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RIGOROUS EXPRESSIONS FOR THE EQUIVALENT EDGE CURRENTS

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Abstract—An exact form for the equivalent edge current is derived by using the axioms of the modified theory of physical optics and the canonical problem of half-plane. The edge current is expressed in terms of the parameters of incident and scattered rays. The analogy of the method with the boundary diffraction wave theory is put forward. The edge and corner diffracted waves are derived for the problem of a black half-strip.

1. INTRODUCTION

The high frequency diffraction techniques of electromagnetic theory are namely based on two methods which can be introduced as geometrical optics (GO) and physical optics (PO) [1, 2]. GO is a ray based method that defines the ray paths and amplitudes of electromagnetic waves according to the solution of two equations that are the high frequency approximations of the Helmholtz equation. These are the eikonal and transport equations. GO was correctly defining the transmitted and reflected rays in the context of the scatterer's geometry but was predicting zero field at the shadow regions. This defect of the theory was eliminated by Keller, who introduced his geometrical theory of diffraction (GTD) in the middle of the twentieth centaury [3]. He extended GO by defining the edge and surface diffracted rays which are the fields that are observed at the shadow boundaries. Since he used the high frequency asymptotic solutions of the canonical problems in order to define the diffraction coefficients, the diffracted fields were

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approaching to infinity at the transition boundaries where the GO fields have discontinuities. Keller's theory had also problems for the geometries that were including caustics. The infinities at the transition regions were removed by the uniform versions of GTD [4, 5]. Recently a uniform theory was also put forward for the diffraction of evanescent plane waves [6]. The method of equivalent currents was developed by the efforts of Ryan et al. [7], Knott et al. [8, 9] and James et al. [10] in order to obtain a valid field expression for the caustic regions. In the method of equivalent currents, an infinitesimal current element, which is related with the incident field, is defined that flows along the diffracting edge of the scatterer. The integration of the element along the edge gives the edge diffracted rays. The edge currents are obtained by using the diffracted fields that are found from the rigorous solutions of canonical problems [11]. The method was further developed by Michaeli who obtained a line integral expression which was also valid for the regions that are out of the Keller's cone [12, 13]. He derived the line integral by using the method of asymptotic reduction in order to directly find the edge diffracted waves. The detailed history of the progress of the equivalent currents method can be found in[14, 15]. The method of equivalent edge currents has important application in the electromagnetic theory of scattering. Zhang and Wu studied the hybridization of this method with the method of moment [16]. An equivalent source version was investigated by Hongo and Naqvi for a conducting disk and a hole in a conducting plane [17].

It is the aim of this paper to derive rigorous expressions for the equivalent edge currents by using the axioms of the modified theory of physical optics (MTPO) [18] and the high-frequency asymptotic solution of the half-plane problem. MTPO is a method that gives the exact diffracted waves for scattering problems by conducting bodies. The defect of PO is the incorrect edge contributions of the scattering integral. MTPO fixes this defect directly in the structure of PO by redefining the surface current according to three axioms. We will express the configuration of the equivalent current in a different way from the commonly used representations in the literature. It will be shown that the scattering angle is not directly equal to the observation angle and the integration of the current along the edge contour gives the edge diffracted waves. It will also be noted that this method is strongly analogous to the boundary diffraction wave (BDW) theory of optical diffraction [19, 20]. The diffraction of plane waves by a black half-strip will be investigated by using the method as an application.

A time factor of $\exp(jwt)$ will be considered and suppressed throughout the paper.

2. THEORY

The canonical problem of a conducting half plane which is illuminated by a plane wave is taken into account. The geometry of the problem is given in Fig. 1.

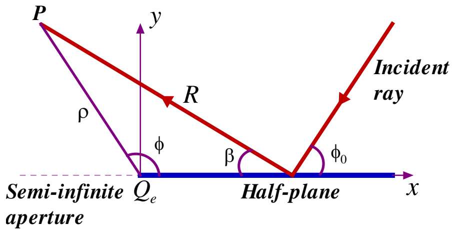

Figure 1. Geometry of the half-plane.

