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NEAR FIELD OF SLOT RADIATORS

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Abstract—In this paper, the cosinusoidal slot aperture distribution is replaced with a two term approximation. Using this two term approximation, the slot fields are evaluated in closed form and explicit expressions are given in terms of sine and cosine integrals. The two term approximation and the near fields derived therefrom agree closely with the cosinusoidal distribution for slot lengths upto 0.65λ, with an error of less than 3.3% with respect to the numerical results for distances $R_{nearest} \geq 0.05\lambda$ from the slot in the near field, where $R_{nearest}$ is the distance of the nearest point on the slot from the field point. The formulation given here is of practical use in estimating mutual coupling in an array or in estimating radiated emissions for Electromagnetic Compatibility (EMC) analysis.

1. INTRODUCTION

Slots are commonly used as radiating elements in antennas or antenna arrays. Mutual coupling between slots needs to be taken into account for accurately predicting the performance of slotted array antennas. In Electromagnetic Compatibility (EMC) analysis, radiation from slots needs to be taken into account for estimating leakage and interference from apertures used for ventilation, seams, display windows etc. In Radiated Emissions measurement, fields radiated from an Equipment Under Test (EUT) are often measured at a distance of 1 m or 3 m, over a frequency range where the far field criterion may not be fully satisfied.

The near field of dipoles with piecewise sinusoidal (PWS) distribution of the form $\sin[k(l-|z|)]$, where $2l$ is the length of the slot and k is the wavenumber, is well known [1, 2]. The near field for similar

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magnetic current distribution in the slot can be obtained by duality [3]. However, it is known that the aperture electric field or magnetic current in the slot is closer to the cosinusoidal distribution of the form $\cos(\frac{\pi}{2l}z)$ [4]. Hence, slots are quite often analysed using the Method of Moments (MoM) with basis functions of the form $\sin[\frac{p\pi}{2l}(l+z)]$ for the aperture magnetic current distribution [5–8]. Although it is known that the far field for such slots with cosinusoidal aperture distribution can be readily evaluated [3], closed form expressions for the near field of slots with cosinusoidal distribution are not available in the literature. Such closed form or analytic expressions help to provide useful insight into the behaviour of the slot by highlighting the singularities or zeroes, by allowing differentiation to provide variation with respect to a particular parameter or for variational solutions, by permitting integration and so on. At the same time, slower and expensive numerical techniques with attendant convergence related issues and the choice of proper method can be avoided.

In the following, the well known cosinusoidal aperture distribution of slots has been approximated by a two term approximation to facilitate the evaluation of near fields in closed form, and the near fields are evaluated in terms of sine and cosine integrals.

2. FORMULATIONS AND EQUATIONS

The geometry of the slot configuration with the co-ordinate system is illustrated in Fig. 1.

The slot is assumed to be along the z-axis in the yz plane. The slot

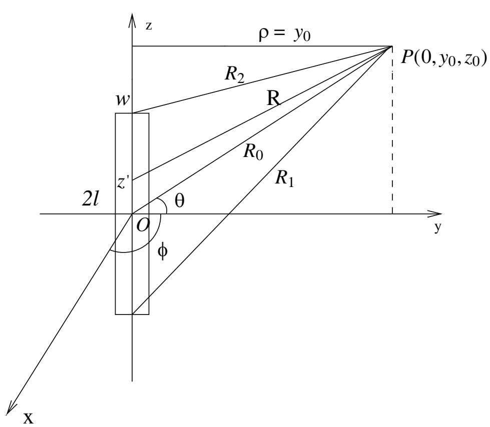


Figure 1. Geometry of the problem.

is assumed to be of length $2l$ and width w. The width w is assumed to be much less than length, i.e., narrow slot approximation is employed. As shown in [9], for $w/2l < 10$, this introduces negligible error in the coupling amplitude and phase, while for wider slots, accurate results can be obtained with two transverse integrations along each slot width. Hence, the slot can be replaced by an equivalent linear magnetic current source having a field pattern with azimuthal symmetry similar to a dipole. Thus, the field point can be conveniently located in the yz plane ($\phi = \pi/2$), without loss of generality.

In Fig. 1, the centre of the slot is at the origin $O(0,0,0)$ and the observation point P is at $(0, y_0, z_0)$. $P \equiv (y_0, \pi/2, z_0)$ in cylindrical co-ordinates and $P \equiv (R_0, \pi/2 - \theta, \pi/2)$ in spherical co-ordinates. The centre to field point distance is R_0.

