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DESIGN RULES FOR A FABRY-PEROT NARROW BAND TRANSMISSION FILTER CONTAINING A METAMATERIAL NEGATIVE-INDEX DEFECT

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Abstract—In this work, we theoretically study the optical properties of a multilayer Fabry-Perot narrow band transmission filter containing a metamaterial negative-index defect. As in the usual Fabry-Perot filter design, the negative-index defect is sandwiched by two quarter-wave dielectric mirrors. Some useful design rules on selecting value of the negative-index of the defect have been numerically elucidated. Such narrow band transmission filtering is achieved when the refractive index of defect is either a negative even integer if the thickness is taken as a quarter of design wavelength; or a negative odd integer if the thickness is taken as a half of design wavelength.

1. INTRODUCTION

Planar dielectric multilayered structures are important in modern photonics. They can be designed as Fabry-Perot interferometers or antireflection coatings. A distributed Bragg reflector (DBR) or photonic crystal (PC) is a periodic layered structure made up of N-pair of alternating dielectrics with refractive indices n_1 and n_2, respectively. It is known that a DBR has some high-reflectance reflection band that is now referred to as the photonic bandgap (PBG). Theory of

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DBR states that PBGs can be obtained for a quarter-wave stack, i.e., $n_1 d_1 = n_2 d_2 = \lambda_0/4$, where d_1 and d_2 are respectively the thicknesses of constituent bilayer and λ_0 is the design wavelength specified near or at the center of the reflection band [1, 2]. In addition, when a defect layer is inserted into between two DBRs to break the periodicity, a Fabry-Perot resonator (FPR) or Fabry-Perot narrow band transmission filter is formed, which is proven to be a useful optical device.

Recently, a multilayer structure containing one type of metamaterials such as the negative-index material (NIM) attracts much attention. Metamaterials (MTMs) were theoretically proposed firstly by Veselago in 1968 [3]. An NIM has double-negative parameters, i.e., both permittivity and permeability are simultaneously negative. A purely left-handed DBR consisting of alternating left-handed layers with different negative refractive indices was reported by Sabah and Uckun [4]. DBRs or one-dimensional PCs with NIMs and usual right-handed materials (RHMs) are also available now [5]. The phenomenon of negative refraction can even be appropriately designed and observed in a one-dimensional PC made up of all RHMs [6]. By making use of the NIM as a defect layer in a DBR, an FPR with NIM-defect is reported recently [7]. In this work, using an NIM-defect sandwiched between two DBRs to design an FPR will be studied. Optical transmittance is investigated as a function of negative refractive index of NIM-defect. Simulation results reveal that transmission peaks exist within the PBG with certain values in the negative refractive index of the NIM-defect when the thickness of which is taken to be quarter- or half-wavelength. Such results have suggested some design rules for the negative refractive index that are of technical use in the design of an FPR containing an NIM-defect.

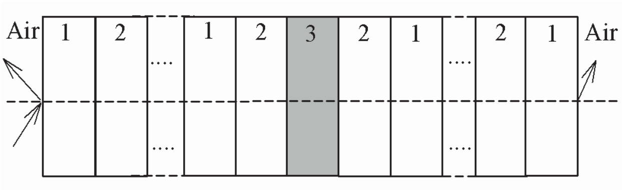

Figure 1. The structure of the narrowband FPR with a defect layer 3 sandwiched by two DGRs in a symmetric configuration.

2. CALCULATION OF TRANSMITTANCE FOR AN FPR

Let us consider a design of multilayer FPR, air/$(12)^{N_L}3(21)^{N_R}$/air depicted in Fig. 1, where the central layer 3 with thickness d_3 and refractive index $n_3 < 0$ is an NIM-defect which is sandwiched between two DBRs made of layer 1 and 2 with refractive indices n_1 and n_2 and thicknesses d_1 and d_2, respectively. The two DBRs, which have periods N_L and N_R, respectively, are taken to be the quarter-wave stacks, i.e., $n_1d_1 = n_2d_2 = \lambda_0/4$, where λ_0 is the design wavelength. The incident and transmitted regions are air with a refractive index of $n_a = 1$. An electromagnetic wave with temporal part $\exp(i\omega t)$ is incident on the most left plane boundary. The normal-incidence transmission coefficient can be calculated by making use of the transfer matrix method (TMM) in a stratified medium [2]. Based on this method, the total system matrix is given by