The incident wave has the scalar expression of

$$u_i = u_0 \exp[jk(x\cos\phi_0 + y\sin\phi_0)] \tag{1}$$

for k is the wave-number. u represents one of the scalar components of the electrical or magnetic fields. The incident diffracted field can be given by the equation of

$$u_{di} = u_0 \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{1}{\cos\frac{\phi-\phi_0}{2}} \frac{e^{-jk\rho}}{\sqrt{k\rho}}. \tag{2}$$

We will suppose a line integral of

$$u_d(P) = \frac{1}{4\pi} \int\limits_C f(l) u_i(Q_e) \frac{e^{-jkR_e}}{R_e} dl \tag{3}$$

where C is the edge contour. Q_e is the diffraction point on the edge. R_e is the distance between the diffraction and observation points. We will only consider the incident diffracted field to outline the philosophy of the theory. The method will be generalized for the soft and hard surfaces in the following sections. Eq. (3) can be rewritten as

$$u_d(P) = \frac{u_0}{4\pi} \int\limits_{-\infty}^{\infty} f(z') \frac{e^{-jkR_e}}{R_e} dz' \tag{4}$$

for the geometry in Fig. 1. R_e is equal to

$$R_e = \sqrt{\rho^2 + (z-z')^2} \tag{5}$$

where ρ is $\sqrt{x^2+y^2}$. The integral in Eq. (4) can be evaluated by using the method of stationary phase. The first derivative of the phase function can be obtained as

$$\frac{\partial R_e}{\partial z'} = -\frac{z-z'}{R_e} \tag{6}$$

which gives the stationary phase point as $z_s = z$ when equated to zero. The stationary phase value evaluation of the integral yields

$$u_d \approx u_0 \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} f(z) \frac{e^{-jk\rho}}{\sqrt{k\rho}}. \tag{7}$$

In fact, there are two edge diffracted waves in a diffraction problem by a conducting surface. The first one is the incident diffracted wave, which compensates the discontinuity of the incident wave at the shadow boundary. The second one is the reflected diffracted wave that overcomes the reflected wave's discontinuity at the reflection boundary. In the present analysis, we consent that the half-plane is black which reflects or transmits the incident wave that hits on it. As mentioned before, the generalization of the theory will be performed in the next sections.

In Fig. 2, a general geometry is given for the incident and edge diffracted rays. β and η are arbitrary angles that combines the diffraction point with the observation point. Their values can be determined in the asymptotic evaluation of the edge diffraction integral.

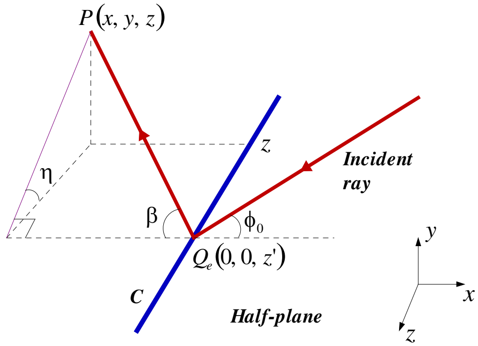

Figure 2. General geometry for the edge diffraction.

As a second step, we will equate Eq. (7) to Eq. (2). $f(z)$ is found to be

$$f(z) = \frac{1}{\cos\frac{\phi-\phi_0}{2}}. \tag{8}$$

It is important to note that $f(z)$ is the stationary phase value of $f(z')$ and we are interested in the structure of $f(l)$. Fig. 2 offers a general geometry for edge diffraction and we will benefit from this figure in order to determine $f(l)$. Eq. (8) can be rewritten as

$$f(z) = \sqrt{\frac{2}{1+\cos(\phi-\phi_0)}} \tag{9}$$

which leads to the equation of

$$f(z) = \sqrt{\frac{2}{1-\cos[\pi-(\phi-\phi_0)]}}. \tag{10}$$

Equation (10) can be written as

$$f(z) = \sqrt{\frac{2}{1-\vec{s}_i\cdot\vec{s}_d}} \tag{11}$$

from which we can obtain the general expression for the function of $f(l)$. $f(l)$ can be given by

$$f(l) = \sqrt{\frac{2}{1-\vec{s}_i\cdot\vec{s}_d}} \tag{12}$$

which yields the equation of

$$f(l) = \sqrt{\frac{2}{1-\cos(\beta+\phi_0)}}. \tag{13}$$

As a result the line integral of edge diffraction can be obtained as

$$u_d(P) = \frac{1}{2\sqrt{2}\pi}\int\limits_C \frac{u_i(Q_e)}{\sqrt{1-\vec{s}_i\cdot\vec{s}_d}}\frac{e^{-jkR_e}}{R_e}dl. \tag{14}$$