Then the total field at P,

$$H_t = H_z \bar{z} + H_\rho \bar{\rho} \tag{1}$$

The magnetic current in the slot $M = (E_s \times n) w = cos(z\pi/2l)\bar{z}$ [4], where E_s is the slot electric field, n is the outward normal to the slot surface and $\bar{z}$ and $\bar{\rho}$ are unit vectors along z and ρ respectively. The aperture electric field is assumed to be uniform across the width. The phase of the electric field is assumed to be uniform over the slot. For more general aperture distributions represented as a summation of higher order basis functions $E_s = \sum_{p=1}^{N} \sin[\frac{p\pi}{2l}(l+z)]$, e.g., [7, 8], the following analysis can be easily applied using superposition.

Then, following the procedure in [1, 9],

$$H_z = \int_{-l}^{l} \frac{M}{j\eta k} \left(\frac{\partial^2}{\partial z^2} + k^2 \right) G dz' \tag{2}$$

$$H_\rho = H_y = \int_{-l}^{l} \frac{M}{j\eta k} \frac{\partial^2 G}{\partial z \partial y} dz' \tag{3}$$

$$E_\phi = -E_x = \int_{-l}^{l} M \frac{\partial G}{\partial y} dz' \tag{4}$$

where $k = 2\pi/\lambda$, λ is the free space wavelength, $\eta = 120\pi$ is the free space impedance, $G = G(R) = \frac{e^{-jkR}}{2\pi R}$ is the Green's function for a slot in an infinite conducting ground plane and $R = \sqrt{(z - z')^2 + \rho^2}$.

Integrating Equation (2) by parts and recalling that $M(\pm l) = 0$, we get

$$H_z = \frac{1}{j\eta k} \left\{ \left[\frac{\pi}{2l} \sin\left(\frac{\pi}{2l} z \right) G \right]_{-l}^{l} + \int_{-l}^{l} \left[k^2 - \left(\frac{\pi}{2l} \right)^2 \right] \cos\left(\frac{\pi}{2l} z \right) \frac{e^{-jkR}}{2\pi R} dz' \right\} \tag{5}$$

Equation (5) cannot be evaluated in closed form. The same is true of Equation (3) or Equation (4).

At the same time, it is well known that dipole impedance with certain approximations for current can be evaluated in closed form [10, 11]. Here, we propose a two term approximation after [10] for the magnetic current (or slot electric field). This leads to closed form expressions for the slot near field as shown next.

2.1. Two Term Approximation for Aperture Distribution

A two term approximation similar to that used in [10] is used for slot magnetic current distribution

$$M(z') = \cos\left(\frac{\pi}{2l}z'\right) \approx a_1\left\{\sin\left[k(l-|z'|)\right] + a_0 k(l-|z'|)\cos[k(l-|z'|)]\right\} \quad (6)$$

However, whereas the constant in [10] is evaluated from a variational analysis subject to the condition of variation in impedance Z_i becoming a minimum, here we approximate the current to a cosinusoid and the constants a_1, a_0 are evaluated subject to

$$M(0) = 1 \qquad \text{and} \qquad \frac{\partial M}{\partial z'}|_{z'=0} = 0$$

Then,

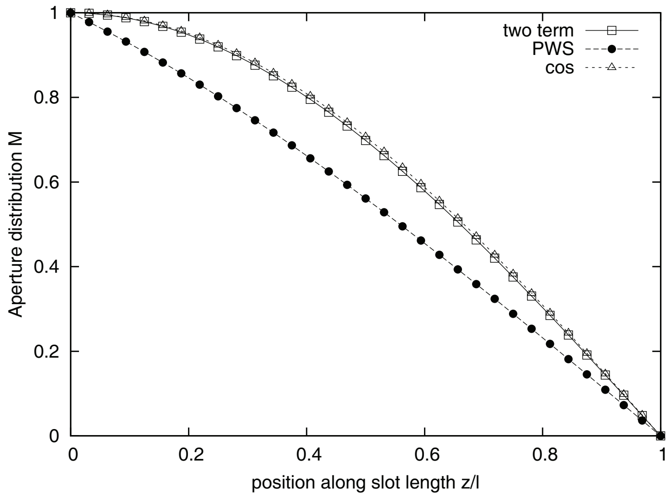
$$a_0 = \frac{1}{kl\tan(kl) - 1} \quad (7)$$

