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FOCAL REGION FIELD OF A PARABOLOIDAL REFLECTOR COATED WITH ISOTROPIC CHIRAL MEDIUM

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Abstract—Maslov's method is used to derive the expressions for high frequency fields around the focal region of a paraboloidal reflector coated with isotropic and homogeneous chiral medium. The field expressions thus calculated are solved numerically, and the results are presented in the paper. Moreover, the dependency of the electric field on the thickness of the coated chiral medium and its properties is also studied. The results of this study are presented in the paper, and the conclusions are drawn accordingly.

1. INTRODUCTION

Asymptotic ray theory (ART), or geometrical optics (GO) is widely used to analyze RF waves at high frequencies in various mediums as given in [1–3]. However, these high frequency techniques fail at caustics. In various applications, such as parabolic reflectors and other focusing systems, the field strength at these regions is of practical importance. Hence, an asymptotic method based on Maslov's theory is used to study the behavior of field pattern around the focal regions [4, 5]. Maslov's method is used by many authors to study the field behavior of various focusing systems [6–17]. In this method, the

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ray is expressed in hybrid coordinates, chosen from the wave vector coordinates $\mathbf{P} = (p_x, p_y, p_z)$ and space coordinates $\mathbf{R} = (x, y, z)$. Maslov's method uses the simplicity of ray theory and the generality of Fourier transform to avoid the singularity at caustics. In the present work, our interest is to apply Maslov's method to find the field around the focal region of a paraboloidal reflector coated with isotropic and homogeneous chiral medium. This work is an extension of the previous work, in which field at the caustic of a two dimensional coated parabolic reflector was studied [13], to three dimensional case in which field is calculated at the focus of a paraboloidal reflector coated with an isotropic and homogeneous chiral medium. Chiral medium is microscopically continuous medium composed of chiral objects, uniformly distributed and randomly oriented [19]. A chiral object is a three dimensional body that cannot be brought into congruence with its mirror image through translation or rotation e.g., helix, animal hands, Aspartame, three dimensional tetrahedron etc. The behavior of electromagnetic waves in chiral medium has been analyzed by many authors [19–25].

In Section 2, the general expressions for fields in free space are developed using GO and Maslov's method. In Section 3, the reflection coefficient of plane waves from a chiral slab backed by perfect electric conductor is discussed. In Section 4, expressions for field around the focal region of a paraboloidal reflector coated with chiral medium are calculated. In Section 5, plots of field around the focal region for various values of geometric and chiral parameters are given. Concluding remarks are given in Section 6.

2. GEOMETRICAL OPTICS AND MASLOV'S METHOD IN FREE SPACE

The GO and Maslov's method have been used to analyze many focusing systems [6, 12, 13], but here it is applied to a paraboloidal reflector coated with chiral medium. Consider a three dimensional wave equation

$$\left(\nabla^2 + k_0^2\right) u(r) = 0 \tag{1}$$

where, $r = (x, y, z)$, $\nabla^2 = \partial^2/\partial_x^2 + \partial^2/\partial_y^2 + \partial^2/\partial_z^2$ and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is wavenumber of the medium. Solution of Eq. (1) may be assumed in the form of Luneberg-Kline series as

$$u(r) = \sum_{m=0}^{\infty} \frac{E^m(r)}{(jk_0)^m} \exp(-jk_0 s) \tag{2}$$

Assuming large values of k_0, in the above series, the higher order terms can be neglected and only the first term is retained. By putting Eq. (2) in Eq. (1) and equating the coefficients of k_0^2, the eikonal equation is obtained as given by [18]

$$(\nabla s(r))^2 - 1 = 0 \tag{3}$$

similarly by equating the coefficients of k_0, the transport equation is obtained