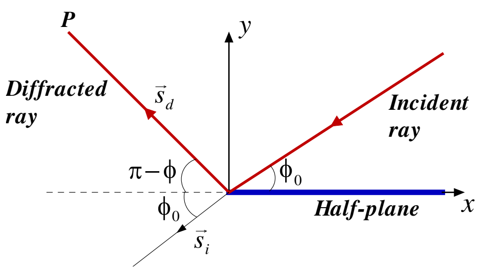

Figure 3. Edge diffraction geometry.

The diffraction integral for the reflected diffracted waves can be constructed in a similar way.

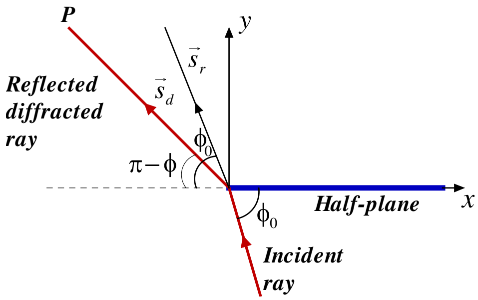

Figure 4. Diffraction geometry for the reflected diffracted waves.

When Fig. 4 is considered, $f(z)$ can be written as

$$f(z) = \sqrt{\frac{2}{1 - \cos[\pi - (\phi + \phi_0)]}} \tag{15}$$

since $\vec{s}_r \cdot \vec{s}_d = \cos[\pi - (\phi - \phi_0)]$. The line integral of diffraction satisfies

$$u_d(P) = \frac{1}{2\sqrt{2}\pi} \int\limits_C \frac{u_i(Q_e)}{\sqrt{1 - \vec{s}_r \cdot \vec{s}_d}} \frac{e^{-jkR_e}}{R_e} dl \tag{16}$$

for the reflected diffracted fields. It is important to note that Eqs. (14) and (16) are in the same form. This is an expected result since the reflected ray can be thought as the transmitted field which hits the half plane in the image direction of the incident wave [18]. The equivalent current can be defined as

$$I_{eq}(Q_e) = \sqrt{2} \frac{u_i(Q_e)}{\sqrt{1 - \vec{s}_r \cdot \vec{s}_d}} \tag{17}$$

according to the line integrals that are derived in this section. The line integral of edge diffraction can be written as

$$u_d(P) = \frac{1}{4\pi} \int\limits_C I_{eq}(Q_e) \frac{e^{-jkR_e}}{R_e} dl \tag{18}$$

when Eqs. (16) and (17) are taken into account. It is apparent that a line integral along the contour of the edge can be constructed for every

GO ray field in a problem of diffraction according to Eq. (18). The equivalent current is a function of the angle of incidence and scattering. The angle of scattering is not determined by the rigorous observation angles of the exact solution but shows an arbitrary direction as in Fig. 2. The exact value of the scattering angle is designated in the evaluation process of the line integral.

3. LINE INTEGRALS FOR SOFT AND HARD SURFACES

Generally one of the two boundary conditions is satisfied by the fields on a conducting object's surface. These are the Dirichlet (soft surface) and Neumann (hard surface) conditions. The Dirichlet condition can be defined as

$$u|_S = 0 \tag{19}$$

whereas the Neumann condition has the expression of

$$\left.\frac{\partial u}{\partial n}\right|_S = 0 \tag{20}$$

for u represents the total field. n is the normal of the surface. The line integrals of edge diffraction can be written as

$$u_{ds}(P) = -\frac{1}{4\pi}\int\limits_C \left[I_{eq}^i(Q_e) - I_{eq}^r(Q_e)\right]\frac{e^{-jkR_e}}{R_e}dl \tag{21}$$

