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GEOMETRY BASED PRECONDITIONER FOR RADIATION PROBLEMS INVOLVING WIRE AND SURFACE BASIS FUNCTIONS

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Abstract—An innovative preconditioner has been developed in this work. It significantly improves the convergence of the iterative solvers applied to electromagnetic radiation problems by a renormalization of the matrix equation. The preconditioner balances the disparities in terms of magnitude and units caused by the strong self-coupling of the antennas, the non-uniformity of the meshes and also by the coexistence of wire and surface basis functions. It can be easily integrated into different electromagnetic solvers with a negligible impact on the computational cost on account of its simple implementation.

1. INTRODUCTION

At the present time, the analysis of electromagnetic radiation and scattering problems represents an active topic of research. The Method of Moments (MoM) [1] is a rigorous numerical solution that is commonly employed to deal with a very wide range of such kind of problems. To expand the induced current density on the composite conducting surface-wire structures, the Rao-Wilton-Glisson (RWG) surface basis functions of [2] and its counterparts defined in [3] for wires and wire-to-surface connections are usually applied.

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Even though the MoM is an effective tool in medium-sized analysis, it leads to an excessive cost in terms of storage and time in case of large scale problems. Because of that, many MoM acceleration strategies based on iterative solutions have emerged to reduce the computational complexity. Among these acceleration techniques, it must be pointed out the Fast Multipole Method (FMM) [4] and its multilevel version, the Multilevel Fast Multipole Algorithm (MLFMA) [5–8], which are extensively used at present. Nevertheless, the FMM and, in general, other methods that tackle the electromagnetic analysis within the framework of iterative schemes usually meet with difficulties when dealing with radiation problems. The convergence of the iterative solutions for this kind of problems is often very slow, essentially due to the localized nature of the excitation defined over a small portion of the mesh representing the antenna. It is also caused by the use of non-uniform meshes to accurately model the details in the neighbourhood of the feed point [9]. The convenience of applying effective preconditioners for radiation problems is thus highlighted.

Many research groups have concentrated their efforts on finding effective preconditioners, specially when the electric field integral equation (EFIE) is applied [8]. Some of the most common techniques are those based on the sparse approximate inverse (SAI) preconditioning [10–12] and the incomplete LU (ILU) factorization type preconditioning [8, 13, 14]. The former have more natural parallelism than the latter, which also present well-known instability problems that sometimes can be overcome by pivoting [15]. Recent works have proposed the preconditioning of the ordinary equation with the transpose complex conjugate of the impedance matrix [16], a multiplicative preconditioner using Calderon identities [17] or a localized preconditioner in the vicinity of a radiation problem antenna [9]. The preconditioning scheme presented in this work acts on three main issues that are responsible for the bad convergence of radiation problems: i) it moderates the strong dominance of the self-coupling of wire or surface antennas; ii) it weights the impedance matrix and the current and voltage vectors to normalize the respective units of wire and surface elements; iii) it moderates the disparities of the impedance matrix values due to the disparate mesh size. These actuations lead to a better guided iterative solution, allowing the FMM to provide accurate predictions for very sensitive radiation parameters such as the input impedance or the mutual couplings, usually pursued when dealing with this kind of problems. Moreover, the simplicity of this scheme and the fact that it can be applied with independence of the solver implementation details are also outstanding features.

The paper is organized as follows: Section 2 reviews the critical concerns of the radiation problems; the preconditioner formulation is detailed, and an improved and more realistic approach for the determination of convergence of the iterative algorithm is provided. Canonical and practical radiation results are presented in Section 3 and, finally, the conclusions are summarized in Section 4.

2. PRECONDITIONER FORMULATION

As it is detailed in [9], radiation problems give rise to an impedance matrix ill-conditioned with disparate eigenvalues. This kind of problems are characterized by non-uniform meshes, non-uniform excitation vector elements and also non-uniform coupling terms. Due to these particular features, the attained accuracy when employing whichever iterative solver in the FMM code is usually worse for radiation than for scattering solutions. The mutual coupling and other electromagnetic parameters of interest are especially sensitive to the bad convergence of the radiation analysis.