$$a_1 = \frac{1}{\sin(kl) + a_0 kl\cos(kl)} \quad (8)$$

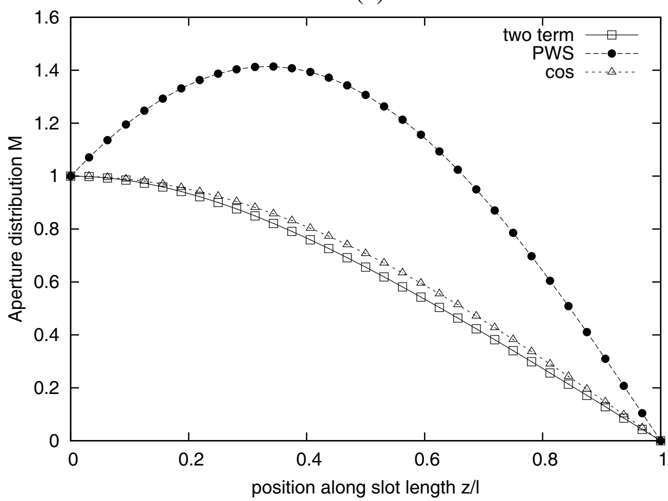
The slot aperture distribution using this approximation is illustrated in Fig. 2 for slot length $2l = 0.3\lambda$ and $2l = 0.75\lambda$ along with the cosinusoidal case. It can be seen that the two distributions are very close to each other. As the practical slot lengths are usually between 0.3λ to 0.65λ [9], mostly slot lengths around these values are taken for illustration purpose in the figures, as being representative of one below and one above $\lambda/2$. The fields for Equation (6) can be evaluated in closed form as shown next.

2.2. Near Field of Slots with Two Term Approximation

Using the two term approximation Equation (6) for slot magnetic current distribution M in Equation (2) and observing that $\frac{\partial}{\partial z} = -\frac{\partial}{\partial z'}$,



(a)



(b)

Figure 2. Aperture distribution for cosinusoidal, piecewise sinusoidal and two term approximation with, (a) slot length $2l = 0.3\lambda$, (b) slot length $2l = 0.75\lambda$.

$\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z'^2}$

$$H_z = \frac{1}{j\eta k}\left\{-\left[\frac{\partial M}{\partial z'}G\right]_{-l}^{l} + \int_{-l}^{l}\left(k^2 + \frac{\partial^2}{\partial z'^2}\right) M \cdot G dz'\right\} \tag{9}$$

This can be shown to be

$$H_z = \frac{1}{j\eta k}\left\{-\left[\frac{\partial M}{\partial z'}\left(G^- - G^+\right)\right]_0^l + \left(-2a_1a_0k^2\right)I\right\} \tag{10}$$

where $G^{\pm} = G(\sqrt{(z \pm z')^2 + y^2})$ and

$$\begin{aligned} I &= \int_0^l \sin[k(l-z')]\cdot(G^- + G^+)dz' && (11)\\ &= \frac{1}{2\pi}\sum_{p=-1}^{1'}\sum_{q=-1}^{1'}\frac{e^{jkq(z_0p+l)}}{2j}\{E_1[kt]\}_0^l && (12)\end{aligned}$$

$z'' = z' + pz_0$, $r = \sqrt{z''^2 + y_0^2}$, $t = r + qz''$ and $E_1(u) = Ci(|u|) - jSi(u)$, where $Ci(u)$ and $Si(u)$ are, respectively, the cosine and sine integrals defined in [12]. The primed symbol $\sum_i'$ indicates that the variable i does not take the value $i = 0$. Then,

$$\begin{aligned} H_z = \frac{1}{j\eta k}\Bigg\{&-\left[\frac{\partial M}{\partial z'}\left(G^- - G^+\right)\right]_0^l \\ &-\frac{2a_1a_0k^2}{2\pi}\sum_{p=-1}^{1'}\sum_{q=-1}^{1'}\frac{e^{jkq(z_0p+l)}}{2j}[E_1(kt)]_0^l\Bigg\} \end{aligned} \tag{13}$$