and

$$u_{dh}(P) = -\frac{1}{4\pi}\int\limits_C \left[I_{eq}^i(Q_e) + I_{eq}^r(Q_e)\right]\frac{e^{-jkR_e}}{R_e}dl \tag{22}$$

for soft and hard surfaces when the exact solution of the diffraction problem by a half-plane is considered [11]. $I_{eq}^i(Q_e)$ and $I_{eq}^r(Q_e)$ can be defined as

$$I_{eq}^i(Q_e) = \sqrt{2}\frac{u_i(Q_e)}{\sqrt{1-\vec{s}_i\cdot\vec{s}_d}} \tag{23}$$

and

$$I_{eq}^r(Q_e) = \sqrt{2}\frac{u_i(Q_e)}{\sqrt{1-\vec{s}_r\cdot\vec{s}_d}} \tag{24}$$

respectively.

4. THEORY OF THE BOUNDARY DIFFRACTION WAVE

The theory of BDW was first invented by the studies of Rubinowicz [21] and Miyamoto et al. [19, 20] in order to introduce a quantitative basis for ideas of Young who mentioned that the scattered field by an edge discontinuity consists of two sub-fields [22]; a field that passes through the aperture unaffected by the discontinuity and a wave which originates from the edge contour. Rubinowicz reduced the diffraction integral of Kirchhoff into a line integral for a spherical wave that passes through an aperture. He showed that the integration of the line integral gives the edge diffracted waves. Miyamoto et al. generalized the theory for arbitrary wave incidence. According to the theory of BDW the diffracted waves can be expressed in terms of a line integral of

$$u_d(P) = \frac{1}{4\pi} \int\limits_C u_i(Q_e) \frac{(\vec{s}_i \times \vec{s}_d) \cdot \vec{t}}{1 - \vec{s}_i \cdot \vec{s}_d} \frac{e^{-jkR_e}}{R_e} dl \tag{25}$$

for $\vec{t}$ is the unit tangential vector of the edge contour. This theory was applied to the problem of diffraction by a half-plane and it was seen that the theory of BDW gives the same result with PO [23, 24]. In fact an equivalent current can be defined as

$$I_{eq} = u_i(Q_e) \frac{(\vec{s}_i \times \vec{s}_d) \cdot \vec{t}}{1 - \vec{s}_i \cdot \vec{s}_d} \tag{26}$$

when compared with Eq. (16). Exact line integrals for the theory of BDW were introduced by Umul, recently [25, 26].

Another important point of the theory is the poles of the integrand. The current component, in Eq. (26), approaches to infinity at $\vec{s}_i \cdot \vec{s}_d = 1$. This expression can be rewritten as

$$\cos(\beta + \phi_0) = 1 \tag{27}$$

according to Fig. 2. Eq. (27) leads to the critical point of $\beta = -\phi_0$, which represents the transition region of the incident ray. In fact this singularity is not a problem according to the BDW theory [19] since it guarantees the existence of the GO field with respect to the Stokes theorem. According to the point of view of the electromagnetic theory, the singularity occurs because the diffraction coefficient of Keller is a high frequency asymptotic expansion of the exact diffracted wave and is not valid at the transition region, where the detour parameter approaches to zero. This parameter is the argument of the Fresnel function that is the exact solution of the diffraction problem by a half-plane [27] and can be approximated to the diffracted field of GTD only for the detour parameter is large enough [28, 29]. The uniform versions of GTD are invented in order to eliminate the related singularity of the diffracted wave at the transition region. In fact the rigorous field expression is uniform and the GO wave can be obtained by a second line integration as shown in [26].

5. APPLICATION: DIFFRACTION BY A FINITE EDGE

A finite edge, the geometry of which is given in Fig. 5, is taken into account. The strip is placed at $S = \{(x, y, z) :\ x = 0,\ y = 0,\ z \in (0, a)\}$. The originality of this problem are the corner diffracted rays, which occur at $z = 0$ and $z = a$. The physical interpretation of these rays is analogous to that of the edge diffracted waves. The corner diffracted rays compensate the discontinuity of the edge diffracted field.