Considering a general radiation problem, the presence of a strong dominance of the antenna self-coupling terms is easily detectable from the observation of the impedance matrix. Under these conditions, large values will define the search directions of an iterative solver such as GMRES [18]. This occurs for both wire or surface antennas, but further conclusions can be extracted when wire antennas are present. In those cases, it may be observed that the wire self-coupling dominance is emphasized as the frequency increases, which suggests the existence of a mesh-size dependence on the impedance matrix elements corresponding to the surface basis functions. That dependence is given by the surface basis common edge length of the RWG surface basis function, and it has been reported in [19] to account for the antenna impedance in small thin strip antennas. An alternative reading may be obtained by focusing the size dependence as a coupling imbalance between wire and surface subdomains. This evidence is the foundation of our preconditioner formulation. The developed preconditioner performs a units harmonization that provides a better balanced system, which in turn leads to an improved convergence of the iterative solver. The details of the formulation are described next.

2.1. Preconditioned Matrix System

The composite surface-wire structure is defined in terms of surface and wire basis functions. The normalization of the involved units starts

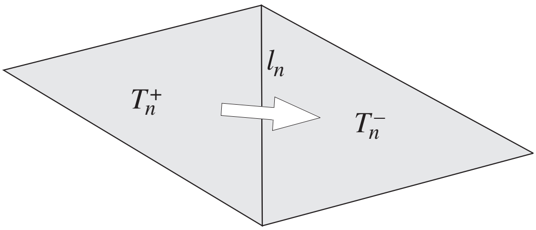

Figure 1. RWG surface basis function. The arrow represents de current flow direction (from triangle T_n^+ to triangle T_n^-).

with the following matrix definition:

$$\mathbf{L} = \mathrm{diag}(l_1, l_2, \ldots, l_n, \ldots, l_N), \tag{1}$$

where N is the total number of unknowns. l_n is the common edge length of the n-th RWG surface basis function, as illustrated in Figure 1, or $l_n = 1$ if it is a wire basis function. $\mathbf{L}$ is a diagonal matrix with non-zero determinant ($l_n \neq 0, \forall n$), and consequently non singular, being $\mathbf{L}\mathbf{L}^{-1} = \mathbf{L}^{-1}\mathbf{L} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Then, the original matrix system of the form $\mathbf{Z}\boldsymbol{I} = \boldsymbol{V}$ may be modified as:

$$\mathbf{L}\mathbf{L}^{-1}\mathbf{Z}\mathbf{L}^{-1}\mathbf{L}\boldsymbol{I} = \boldsymbol{V}, \tag{2}$$

$$\mathbf{L}^{-1}\mathbf{Z}\mathbf{L}^{-1}\mathbf{L}\boldsymbol{I} = \mathbf{L}^{-1}\boldsymbol{V}. \tag{3}$$

By defining the following expressions:

$$\boldsymbol{V}_{eq} = \mathbf{L}^{-1}\boldsymbol{V};\ \boldsymbol{I}_{eq} = \mathbf{L}\boldsymbol{I};\ \mathbf{Z}_{eq} = \mathbf{L}^{-1}\mathbf{Z}\mathbf{L}^{-1}, \tag{4}$$

an equivalent matrix equation can be straightforwardly identified, as expressed below:

$$\mathbf{Z}_{eq}\boldsymbol{I}_{eq} = \boldsymbol{V}_{eq}. \tag{5}$$

In the equivalent system, the original impedance matrix $\mathbf{Z}$ is left and right preconditioned, and current and voltage coefficients are conveniently weighted to obtain an equation with normalized units. To be more precise, the new system is expressed in the impedance, current and voltage standard units, i.e., Ohms, Amperes and Volts, respectively, while in the original one these units are combined with Ohms $\cdot$ m^2, Amperes/m and Volts $\cdot$ m due to the surface basis formulation and the method implementation. By correcting the units and magnitudes disparities in the preconditioned system, the GMRES is properly driven and the convergence issue associated to radiation problems is overcome in a simple, low-cost and effective manner. The preconditioner numerical complexity is $O(N)$ with a very low leading constant, and it can be easily parallelized. Besides, the impedance matrix symmetry obtained when using the EFIE and the Galerkin's testing procedure is maintained with this preconditioner, which may imply some implementation advantages.