$$2\nabla E \cdot \nabla s + E\nabla^2 s = 0 \tag{4}$$

in the above equation only E^0 has been retained and is denoted with E. Wave vector is define as $\mathbf{p} = \nabla s$ and Hamiltonian $H(\mathbf{r}, \mathbf{p}) = (\mathbf{p} \cdot \mathbf{p} - 1)/2$. So the eikonal equation becomes $H(\mathbf{r}, \mathbf{p}) = 0$. eikonal equation can be solved by the method of characteristic as follows

$$\frac{dx}{dt} = p_x,\ \frac{dy}{dt} = p_y,\ \frac{dz}{dt} = p_z,\ \frac{dp_x}{dt} = 0,\ \frac{dp_y}{dt} = 0,\ \frac{dp_z}{dt} = 0 \tag{5}$$

where, t is the parameter along the ray. The solution of Eq. (5) is

$$x = \xi + p_x t,\ y = \eta + p_y t,\ z = \zeta + p_z t,\ p_x = p_{x_0},\ p_y = p_{y_0},\ p_z = p_{z_0} \tag{6}$$

where, (ξ, η, ζ) and (p_{x0}, p_{y0}, p_{z0}) are the initial values of (x, y, z) and (p_x, p_y, p_z) respectively. The phase function is given by

$$s = s_0 + \int_0^t dt = s_0 + t. \tag{7}$$

Applying Gauss's theorem to a paraxial ray tube, the solution of Eq. (4) is given by [18]

$$u(r) = E(r_0)J^{-1/2} \exp(-jk_o(s_0 + t)) \tag{8}$$

where, $E(r_0)$ is the initial value of the field amplitude and $J = D(t)/D(0)$ is the Jacobian for transformation from ray coordinates (ξ, η, ζ) to Cartesian coordinate (x, y, z). As the GO solution is not valid at the focal point ($J = 0$), so Maslov's method is used to find the field around the caustic region of a focussing system as analyzed by [6–20]. The equation which is valid around the focal point of a paraboloidal reflector is given as [6]

$$\begin{aligned} u(r) = \frac{k_o}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(r_0) \left[\frac{D(t)}{D(0)} \frac{\partial(p_x, p_y)}{\partial(x, y)} \right]^{-\frac{1}{2}} \exp(-jk_o(s_0 + t \\ -x(p_x, p_y, z)p_x - y(p_x, p_y, z)p_y + xp_x + yp_y))dp_x dp_y \end{aligned} \tag{9}$$

The expression $\frac{D(t)}{D(0)}\frac{\partial(p_x,p_y)}{\partial(x,y)}$ can simply calculated as,

$$\frac{D(t)}{D(0)}\frac{\partial(p_x,p_y)}{\partial(x,y)} = \frac{1}{D(0)}\frac{\partial(p_x,p_y,z)}{\partial(\xi,\eta,\zeta)}. \tag{10}$$

3. REFLECTION OF PLANE WAVES FROM A CHIRAL SLAB BACKED BY PERFECT ELECTRIC CONDUCTOR

In this paper, we want to find the reflected field around the focal region of a paraboloidal reflector coated with chiral medium. To achieve this the reflection of plane waves from a chiral slab backed by perfect electric conducting (PEC) plane is discussed as in [13, 19]. As shown in Figure 1 the region $z \leq 0$ is occupied by free space. The constitutive relations in this medium are given as

$$\mathbf{D} = \epsilon_0\mathbf{E}, \quad \mathbf{B} = \mu_0\mathbf{H}$$

where, $\mathbf{D}$ and $\mathbf{B}$ are electric and magnetic flux densities respectively, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields represented, ϵ_0 is the permittivity and μ_0 is the permeability of the free space. The perfect electric conductor (PEC) is placed at $z = d$ as shown in the Figure 1. Region $0 \leq z \leq d$ is occupied by the chiral medium defined by Drude-Born-Fadorov (DBF) constitutive relations [19] as follows

$$\mathbf{D} = \epsilon(\mathbf{E} + \beta\nabla\times\mathbf{E}), \quad \mathbf{B} = \mu(\mathbf{H} + \beta\nabla\times\mathbf{H})$$

where, ϵ is the permittivity and μ is the and permeability of the coated chiral medium. β is the chirality parameter of this medium. The

