave number.

The diffracted field from the PMC half plane can be evaluated by using Eq. (2). In this section, firstly, the vector potential concerning to the PMC half plane will be obtained. $U(Q)$ is the sum of the incident $U_i(Q)$ and reflected $U_r(Q)$ wave according to the *extended* TBDW [18].

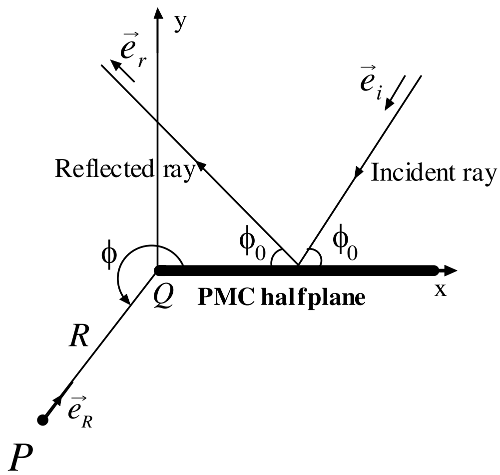

Figure 2. Geometry of the scattering from PMC half plane.

Since the secondary source point Q is located at the origin, $x' = y' = 0$ for this case. Gradient of $U(Q)$ can be given as

$$\nabla_Q U(Q) = -jku_i(\vec{e}_i + \vec{e}_r) \tag{6}$$

at this point. Hence, the vector potential of the PMC half plane problem can be found as

$$\vec{W}(Q,P) = u_i \frac{1}{4\pi} \frac{e^{-jkR}}{R} \left(\frac{\vec{e}_R \times \vec{e}_i}{1 + \vec{e}_R \cdot \vec{e}_i} + \frac{\vec{e}_R \times \vec{e}_r}{1 + \vec{e}_R \cdot \vec{e}_r} \right) \tag{7}$$

by using Eqs. (5) and (6) in Eq. (3). The related unit vectors in Eq. (7) can be written as

$$\begin{aligned} \vec{e}_i &= -\cos\phi_0 \vec{e}_x - \sin\phi_0 \vec{e}_y \\ \vec{e}_r &= -\cos\phi_0 \vec{e}_x + \sin\phi_0 \vec{e}_y \\ \vec{e}_R &= -\cos\phi \vec{e}_x - \sin\phi \vec{e}_y \end{aligned} \tag{8}$$

by considering the geometry in Figure 2. Hence, the diffracted field integral can be found by evaluating Eqs. (7) and (8) in Eq. (2). One obtains

$$U_B(P) = -u_i \frac{1}{4\pi} \left[\tan\left(\frac{\phi - \phi_0}{2}\right) + \tan\left(\frac{\phi + \phi_0}{2}\right) \right] \int\limits_{\Gamma} \frac{e^{-jkR}}{R} dl. \tag{9}$$

Where, $\vec{l}$ is equal $-\vec{e}_z$ for this problem. R is the amplitude of the position vector $\vec{R}$ (See Figure 2) and given as

$$R = \left[x^2 + y^2 + (z - z')^2\right]^{1/2} \tag{10}$$

Therefore, the diffracted field integral can be rewritten as

$$U_B(P) = -u_i \frac{1}{4\pi} \left[\tan\left(\frac{\phi - \phi_0}{2}\right) + \tan\left(\frac{\phi + \phi_0}{2}\right) \right] \int\limits_{z'=-\infty}^{\infty} \frac{e^{-jkR}}{R} dz' \tag{11}$$

by considering $dl = dz'$. The integral expression in Eq. (11) defines a Hankel function [19]

$$H_0^{(2)}(k\rho) = \frac{2j}{\pi} \int\limits_0^{\infty} e^{-jk\rho ch\beta} d\beta \tag{12}$$

utilizing the variable change of $(z - z') = \rho sh\beta$. As a result, the diffracted field integral is obtained as

$$U_B(P) = -u_i \frac{1}{4j}\left[\tan\left(\frac{\phi - \phi_0}{2}\right) + \tan\left(\frac{\phi + \phi_0}{2}\right)\right] H_0^{(2)}(k\rho). \quad (13)$$