Similarly, from Equation (3),

$$H_\rho = H_y = \frac{-1}{j\eta k}\frac{\partial}{\partial y}\int_0^l \sum_{p=-1}^{1'} M\frac{\partial}{\partial z'}G(r)dz' \tag{14}$$

Integrating by parts and since $M(\pm l) = 0$

$$H_\rho = \frac{1}{2\pi j\eta k}\frac{\partial}{\partial y}\int_0^l \sum_{p=-1}^{1'}\frac{\partial M}{\partial z'}G(r)dz' \tag{15}$$

$$H_\rho = \frac{1}{2\pi j\eta k}\sum_{p=-1}^{1'}\sum_{q=-1}^{1'}(-p)\left[a_cC' + b_cC_z'\right] \tag{16}$$

where

$$\begin{aligned} C' &= \frac{\partial}{\partial y}\int_0^l e^{jk(z_0p+l)}e^{-jk(z'+z_0p)}.\frac{e^{-jkr}}{r}dz' && (17)\\ C_z' &= \frac{\partial}{\partial y}\int_0^l e^{jk(z_0p+l)}(z'+pz_0-pz_0)e^{-jk(z'+z_0p)}\frac{e^{-jkr}}{r}dz' && (18)\end{aligned}$$

and $a_c = [-ka_1(1+a_0) - j(a_1 a_0 k^2 l\, q)]/2$ and $b_c = jqa_1 a_0 k^2/2$. In the above, z' has been written as $z' + pz_0 - pz_0$ for the ease of further analysis. Then,

$$C' = \left[qe^{jkq(z_0 p+l)} \frac{y_0}{r} \frac{e^{-jkt}}{t} \right]_{z'=0}^{l} \tag{19}$$

Using ([13], Equation (18)), it can be shown that,

$$C_z' = I_1 - pz_0 C' \tag{20}$$

$$I_1 = e^{jkq(z_0 p+l)} y_0 \left\{ \frac{e^{-jkt}}{r} + jkE_1(kt) \right\}_{z'=0}^{l} \tag{21}$$

H_ρ can be written in a compact form suitable for programming as

$$H_\rho = \frac{1}{2\pi j\eta k} \sum_{p=-1}^{1'} \sum_{q=-1}^{1'} (-p) \left[(a_c - b_c p z_0) C' + b_c I_1 \right]. \tag{22}$$

This can also be written in a form similar to dipole fields with $R_1 = \sqrt{(z_0+l)^2 + y_0^2}$, $R_2 = \sqrt{(z_0-l)^2 + y_0^2}$ (Fig. 1).

$$\begin{aligned} H_\rho = \frac{1}{j2\pi\eta k} \Bigg\{ & \frac{-ka_1(1+a_0)}{y_0} \left[(z_0+l)\frac{e^{-jkR_1}}{R_1} + (z_0-l)\frac{e^{-jkR_2}}{R_2} \right. \\ & \left. -2z_0 \cos(kl) \frac{e^{-jkR_0}}{R_0} \right] + \frac{ja_1 a_0 k^2}{y_0} \left[(z_0+l)e^{-jkR_1} + (z_0-l)e^{-jkR_2} \right. \\ & \left. -2z_0 \cos(kl) e^{-jkR_0} + j2lz_0 sin(kl) \frac{e^{-jkR_0}}{R_0} \right] \\ & + \frac{a_1 a_0 k^3 y_0}{2} \sum_{p=-1}^{1'} \sum_{q=-1}^{1'} pqe^{jkq(z_0 p+l)} [E_1(kt)]_0^l \Bigg\} \end{aligned} \tag{23}$$

Proceeding as above from Equation (4), it can be shown that

$$E_\phi = -E_x = \frac{1}{2\pi} \sum_{p=-1}^{1'} \sum_{q=-1}^{1'} \left[(f_c - g_c p z_0) C' + g_c I_1 \right] \tag{24}$$

where $f_c = [-jqa_1 + a_1 a_0 kl]/2$ and $g_c = -a_1 a_0 k/2$. Or

$$\begin{aligned} E_\phi \ = \ & \frac{1}{2\pi}\left\{\frac{-ja_1}{y_0}\left[e^{-jkR_1}+e^{-jkR_2}-2\cos(kl)e^{-jkR_0}\right]-\frac{a_1a_0k}{y_0}\right. \\ & \left[R_1e^{-jkR_1}+R_2e^{-jkR_2}-2R_0\cos(kl)e^{-jkR_0}+j2l\sin(kl)e^{-jkR_0}\right] \\ & \left.-\frac{ja_1a_0k^2y_0}{2}\sum_{p=-1}^{1'}\sum_{q=-1}^{1'}e^{jkq(z_0p+l)}[E_1(kt)]_0^l\right\} \end{aligned} \quad (25)$$