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \mathbf{D}_0^{-1}\left[\mathbf{D}_1\mathbf{P}_1\mathbf{D}_1^{-1}\mathbf{D}_2\mathbf{P}_2\mathbf{D}_2^{-1}\right]^{N_L} \left[\mathbf{D}_3\mathbf{P}_3\mathbf{D}_3^{-1}\right]\left[\mathbf{D}_2\mathbf{P}_2\mathbf{D}_2^{-1}\mathbf{D}_1\mathbf{P}_1\mathbf{D}_1^{-1}\right]^{N_R}\mathbf{D}_0, \quad (1)$$

where N_L and N_R are the periods of the left and right DBRs, respectively. The transmission coefficient t is given by $t = 1/M_{11}$ and then the transmittance is $T = t^*t$.

In Eq. (1) the dynamical matrix for the medium ℓ is given by

$$\mathbf{D}_\ell = \begin{pmatrix} 1 & 1 \\ n_\ell & -n_\ell \end{pmatrix}, \quad \ell = 0,\ 1,\ 2,\ \text{and}\ 3, \quad (2)$$

and the propagation matrix in each layer is expressed as

$$\mathbf{P}_\ell = \begin{pmatrix} \exp(i\phi_\ell) & 0 \\ 0 & \exp(-i\phi_\ell) \end{pmatrix}, \quad \ell = 1,\ 2,\ \text{and}\ 3, \quad (3)$$

where

$$\phi_\ell = k_0 n_\ell d_\ell, \quad \ell = 1,\ 2,\ \text{and}\ 3 \quad (4)$$

where $k_0 = \omega/c$ the free-space wavenumber and $\ell = 0$ indicates the air. In the case of equal-period, $N_L = N_R = N$, an analytical expression for the transmittance at $\lambda = \lambda_0$ can be obtained, with the result

$$T(\lambda_0) = \frac{4}{4\cos^2\left(\frac{2\pi}{\lambda_0}n_3d_3\right) + \left[\frac{1}{n_3}\left(\frac{-n_2}{n_1}\right)^{2N} + n_3\left(\frac{-n_1}{n_2}\right)^{2N}\right]^2 \sin^2\left(\frac{2\pi}{\lambda_0}n_3d_3\right)}. \quad (5)$$

3. NUMERICAL RESULTS AND DISCUSSION

In what follows we present the calculated optical transmittance as a function of negative index of NIM-defect. The materials in DBRs are $n_1 = 1.34$ (Na_3AlF_6) and $n_2 = 2.6$ (ZnSe), respectively [8]. These two constituent layers are taken to be quarter-wave stacks, i.e., $n_1 d_1 = n_2 d_2 = \lambda_0/4$. In addition, we set the design wavelength to be $\lambda_0 = 500\,\text{nm}$.

The calculated transmittance as a function of n_3 is shown in Fig. 1, in which the defect thickness $d_3 = \lambda_0/4$ and the numbers of periods of DBRs $N_L = N_R = 8$ are taken in our calculation. It can be seen that the transmittance peaks appear when n_3 is equal to a negative even integer. An example for the wavelength-dependent transmittance at $n_3 = -2$ and $d_3 = \lambda_0/4$ is shown in Fig. 3, in which

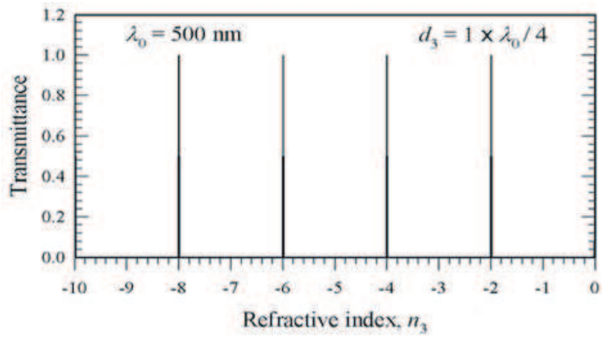

Figure 2. Calculated transmittance versus n_3 for an FPR with an NIM-defect.