Where, ρ is equal the $[x^2 + y^2]^{1/2}$. Debye's asymptotic expansion of the second kind Hankel function can be given as

$$H_0^{(2)}(k\rho) \approx \sqrt{\frac{2}{\pi}} \frac{e^{-j[k\rho - (\pi/4)]}}{\sqrt{k\rho}} \quad (14)$$

for $k\rho \to \infty$. Hence, the diffracted field can be found as

$$U_B(P) \approx -\frac{u_i}{2\sqrt{2\pi}}\left[\tan\left(\frac{\phi - \phi_0}{2}\right) + \tan\left(\frac{\phi + \phi_0}{2}\right)\right] \frac{e^{-jk\rho - j\pi/4}}{\sqrt{k\rho}} \quad (15)$$

for the PMC half plane problem. The expression of the diffracted field in Eq. (15) approaches to infinity at the transition regions (reflection and shadow boundary). Therefore, uniform diffracted field will be found. The first part of the diffracted field can be written as

$$U_{Bi}(P) \approx -\frac{u_i e^{-j\pi/4}}{2\sqrt{\pi}} \sin\left(\frac{\phi - \phi_0}{2}\right) \frac{e^{-j2k\rho\cos^2\left(\frac{\phi - \phi_0}{2}\right)}}{\sqrt{2k\rho}\cos\left(\frac{\phi - \phi_0}{2}\right)} e^{jk\rho\cos(\phi - \phi_0)} \quad (16)$$

by utilizing the trigonometric identity of $1 = 2\cos^2(A) - \cos(2A)$. Then, Eq. (16) can be rewritten as

$$U_{Bi}(P) \approx u_i \hat{F}(\xi_i) \sin\left(\frac{\phi - \phi_0}{2}\right) e^{jk\rho\cos(\phi - \phi_0)}. \quad (17)$$

The argument of the Fresnel function represents the detour parameter [20, 21]. The detour parameter gives the phase difference between the incident (or reflected) and diffracted fields. Thus, the detour parameter associated with the incident field can be easily obtained as

$$\xi_i = -\sqrt{2k\rho}\cos[(\phi - \phi_0)/2] \quad (18)$$

by considering Eqs. (5) and (15). $\hat{F}(\xi_i)$ is the Fresnel function and can be given as

$$\hat{F}(\xi_i) = \frac{e^{-j(\xi_i^2 + \pi/4)}}{2\sqrt{\pi}\xi_i}. \quad (19)$$

So, the first uniform part of the diffracted field can be written as

$$U_{Bi}(P) \approx u_i\, F(|\xi_i|)\mathrm{sgn}(\xi_i)\sin\left(\frac{\phi-\phi_0}{2}\right) e^{jk\rho\cos(\phi-\phi_0)} \tag{20}$$

by using the asymptotic relation for the Fresnel function valid for large arguments, i.e., $\hat{F}(\xi_i) \approx F(|\xi_i|)\mathrm{sgn}(\xi_i)$. Here, $\mathrm{sgn}(\xi_i)$ shows the signum function, which is equal to -1 for $\xi_i < 0$ and 1 for $\xi_i > 0$. Fresnel integral $F(\xi_i)$ can be given as

$$F(\xi_i) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int\limits_{\xi_i}^{\infty} e^{-jt^2} dt. \tag{21}$$

Similarly, the second uniform part of the diffracted field can be found as

$$U_{Br}(P) \approx -u_i\, F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right) e^{jk\rho\cos(\phi+\phi_0)}. \tag{22}$$

Here, ξ_r is the detour parameter associated with the reflected field and can be given as

$$\xi_r = -\sqrt{2k\rho}\cos[(\phi+\phi_0)/2] \tag{23}$$

for this problem. The uniform total diffracted field can be found as

$$\begin{aligned} U_B(P) &= U_{Bi}(P) + U_{Br}(P) \\ &= u_i \left[F(|\xi_i|)\mathrm{sgn}(\xi_i)\sin\left(\frac{\phi-\phi_0}{2}\right) e^{jk\rho\cos(\phi-\phi_0)} \right. \\ &\quad \left. +\ F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right) e^{jk\rho\cos(\phi+\phi_0)} \right] \end{aligned} \tag{24}$$

As a result, *extended* TBDW total scattered fields can be obtained as

$$\begin{aligned} U_{Bt}^{(pmc)} &= u_i \left\{ e^{jk\rho\cos(\phi-\phi_0)} u(-\xi_i) + e^{jk\rho\cos(\phi+\phi_0)} u(-\xi_r) \right. \\ &\quad + \left[F(|\xi_i|)\mathrm{sgn}(\xi_i)\sin\left(\frac{\phi-\phi_0}{2}\right) e^{jk\rho\cos(\phi-\phi_0)} \right. \\ &\quad \left.\left. +\ F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right) e^{jk\rho\cos(\phi+\phi_0)} \right] \right\} \end{aligned} \tag{25}$$

by using the Eqs. (5) and (24) for PMC half plane problem.