The H_z, H_ρ and E_ϕ components determine the near field of slot radiators in closed form. By comparing with [1], it can be seen that the first part of each of the fields H_z (Equation (13)), H_ρ (Equation (23)) and E_ϕ (Equation (25)) is from the dipole distribution $\sin[k(l-|z|)]$.

The above fields can be further corrected for the difference in the two distributions by using a multiplicative constant γ [9], that is a ratio of the first moment of the two distributions.

$$\gamma = \frac{2kl}{\pi\{a_1(1-a_0)[1-\cos(kl)]+a_1a_0kl\sin(kl)\}} \quad (26)$$

The near fields for higher order modes in the slot or for higher basis functions $\sin[\frac{p\pi}{2l}(l+z)]$ can be found by superposition.

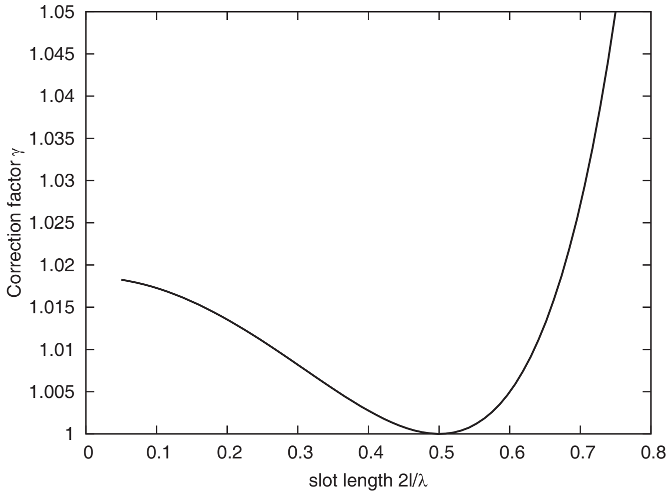


Figure 3. Correction factor for two term approximation.

3. NUMERICAL RESULTS

The two term approximation has been plotted in Fig. 2 for $2l = 0.3\lambda$ and $2l = 0.75\lambda$. The PWS and cosinusoidal distributions are also plotted in the figure. The cosinusoidal distribution and the two term approximation can be seen to be very close to each other. The PWS distribution differs significantly from the cosinusoidal, both for slot lengths lesser than as well as greater than 0.5λ.

The correction factor γ for two term approximation, that is the ratio of the first moments of the two distributions is plotted in Fig. 3. The correction factor is almost one for slot lengths shorter than 0.5λ, showing excellent agreement between the approximate and ideal distributions, while it can be seen to depart from unity rapidly above 0.5λ. The correction required is less than 2% up to a slot length of 0.65λ and less then 5% upto $2l = 0.75\lambda$.

The relative error in near H-field magnitude $(\sqrt{|H_z|^2 + |H_\rho|^2})$ obtained using numerical integration of Equations (2), (3) and that obtained analytically from Section 2.2 is plotted in Fig. 4 over $0° \leq \theta \leq 90°$ for $2l = 0.3\lambda$ and $2l = 0.65\lambda$ with reference to the numerically evaluated field magnitude at $R_0 = 0.65\lambda$ and $R_0 = 13\lambda$, i.e., at one and 20 slot lengths respectively. The correction factor is taken into account while plotting the above. The numerical results are evaluated by dividing the slot into cells and employing three point gaussian quadratue over each cell. It is seen that the error remains less than 1% over the entire angular range $0° \leq \theta \leq 90°$, for slot length $2l \leq 0.65\lambda$ at distance $R_0 \geq 2l$. The error increases for slot length $2l > 0.5\lambda$ where the approximate aperture distribution differs more significantly from the cosinusoidal one. The error is more for wider angles as compared to $\theta = 0°$. For wider angles (e.g., $\theta = 90°$), the error can be seen to increase with distance in the near field, but remains constant in the far field (around 0.6% for $\theta = 90°$) as seen from Fig. 5.