2.2. Weighted System Residue

The residue norm is usually employed as a convergence measure of the iterative solution. For the original system it can be found as follows:

$$r_o = \frac{\| \boldsymbol{V} - \mathbf{Z}\boldsymbol{I}\|}{\| \boldsymbol{V}\|}, \tag{6}$$

which will be referred as *original* residue.

A standardization procedure must be also applied to the calculation of this residue norm. The coupling magnitude imbalance and the units disparity due to the combination of wire and surface subdomains make the norm calculation meaningless, because it will be dominated by the wire elements, regardless of the surface contributions. To overcome this issue, a more *realistic* residue with the units normalized to Volts must be defined. It can be evaluated as follows:

$$r_w = \frac{\left\|\mathbf{L}^{-1} \left(\boldsymbol{V} - \mathbf{Z}\boldsymbol{I}\right)\right\|}{\|\mathbf{L}^{-1} \boldsymbol{V}\|}. \tag{7}$$

This residue calculation (hereinafter, *weighted* residue) defines the iterative solver convergence rate properly for the original system, thus allowing a consistent comparison between the alternative solutions.

3. NUMERICAL RESULTS

In this section, different numerical examples have been included in order to demonstrate the improvements derived from the utilization of the proposed preconditioner. The EFIE formulation has been adopted in the FMM code and a restarted GMRES solver has been used in this work (although the proposed preconditioner is general, and it could be applied to any other surface integral equation formulation or iterative method.) A group size of 1λ and 16 multipole terms have been considered in all the examples. The delta-gap model has been applied to obtain the excitations of the antenna sources.

The first tested geometry consists on a hollow cylinder of 12 160 surface unknowns and a $\lambda/2$-length dipole pair modeled employing 10 wire basis functions. The cylinder dimensions and the position

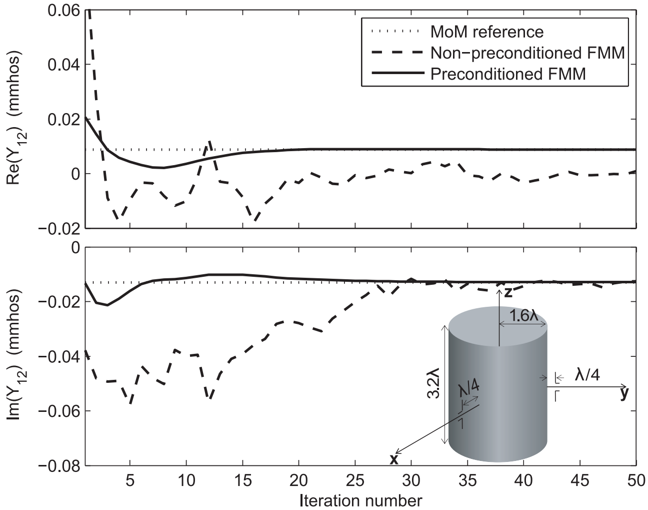

Figure 2. Real (up) and Imaginary (down) parts of Y_{12}. Prediction for the cylinder and the dipole pair of both non-preconditioned and preconditioned FMM with regard to MoM reference.

of dipoles are indicated in Figure 2. The frequency of the analysis is 30 GHz and the results have been obtained after 50 iterations of the GMRES (restart 25). The estimation of the mutual admittance Y_{12} provided by a reference MoM code and both non-preconditioned and preconditioned FMM approaches is shown in Figure 2. The MoM solution has been obtained by factorizing and solving the matrix system. Looking at the representation of Y_{12}, it is clear that the non-preconditioned FMM prediction comes up slowly to the reference value without reaching it even after the allowed number of iterations. Instead, a better agreement with the reference result is achieved in only a few iterations with the preconditioned FMM method.