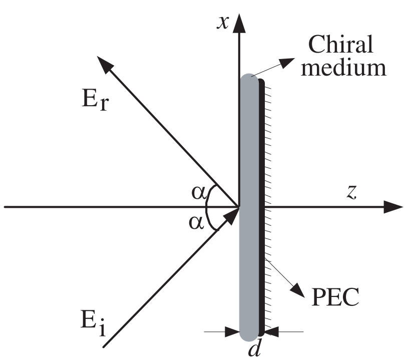

Figure 1. Reflection of plane waves from chiral slab backed by PEC plane.

incident electric field ($\mathbf{E}_i$) and the reflected electric field ($\mathbf{E}_r$) makes an angle α with the normal to the surface and can be expressed as

$$\mathbf{E}_i = \left\{ A_\perp \mathbf{a}_y + A_\parallel \left(-\frac{k_{0z}}{k_0}\mathbf{a}_x + \frac{k_{0x}}{k_0}\mathbf{a}_z \right) \right\} \exp(jk_{0z}z + jk_{0x}x) \quad (11)$$

and the reflected field as

$$\mathbf{E}_r = \left\{ B_\perp \mathbf{a}_y + B_\parallel \left(\frac{k_{0z}}{k_0}\mathbf{a}_x + \frac{k_{0x}}{k_0}\mathbf{a}_z \right) \right\} \exp(-jk_{0z}z + jk_{0x}x) \quad (12)$$

where, $A_\perp$, $B_\perp$ and $A_\parallel$, $B_\parallel$ are the perpendicular and parallel components w.r.t the plane of incident respectively. $k_0 = \omega\sqrt{\epsilon_0\mu_0}$, $k_{0z} = k_0 \cos\alpha$ and $k_{0x} = k_0 \sin\alpha$. Field in the chiral layer can conveniently be written in terms of Beltrami fields as

$$\mathbf{E} = \mathbf{Q}_L - j\eta\mathbf{Q}_R \quad (13)$$

$$\mathbf{H} = \mathbf{Q}_R - j\mathbf{Q}_L/\eta \quad (14)$$

where

$$\begin{aligned}
\mathbf{Q}_R &= A_1 \left\{ \mathbf{a}_y + j\left(\frac{k_{1z}}{k_1}\mathbf{a}_x + \frac{k_{0x}}{k_1}\mathbf{a}_z \right) \right\} \exp(-jk_{1z}z + jk_{0x}x) \\
&\quad + B_1 \left\{ \mathbf{a}_y + j\left(-\frac{k_{1z}}{k_1}\mathbf{a}_x + \frac{k_{0x}}{k_1}\mathbf{a}_z \right) \right\} \exp(jk_{1z}z + jk_{0x}x) \quad (15) \\
\mathbf{Q}_L &= A_2 \left\{ \mathbf{a}_y + j\left(\frac{k_{2z}}{k_2}\mathbf{a}_x + \frac{k_{0x}}{k_2}\mathbf{a}_z \right) \right\} \exp(-jk_{2z}z + jk_{0x}x) \\
&\quad + B_2 \left\{ \mathbf{a}_y + j\left(-\frac{k_{2z}}{k_2}\mathbf{a}_x + \frac{k_{0x}}{k_2}\mathbf{a}_z \right) \right\} \exp(jk_{2z}z + jk_{0x}x) \quad (16)
\end{aligned}$$