The error calculated for PWS [9] and two term approximations is shown in Fig. 6 for $2l = 0.3\lambda$ and $2l = 0.65\lambda$ at a distance of $2l$ and $40l$ respectively. It can be seen that the two term approximation results agree better with the numerically computed fields from cosinusoidal distribution.

To explore the error in the near field, the field is calculated at $\rho = 0.05\lambda$ parallel to the slot $(0 \leq z \leq l\ )$ for slot length up to 0.65λ. The error in both, H field and E field is shown in Fig. 7(a). The error is seen to remain less than 3.3% for magnetic field and less than 1.6% for electric field from the figure. Thus, in general, with this two term approximation and the above formulation, the error in calculated field is less than 3.3% with respect to the assumed cosinusoidal one for slot

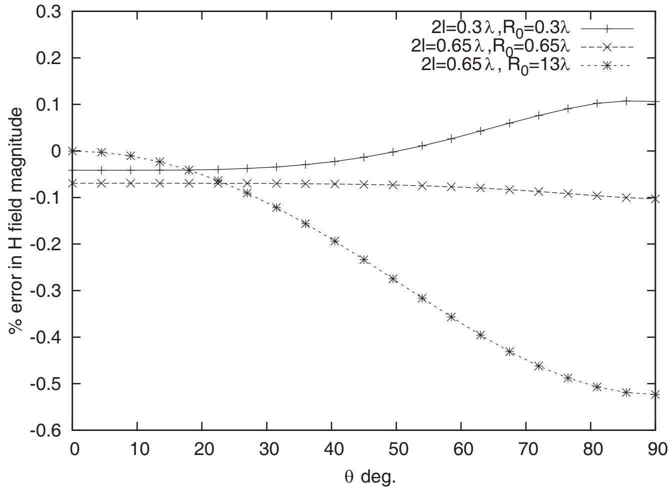


Figure 4. % error in magnitude of near magnetic field calculated from two term approximation for slot length $2l = 0.3\lambda$ and $2l = 0.65\lambda$.

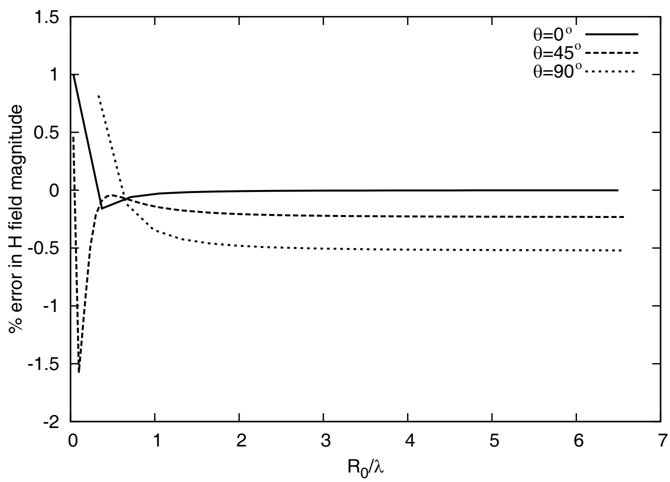


Figure 5. Variation of % error in magnitude of magnetic field calculated from two term approximation with distance R_0/λ for slot length $2l = 0.65\lambda$ at various angles θ.

length up to 0.65λ at all distances $R_{nearest} \geq 0.05\lambda$, where $R_{nearest}$ is the distance of the nearest point on the slot from the field point P. The variation in error with position along the slot for calculated magnetic

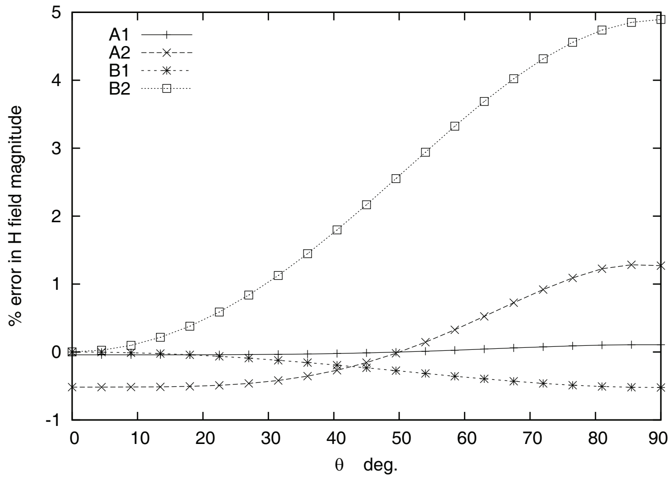


Figure 6. Variation in % H field magnitude for PWS and two term approximation, A1: $2l = 0.3\lambda, R_0 = 0.3\lambda$, two term approximation, A2: $2l = 0.3\lambda, R_0 = 0.3\lambda$, PWS; B1: $2l = 0.65\lambda, R_0 = 13\lambda$, two term approximation, B2: $2l = 0.65\lambda, R_0 = 13\lambda$, PWS.