The surface is illuminated by a plane wave which has the expression of $u_0 \exp\left[jk(x \sin\theta_0 + z\cos\theta_0)\right]$. We will use the alternative form of the equivalent currents, derived in the Appendix. For the sake of simplicity, only the incident diffracted fields will be evaluated. The line integral expression of the diffracted can be written as

$$u_d(P) = \frac{1}{2\sqrt{2}\pi} \int\limits_C u_i(Q_e) \frac{\sqrt{1 - \vec{s}_r \cdot \vec{s}_d}}{\vec{n}_e \cdot (\vec{s}_d - \vec{s}_i)} \frac{e^{-jkR_e}}{R_e} dl \tag{28}$$

according to Eq. (14). Eq. (28) can be arranged as

$$u_d(P) = \frac{u_0}{2\sqrt{2}\pi} \int\limits_{z'=0}^{a} e^{jkz'\cos\theta_0} \frac{\sqrt{1 - \vec{s}_r \cdot \vec{s}_d}}{\vec{n}_e \cdot (\vec{s}_d - \vec{s}_i)} \frac{e^{-jkR_e}}{R_e} dz' \tag{29}$$

for R_e is equal to

$$R_e = \sqrt{\rho^2 + (z - z')^2} \tag{30}$$

for $C \cdot \rho$ is equal to $\sqrt{x^2 + y^2} \cdot \vec{s}_i$ and $\vec{s}_d$ can be defined by

$$\vec{s}_i = -\sin\theta_0 \vec{e}_x - \cos\theta_0 \vec{e}_z \tag{31}$$

and

$$\vec{s}_d = \sin\beta \sin\eta \vec{e}_x - \sin\beta\cos\eta \vec{e}_y - \cos\beta \vec{e}_z \tag{32}$$

according to the geometry in Fig. 5. $\vec{s}_r$ is equal to $\sin\theta_0 \vec{e}_x - \cos\theta_0 \vec{e}_z$. The scalar product of $\vec{s}_r \cdot \vec{s}_d$ can be determined as

$$\vec{s}_r \cdot \vec{s}_d = \sin\beta \sin\theta_0 \sin\eta + \cos\beta\cos\theta_0. \tag{33}$$

$\vec{n}_e$ is $-\vec{e}_y$. The integral, in Eq. (29), can be evaluated asymptotically [30]. The integral reads

$$u_d(P) = \frac{u_0}{2\sqrt{2\pi}} \int\limits_{z'=0}^{a} e^{jkz'\cos\theta_0} \frac{\sqrt{1-\sin\beta\sin\theta_0\sin\eta-\cos\beta\cos\theta_0}}{\sin\beta\cos\eta} \frac{e^{-jkR_e}}{R_e} dz'. \quad (34)$$

The phase function of the integral, in Eq. (36), can be given by

$$g(z') = z'\cos\theta_0 - R_e. \quad (35)$$

The first derivative of the phase function gives

$$g'(z') = \cos\theta_0 + \frac{z-z'}{R_e}. \quad (36)$$

The value of β at the stationary point is equal to θ_0 according to Eq. (36). Eq. (34) can be written as

$$u_d(P) = \frac{u_0}{2\sqrt{2\pi}} \frac{e^{jk(z\cos\theta_0-\rho\sin\theta_0)}}{R_{es}\sqrt{1+\sin\eta}} \int\limits_{z'=-\infty}^{\infty} e^{-jk\frac{\sin^2\theta_0}{2R_{es}}(z'-z_s)^2} dz' \quad (37)$$

at the stationary phase point [18]. R_{es} and z_s are the stationary phase values of R_e and z, respectively. As a result the diffracted field by the edge contour of C is found to be

$$u_d(P) \approx \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{e^{-jk\rho\sin\theta_0}}{\sqrt{k\rho\sin\theta_0}} \frac{1}{\cos(\sigma/2)} e^{jkz\cos\theta_0} \quad (38)$$

for σ is equal to $(\pi/2) - \eta$. ρ is $R_{es}\sin\theta_0$ at the stationary point. The field approaches to infinity at $\eta = -\pi/2$. This coordinate is the place of the shadow boundary where the incident field discontinuously goes to zero. The diffracted field has a cylindrical wave nature. It is important to note that η is equal to $\phi - (3\pi/2)$. The place of the transition region is at $\phi = \pi$ in the cylindrical coordinates.