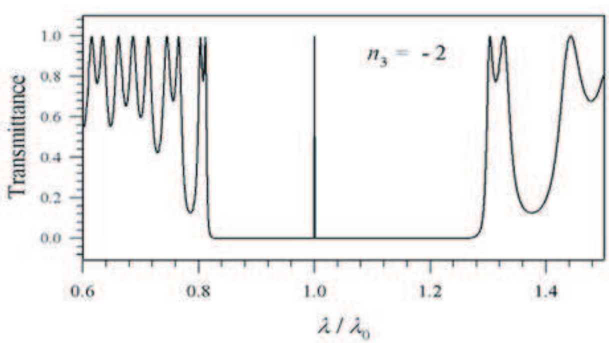

Figure 3. Calculated wavelength-dependent transmittance at $n_3 = -2$.

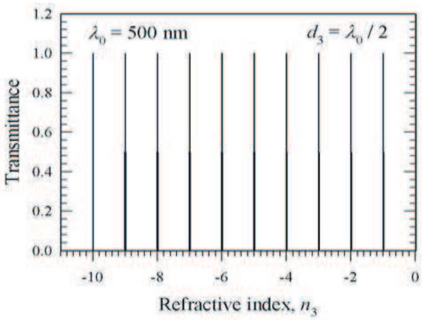

Figure 4. Calculated transmittance versus n_3 for an FPR with an NIM-defect.

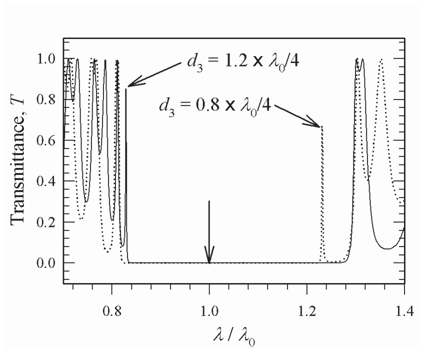

Figure 5. Calculated wavelength-dependent transmittance at $n_3 = -2$ when d_3 is not exactly equal to the quarter-wavelength $\lambda_0/4$. The solid curve is for $d_3 = 1.2\lambda_0$, whereas the dotted curve is for $d_3 = 0.8\lambda_0$.

the transmittance peak exists within the PBG and is located at design wavelength $\lambda = \lambda_0$. The results of Figs. 2 and 3 clearly reveal that an FPR with resonant wavelength equal to the design wavelength can be designed and achievable with an NIM-defect whose thickness is equal to the quarter-wavelength.

If now the defect thickness is half-wavelength ($d_3 = \lambda_0/2$) instead, the result of calculated transmittance is depicted in Fig. 4. It is seen that the transmittance peaks occur when the value in n_3 is equal to a negative integer. These resonant conditions can be well explained from Eq. (5). If the defect thickness is $d_3 = \lambda_0/4$ at designed frequency, then, from Eq. (5), we see that $T = 1$ when n_3 is a negative even integer. On the other hand, transmission peaks will also occur for $d_3 = \lambda_0/2$ according to Eq. (5). In conclusion, transmission peak occurs when the magnitude of the optical length of defect is equal to the half wavelength. And an ultra narrow band transmission filter can be achieved.

Although the transmittance spectrum in Fig. 3 has been reported in [7], the above design rules for the negative refractive index of an NIM-defect, however, was not explored there. In addition, another basic distinction between the NIM-defect and usual RHM-defect can be seen in Fig. 5, where a phase-shifted FPR is designed by changing the NIM-defect thickness such that it is not equal to $\lambda_0/4$. For a thinner defect, say $d_3 = 0.8\lambda_0/4$, the transmission-peak wavelength is moved to be higher than the design wavelength (the dotted curve), while it is shifted to be smaller than the design wavelength when d_3 is larger than $\lambda_0/4$ (the solid curve). This shifting tendency is consistent with that of an FPR with RHM-defect. However, with NIM-defect in FPR the transmission peak for is substantially depressed in addition to the shift in the peak position. For $d_3 = 0.8\lambda_0/4$, the peak height is reduced to about 0.65 and to 0.84 at $d_3 = 1.2\lambda_0$. In a usual RHM-defect, the peak height will never be depressed, remaining as one. The result in Fig. 5 indicates that the peak height is very sensitive to the thickness variation in the thickness of the NIM-defect. Care therefore should be taken when designing a phase-shifted FPR using the NIM-defect.

4. CONCLUSION

By making use of the quarter-wave stacks, a multilayer symmetric FPR containing an NIM-defect in the visible region is designed. Useful design rules for selecting negative index in the NIM defect layer have been provided. The transmission peak height is strongly dependent on the variation of the defect thickness. This dependence is not seen in the usual FPR using the RHM-defect.

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