Both original and weighted residues of Equations (6) and (7), respectively, are plotted in Figure 3. This figure clearly illustrates the chaotic behavior of the original residue in contrast to the smooth curves of the weighted one. In addition, it can be observed that the weighted residue performance is directly correlated with the ability of the proposed preconditioner to provide an improved guide to the iterative solver, unlike the original residue. This lack of correlation between the original residue and the accuracy of the final result becomes clearer looking at Figures 2 and 3. From the original residue representation of Figure 3, we may think that a better result is being obtained with the non-preconditioned equation for the first 10 iterations. However,

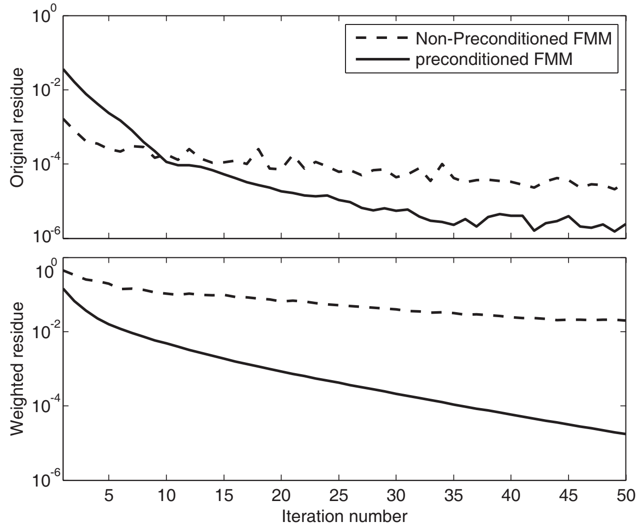

Figure 3. Original (up) and weighted (down) residues of both non-preconditioned and preconditioned FMM for the cylinder and the dipole pair.

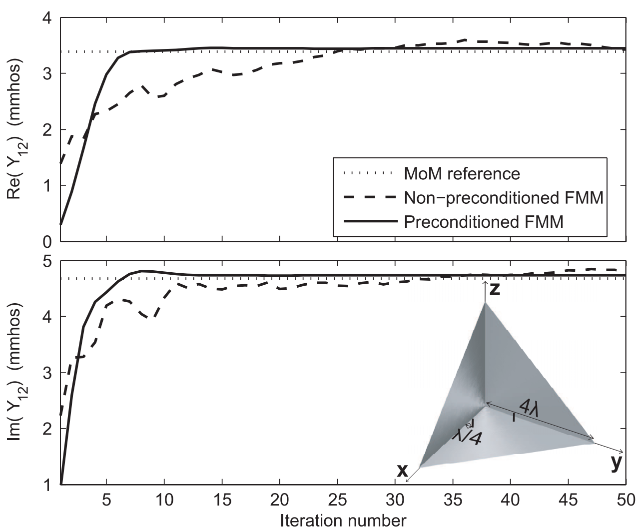

Figure 4. Real (up) and Imaginary (down) parts of Y_{12}. Prediction for the corner reflector and the monopole pair of both non-preconditioned and preconditioned FMM with regard to MoM reference.

looking at Figure 2, it is clear that this is a false perception: the mutual coupling prediction is more accurate using the preconditioned equation. In contrast, the weighted residue of Figure 3 shows a highly correlated behavior with the actual situation reflected by the mutual admittance result. Herein, the weighted residue is a more eligible candidate to predict the convergence of the iterative resolution of the system.

In order to give insight on the preconditioner behavior, a PEC trihedral corner reflector with three mutually perpendicular triangular sides has been considered next. Two $\lambda/4$ wire monopoles have been placed over the geometry base as shown in Figure 4. The frequency of the simulation and the GMRES parameters are the same as in the preceding example. The total number of unknowns is made up of 18 336 surface basis functions, 4 wire basis and 2 junction basis. While in the previous analysis the cylinder reduces the mutual coupling between the antennas, in this example the presence of the trihedral structure reinforces it. Under these conditions, the preconditioned FMM method is still providing a better prediction of the mutual admittance. As it can be observed in Figure 4, the result of the preconditioned technique shows better concordance with the reference data. Its solution becomes stable after a few iterations, unlike the non-preconditioned approach, where visible fluctuations around the reference value can be noticed. According to this result and the residues shown in Figure 5, it becomes clear again that the weighted residue is a more advantageous indication