The detour parameter of the field can be given by

$$\xi = \sqrt{2k\rho\sin\theta_0}\cos\frac{\phi}{2}. \quad (39)$$

The trigonometric relation of

$$-k\rho\sin\theta_0 = k\rho\sin\theta_0\cos\phi - \xi^2 \quad (40)$$

is valid. Eq. (40) will be used in the phase of Eq. (38) instead of $-k\rho\sin\theta_0$. The term of $k\rho\sin\theta_0\cos\phi$ is equal to $kx\sin\theta_0$ according to Fig. 5. As a result, the uniform version of Eq. (38) can be written as

$$u_d(P) \approx u_0 e^{jk(x\sin\theta_0 + z\cos\theta_0)} sign(\xi) F\left[|\xi|\right] \tag{41}$$

when Refs. [6, 31] are taken into account. $sign(x)$ is the signum function, which is equal to 1 for $x > 0$ and zero, otherwise. $F[x]$ represents the Fresnel integral, which is equal to

$$F[x] = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int_x^\infty e^{-jt^2} dt. \tag{42}$$

The transition region of the incident diffracted field occurs at $\phi = \pi$, which is the coordinate of the discontinuity of the incident wave.

The corner diffracted waves are evaluated from the edge points of the integral, given in Eq. (34). First of all the edge point at $z' = 0$ will be taken into account. β is equal to $\pi - \theta$ at this point. R_e is r. The corner diffracted wave can be found as

$$u_{c1} = -\frac{u_0}{jk2\sqrt{2}\pi} \frac{\sqrt{1 - \sin\theta\sin\theta_0\cos\phi + \cos\theta\cos\theta_0}}{\sin\theta\sin\phi(\cos\theta + \cos\theta_0)} \frac{e^{-jkr}}{r} \tag{43}$$

according to [16]. The second corner diffracted field reads

$$u_{c2} = \frac{u_0}{jk2\sqrt{2}\pi} \frac{\sqrt{1 - \sin\beta_c\sin\theta_0\cos\phi - \cos\beta_c\cos\theta_0}}{\sin\beta_c\sin\phi(\cos\theta_0 - \cos\beta_c)} \frac{e^{-jkR_{ec}}}{R_{ec}} \tag{44}$$

where R_{ec} is equal to $\sqrt{\rho^2 + (z-a)^2}$. β_c is $\sin^{-1}(\rho/R_{ec})$. It is apparent that the diffracted fields, in Eqs. (43) and (44), are not uniform, because they are evaluated directly by using the edge point technique, which gives the GTD fields. There are two detour parameters in question. The first one is the detour parameter of the edge diffracted

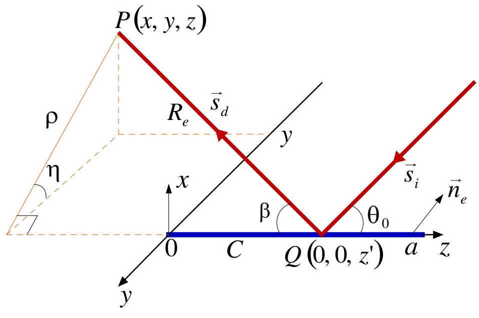

Figure 5. Geometry of the finite edge.

wave, which is given in Eq. (39). The second one is related with the corner diffracted wave that can be defined as

$$\xi_c = -\sqrt{2kr}\cos\frac{\theta+\theta_0}{2} \tag{45}$$

since $\theta \in [0, \pi]$ in the spherical coordinates. The uniform corner diffracted field can be obtained as

$$u_{c1} = \frac{u_0}{\sqrt{2}}\frac{f(\theta,\,\phi)\sqrt{\sin\theta_0}}{\sqrt{\sin\theta}\sin(\phi/2)\cos\frac{\theta-\theta_0}{2}} sign(\xi_c)F[|\xi_c|]sign(\xi)F[|\xi|]e^{jk(x\sin\theta_0+z\cos\theta_0)} \tag{46}$$

where $f(\theta,\,\phi)$ is equal to

$$f(\theta,\,\phi) = \sqrt{1-\sin\theta\sin\theta_0\cos\phi+\cos\theta\cos\theta_0}. \tag{47}$$

u_{c2} can be made uniform in a similar way. In this study we will only deal with the edge diffracted wave and u_{c1}.