field parallel to the slot length for slot length $2l = 0.3\lambda$ and $2l = 0.65\lambda$ is shown in Fig. 7(b). The maximum error in the magnetic field is seen to occur on the centreline, $z/l = 0$, for $2l < \lambda/2$ and gradually shifts towards the ends for slot length $2l > \lambda/2$. The near electric field is, in general, approximated better than the magnetic field. The error in magnetic field for 7(a) is calculated with $z/l = 0$ for $2l \leq 1$ and with $z/l = 0.28$ for $2l > \lambda/2$. The error in electric field is maximum around $z/l = 0.9$ for $2l > \lambda/2$.

For EMC measurements as per, say, MIL-STD-461F [14], the electric field is measured from an equipment under test (EUT) at a distance of 1m from 10 kHz to 1 GHz or more, for RE102 test. The error in electric field radiated from a 20 cm slot in an EUT (corresponding to $2l/\lambda$ from 6.67×10^{-6} to 0.667) at a distance of 1 m (corresponding to R_0/λ from 3.33×10^{-5} to 3.33) calculated from the two term approximation with respect to the assumed cosinusoidal one is shown in Fig. 8. Such a slot could be for display or ventilation or due to improper bonding of seams etc. The error is quite less and acceptable as seen from Fig. 8.

The error calculated from far field analysis [3] assuming cosinusoidal distribution is compared with that from the near field analysis (no far field approximation) assuming two term approximation for the aperture field, in Fig. 9. The error from two term approximation with exact fields for this distribution as derived above, is seen to be lesser than that for far field analysis, both, for slot length $2l = 0.3\lambda$ and for $2l = 0.65\lambda$.

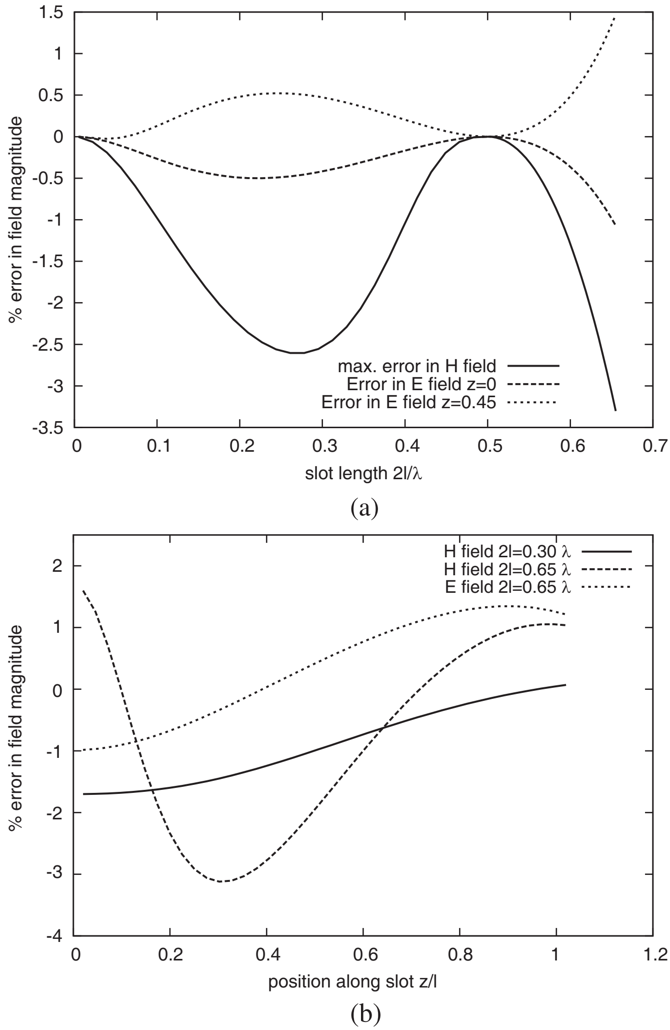


Figure 7. The error in field magnitude parallel to the slot at $\rho = 0.05\lambda$, (a) variation in error with slot length $2l/\lambda$, (b) variation in error parallel to the length of the slot for slot length $2l = 0.3\lambda$ and $2l = 0.65\lambda$.