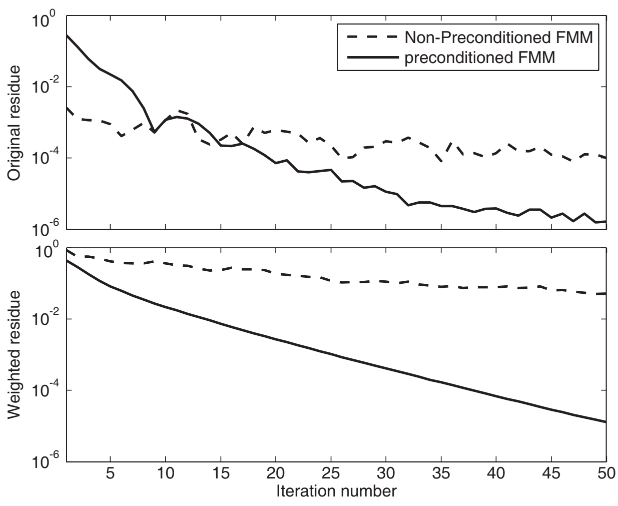

Figure 5. Original (up) and weighted (down) residues of both non-preconditioned and preconditioned FMM for the corner reflector and the monopole pair.

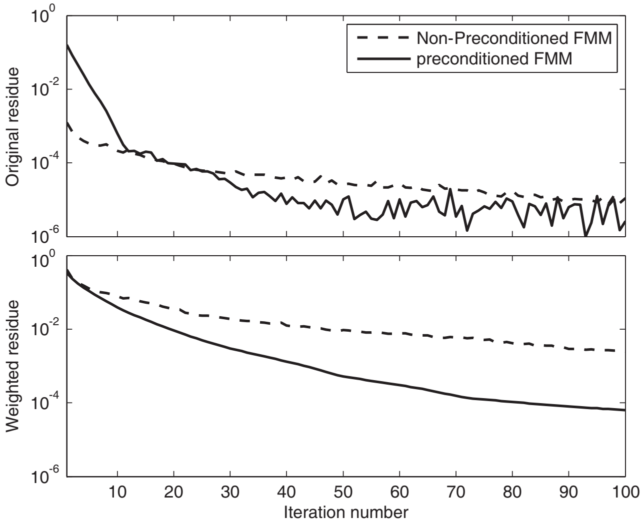

Figure 6. Original (up) and weighted (down) residues of both non-preconditioned and preconditioned FMM for the truck cab.

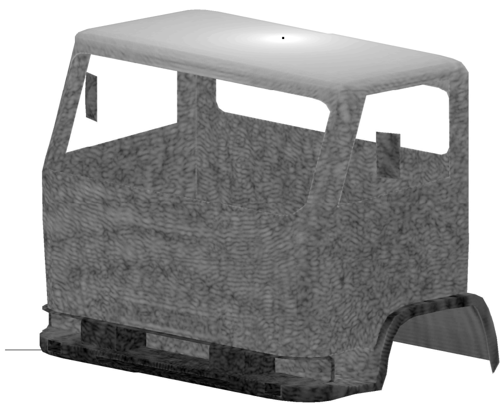

Figure 7. Current density over the truck cab surface obtained with the preconditioned method.

of both the convergence rate and the accuracy of the result. The validity of the proposed procedure for problems involving wire, surface and also junction basis functions is confirmed with this example.

Finally, the analysis at 9 GHz of a truck cab with more than two million unknowns is presented (2 285 398 surface basis, 4 wire basis and 1 junction basis). A monopole antenna placed on the cab roof has been considered as the radiating element. It is a more realistic example that involves surface, wire and junction basis functions. 100 extern iterations of the GMRES and a restart parameter of 100 have been considered in this simulation. As it is shown in Figure 6, the use of the preconditioner entails a significant improvement of the convergence rate. The surface current density pattern obtained with the preconditioned FMM method is represented in Figure 7.

4. CONCLUSIONS

It has been presented in this work a preconditioner that stands out for its simple implementation and its ability to overcome the limitations usually attributed to the FMM in radiation problems. The preconditioner reserves the impedance matrix symmetry given by the EFIE formulation and it has a negligible impact on the global cost, despite of the method employed to obtain the solution. In fact, it has an $O(N)$ numerical complexity and it could be straightforwardly parallelized. The proposed preconditioning technique is not supported only by empirical observations. It is based on the existing units imbalance associated to the coexistence of wire and surface basis functions, and also on the coupling magnitude disparities, related to the non-uniformity of the meshes and the excitation of radiation problems.