6. NUMERICAL RESULTS

In this section, the edge and corner diffracted fields will be plotted numerically. The edge diffracted wave compensates the GO field at the transition region since the GO wave is discontinuous at this region. The diffracted field's amplitude is the half of the Go wave at the transition zone. A similar compensation occurs for the edge diffracted wave since it has a discontinuity at the corner of the scatterer. For this reason the function of the corner diffracted field is analogous to the edge diffracted wave with a dimension difference. Edge diffracted field exists for the line discontinuities. The corner diffracted field is taken into account for a point discontinuity. For this reason it is a spherical wave in nature as can be seen in Eqs. (43) and (44). The discontinuity of the edge diffracted wave is shown in Fig. 6.

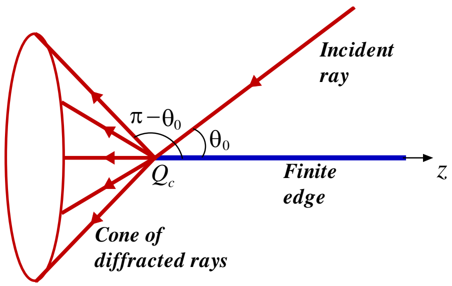

Figure 6. Discontinuity of the edge diffracted wave.

In Fig. 6, it is shown that the edge diffracted waves form a cone which is named as Keller's cone. This cone will be continuous along the edge contour since the same diffraction will occur on the points of the edge that follow each other. But after the corner point of the edge, there is no point that will cause an edge diffracted wave. For this reason there will be discontinuity at the edge diffracted wave for $\theta > \pi - \theta_0$, which determines a transition region for the process of corner diffraction at $\theta = \pi - \theta_0$. The edge diffracted wave exists in the space for $\theta < \pi - \theta_0$. This phenomenon is similar to that of the discontinuity of the GO field. We expect that the amplitude of the corner diffracted wave is half of the amplitude of the edge diffracted wave at $\theta = \pi - \theta_0$ which is the transition region.

Figure 7 shows the variation of the edge diffracted wave with respect to the angle of ϕ, which changes in the plane of (x, y). The field, given in Eq. (41), is plotted. ρ is equal to 6λ. The angle of incidence (θ_0) is $60°$. The amplitude of the incident wave is one. According to Fig. 7, the shadow boundary occurs at $\phi = 180°$, which is a harmonious value with the geometry, given in Fig. 5. There is no incident wave for $\phi > 180°$ since the plane of the edge contour cloaks the incident field. It is important to note that the amplitude of the diffracted wave is equal to 0.5 at the shadow boundary. This value is the half of the incident wave's amplitude.

Figure 8 depicts the variation of the total, edge and corner diffracted waves versus θ. The corner diffracted field is plotted, using

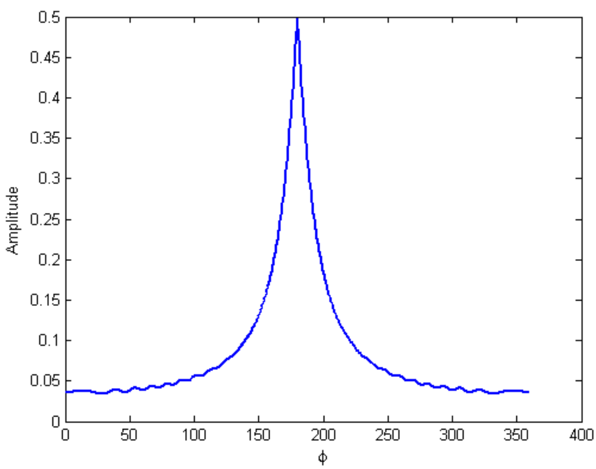

Figure 7. Edge diffracted wave.