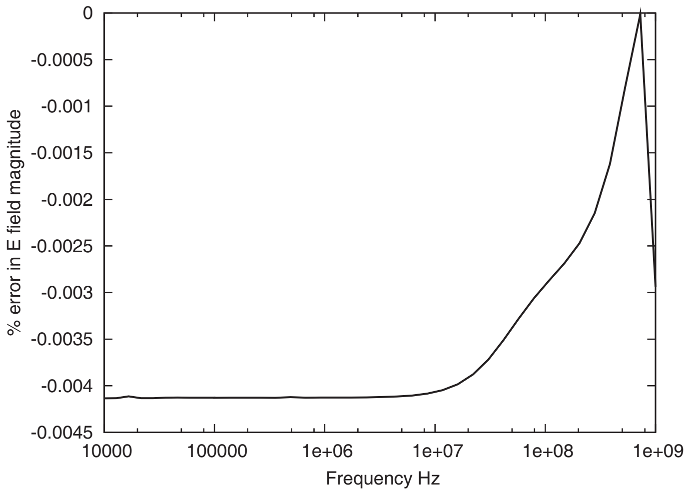


Figure 8. The error in radiated electric field from a 20 cm slot at 1 m distance.

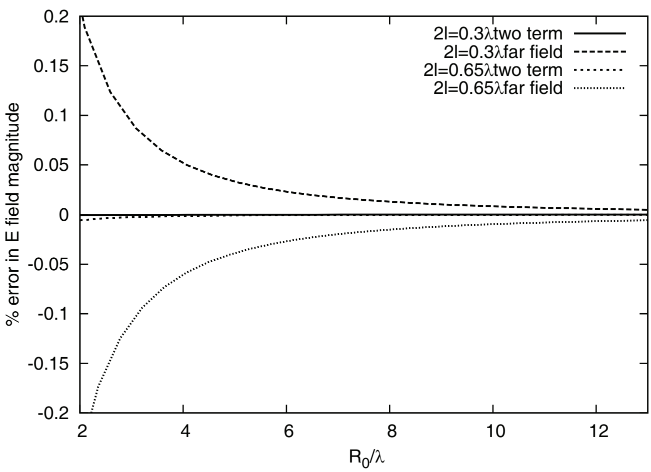


Figure 9. % error in E field for cosinusoidal distribution from far field analysis and from two term approximation from near field analysis.

3.1. Discussion of Results

From the foregoing analysis, it can be seen that the two term approximation for the cosinusoidal aperture distribution gives practically useable results for near fields derived in closed form. The error in E field is found to be lesser than that in H field. The error calculated from this method is better than that from other approximations in use, either for the aperture field like the piecewise sinusoidal approximation [9] or for the the fields derived, like the far field approximation. Hence, the above formulation provides a fast and accurate closed form solution to the near fields of slot radiators with cosinusoidal distribution up to a slot length of $2l \leq 0.65\lambda$. The approximation is particularly seen to be very good for slot length $2l \leq 0.5\lambda$. For longer slots at distances closer than 0.05λ, the error increases rapidly to about 3.3% for $2l = 0.65\lambda$ and for still longer slots, the above approximation along with the fields derived from it can no longer be used satisfactorily.

4. CONCLUSION

The two term approximation developed for approximating the aperture distribution of slots is seen to agree closely with the cosinusoidal one for slot lengths $2l \leq 0.65\lambda$. The near fields for this two term approximation were derived in closed form in terms of sine and cosine integrals and are seen to agree with less than 3.3% error for slot lengths $2l \leq 0.65\lambda$ at distance $R_{nearest} \geq 0.05\lambda$, where $R_{nearest}$ is the distance of the nearest point on the slot from the field point. As the practical slot lengths are mostly in this range, the above formulation should be of use in estimating the near fields of slots in closed form for mutual coupling or in estimating radiated emissions for EMC analysis.

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