4. NUMERICAL RESULTS

In this section, *extended* TBDW total scattered fields will be compared with the exact solution of the perfectly magnetic conducting (PMC) half plane problem. The exact solution can be given for PMC half plane problem as [22]

$$U_t^{(pmc)}(P) = 2u_i \sum_{n=0}^{\infty} \frac{\varepsilon_n}{2} e^{jn\pi/4} J_{n/2}(k\rho) \cos\left(\frac{n\phi}{2}\right) \cos\left(\frac{n\phi_0}{2}\right) \tag{26}$$

where $\varepsilon_n = 1$ for $n = 0$, $\varepsilon_n = 2$ for $n \neq 0$.

Figure 3 demonstrates the variation of *extended* TBDW total scattered fields from PMC half plane, given in Eq. (25), and the exact solution of the Helmholtz equation, given in Eq. (26), versus observation angle. Here, u_i is the selected as unit amplitude, $k\rho$ is taken as 30. It is seen from the Figure 3 that the *extended* TBDW scattered fields are very close to the exact solution.

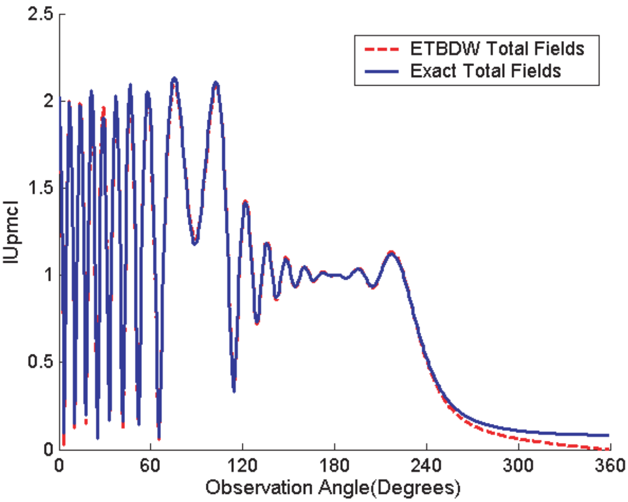

Figure 3. Comparison of total scattered fields from PMC half plane (Oblique incidence $\phi_0 = \pi/3$).

Figure 4 demonstrates the variation of *extended* TBDW total scattered fields, given in Eq. (25), and the exact solution of the Helmholtz equation, given in Eq. (26), versus observation angle. u_i is the selected as unit amplitude, $k\rho$ is taken as 30. It is seen from the Figure 4 that the *extended* TBDW scattered fields are very close

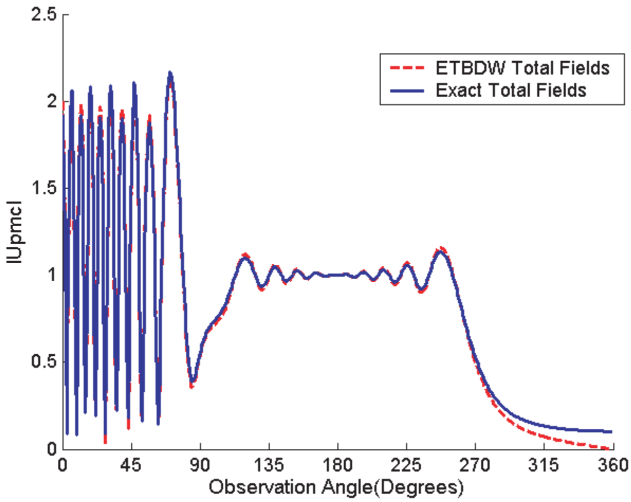

Figure 4. Comparison of total scattered fields from PMC half plane (Normal incidence $\phi_0 = \pi/2$).

to the exact solution. It should be noted that the same observation is valid for all the angles of the edge incidence (ϕ_0) and holds true also for all values of ($k\rho$).

5. CONCLUSION

In this study, the scattered fields from PMC surfaces are examined with the *extended* TBDW. The vector potential of the *extended* TBDW is defined for PMC surfaces. The problem of scattering from a PMC half plane is introduced with the *extended* TBDW. To derive the uniform diffracted fields, the uniform theory of diffraction and the high-frequency asymptotic expansion of the Fresnel function are taken into account. The total uniform scattered fields are then compared numerically with the exact solution of the same problem. It is seen from the numerical results that the *extended* TBDW scattered fields approximate the exact solution successfully.

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