Eq. (46). Total field is the sum of Eqs. (41) and (46). ρ has the same value with Fig. 7. ϕ is taken as 90°. (θ_0) is equal to 60°. It can be observed from Fig. 8 that the edge diffracted wave goes discontinuously to zero at 120° which is $\pi - \theta_0$ in Fig. 6. Its value is equal to 0.5 at the transition region. The amplitude of the corner diffracted wave is 2.5 at this zone. The graphics are harmonious with the physical considerations.

7. CONCLUSION

In this paper, new expressions are derived for the theory of equivalent edge currents by using the method of MTPO and the diffraction coefficients of the half plane problem. The line integrals for hard and soft surfaces are expressed and the relation of the equivalent edge currents and the theory of BDW is stressed. The new expressions are applied to the problem of diffraction of plane waves by a finite edge and the diffracted waves are plotted numerically. It is shown that the results are in harmony with the physical expectations which are considered according to the theory of edge diffraction. It is observed that the corner diffracted waves compensate the discontinuity of the edge diffracted field in the transition region, which occurs at the corner of the geometry.

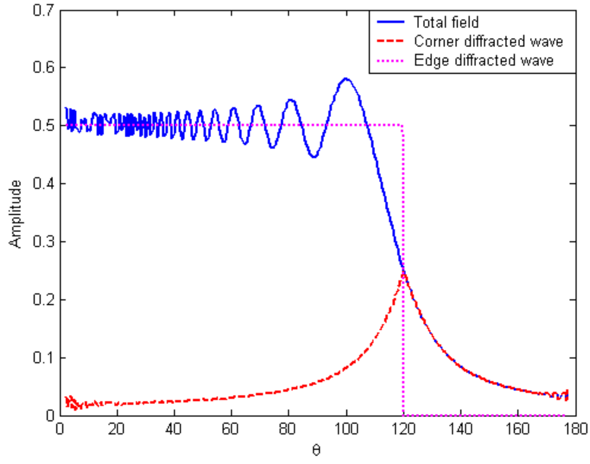

Figure 8. Compensation of the edge diffracted wave.

APPENDIX A.

In this section, we will derive an alternative expression for the equivalent current representation, given in Eq. (23). With this aim, we will take into account the diffraction geometry of Fig. A1.

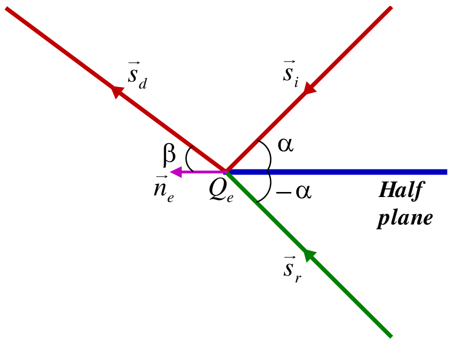

Figure A1. Geometry of edge diffraction.

The equivalent can be written as

$$I_{eq} = \sqrt{2}\frac{u_i(Q_e)}{\sqrt{1-\cos(\beta+\alpha)}} \tag{A1}$$

in terms of the angles of diffraction. Eq. (A1) can be rearranged as

$$I_{eq} = \frac{u_i(Q_e)}{\sin\frac{\beta+\alpha}{2}} \tag{A2}$$

which also yields the equation of

$$I_{eq} = u_i(Q_e)\frac{\sin\frac{\beta-\alpha}{2}}{\sin\frac{\beta+\alpha}{2}\sin\frac{\beta-\alpha}{2}}. \tag{A3}$$

Eq. (A3) can be rewritten as

$$I_{eq} = 2u_i(Q_e)\frac{\sin\frac{\beta-\alpha}{2}}{\cos\beta-\cos\alpha} \tag{A4}$$

which gives

$$I_{eq} = \sqrt{2}u_i(Q_e)\frac{\sqrt{1-\cos(\beta-\alpha)}}{\cos\beta-\cos\alpha}. \tag{A5}$$

Eq. (A5) can be written as

$$I_{eq} = \sqrt{2} u_i(Q_e) \frac{\sqrt{1 - \vec{s}_r \cdot \vec{s}_d}}{\vec{n}_e \cdot (\vec{s}_d - \vec{s}_i)} \tag{A6}$$

in terms of the unit vectors, defined in Fig. A1.

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