The results corresponding to challenging radiation problems have shown that the preconditioned method provides faster convergence and better agreement with references than the non-preconditioned one. Moreover, the proposed weighted residue has demonstrated to be a well-suited measure for the convergence rate, correlated with the accuracy of the solution. Then, the full process represents an effective tool to obtain an improved convergence in the FMM analysis of radiation problems.

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REFERENCES

1. Harrington, R. F., *Field Computation by Moment Method*, IEEE Press, New York, 1993.
2. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, Vol. 30, No. 3, 409–418, 1982.
3. Hwu, S. U., D. R. Wilton, and S. M. Rao, "Electromagnetic scattering and radiation by arbitrary conducting wire/surface configurations," *IEEE Int. Antennas and Porpagat. Symp.*, Vol. 26, 890–893, 1998.
4. Coifman, R., V. Rokhlin, and S. Wanzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas Propagat. Mag.*, Vol. 35, No. 3, 7–12, 1993.
5. Song, J. M., C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propagat.*, Vol. 45, 1488–1493, 1997.
6. Pan, X. M. and X. Q. Sheng, "A highly efficient parallel approach of multi-level fast multipole algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 8, 1081–1092, 2006.
7. Wang, P. and Y. J. Xie, "Scattering and radiation problem of surface/surface junction structure with multilevel fast multipole algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 15, 2189–200, 2006.
8. Fan, Z. H., D. Z. Ding, and R. S. Chen, "The efficient analysis of electromagnetic scattering from composite structures using hybrid CFIE-IEFIE," *Progress In Electromagnetics Research B*, Vol. 10, 131–143, 2008.
9. Hesford, A. J. and W. C. Chew, "On preconditioning and the eigensystems of electromagnetic radiation problems," *IEEE Trans. Antennas Propagat.*, Vol. 56, No. 8, 2413–2420, 2008.
10. Lee, J., J. Zhang, and C.-C. Lu, "Sparse inverse preconditioning of multilevel fast multipole algorithm for hybrid integral equations in electromagnetics," *IEEE Trans. Antennas Propagat.*, Vol. 52, No. 9, 2277–2287, 2004.
11. Rui, P. L. and R. S. Chen, "An efficient sparse approximate inverse preconditioning for FMM implementation," *Microwave and Optical Technology Letters*, Vol. 49, No. 7, 1746–1750, 2007.
12. Ding, D. Z., R. S. Chen, and Z. Fan, "An efficient sai preconditioning technique for higher order hierarchical MLFMM implementa-

tion," *Progress In Electromagnetics Research*, PIER 88, 255–273, 2008.

13. Sertel, K. and J. L. Volakis, "Incomplete LU preconditioner for FMM implementation," *Microwave and Optical Technology Letters*, Vol. 26, No. 7, 265–267, 2000.
14. Heldring, A., J. M. Rius, and L. Ligthart, "New block ILU preconditioner scheme for numerical analysis of very large electromagnetic problems," *IEEE Trans. Magn.*, Vol. 38, No. 2, 337–346, 2002.
15. Benzi, M., "Preconditioning techniques for large linear systems: A survey," *J. Comput. Phys.*, No. 182, 418–477, 2002.
16. Ergül, Ö. and L. Gürel, "Efficient solution of the electric-field integral equation using the iterative LSQR algorithm," *IEEE Antennas Wireless Propagat. Lett.*, Vol. 7, 36–39, 2008.
17. Andriulli, F. P., et al., "A multiplicative calderon preconditioner for the electric field integral equation," *IEEE Trans. Antennas Propagat.*, Vol. 56, No. 8, 2398–2412, 2008.
18. Saad, Y., *Iterative Methods for Sparse Linear Systems*, PWS Publishing Company, a division of International Thomsom Publishing Inc., Boston, MA, 1996.
19. Makarov, S., "MoM antenna simulations with matlab: RWG basis functions," *IEEE Antennas Propagat. Mag.*, Vol. 43, No. 5, 100–107, 2001.