In the above relationships $k = \omega\sqrt{\epsilon\mu}$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\eta = \sqrt{\mu/\epsilon}$, $k_1 = 1/(1 - k\beta)$, $k_2 = 1/(1 + k\beta)$, $k_{1z}^2 + k_{0x}^2 = k_1^2$ and $k_{2z}^2 + k_{0x}^2 = k_2^2$. To find the expressions for reflection coefficient we apply boundary conditions at $z = 0$ which yields the following equations

$$\begin{aligned}
\begin{bmatrix} B_\perp \\ B_\parallel \end{bmatrix} &= [r] \begin{bmatrix} A_\perp \\ A_\parallel \end{bmatrix} + [T] \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \\
\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} &= [R] \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + [t] \begin{bmatrix} A_\perp \\ A_\parallel \end{bmatrix} \quad (17)
\end{aligned}$$

where, $[r]$, $[T]$, $[R]$ and $[t]$ are 2×2 matrices. Elements of the matrices, which are Fresnel coefficients, are given as

$$
\begin{aligned}
r_{11} &= -\left[(\eta_0^2 - \eta^2)(\xi_1 + \xi_2) + 2\eta_0\eta\left(\xi_1\xi_2 - 1\right)\right]/D \\
r_{22} &= \left[\left(\eta_0^2 - \eta^2\right)\left(\xi_1 + \xi_2\right) - 2\eta_0\eta\left(\xi_1\xi_2 - 1\right)\right]/D \\
r_{12} &= 2j\eta_0\eta\left(\xi_1 - \xi_2\right)/D \\
r_{21} &= -r_{12} \\
t_{11} &= 2\eta\left(\eta\xi_2 + \eta_0\right)/D \\
t_{22} &= -2\left(\eta_0\xi_1 + \eta\right)/D \\
t_{12} &= -2j\eta\left(\eta_0\xi_2 + \eta\right)/D \\
t_{21} &= 2j\eta\left(\eta\xi_1 + \eta_0\right)/D \\
R_{11} &= \left[\left(\eta_0^2 + \eta^2\right)\left(\xi_1 - \xi_2\right) + 2\eta_0\eta\left(\xi_1\xi_2 - 1\right)\right]/D \\
R_{22} &= \left[-\left(\eta_0^2 - \eta^2\right)\left(\xi_1 - \xi_2\right) + 2\eta_0\eta\left(\xi_1\xi_2 - 1\right)\right]/D \\
R_{12} &= -2j\eta\xi_2\left(\eta_0^2 - \eta^2\right)/D \\
R_{21} &= 2j\xi_1\left(\eta_0^2 - \eta^2\right)/\eta D \\
T_{11} &= 4\eta_0\xi_1\left(\eta\xi_2 + \eta_0\right)/D \\
T_{22} &= -4\eta\eta_0\xi_2\left(\eta\xi_1 + \eta_0\right)/D \\
T_{12} &= -4j\eta\eta_0\xi_2\left(\eta\xi_1 + \eta_0\right)/D \\
T_{21} &= 4j\eta_0\xi_1\left(\eta_0\xi_2 + \eta\right)/D
\end{aligned}
$$

where

$$
\begin{aligned}
D &= \left(\eta_0^2 - \eta^2\right)\left(\xi_1 + \xi_2\right) + 2\eta_0\eta\left(\xi_1\xi_2 + 1\right) \\
\xi_1 &= \sec\alpha\sqrt{1 - \left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} \\
\xi_2 &= \sec\alpha\sqrt{1 - \left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}
\end{aligned}
$$

Applying boundary conditions at $z = d$ yields the following equations

$$
\begin{bmatrix} A_1 \exp\left(-jk_{1z}d\right) \\ A_2 \exp\left(-jk_{2z}d\right) \end{bmatrix} = \frac{1}{k_2k_{1z} + k_1k_{2z}} \begin{bmatrix} k_2k_{1z} - k_1k_{2z} & 2j\eta k_1k_{2z} \\ -2j\frac{k_2k_{1z}}{\eta} & k_1k_{2z} - k_2k_{1z} \end{bmatrix} \times \begin{bmatrix} B_1 \exp\left(jk_{1z}d\right) \\ B_2 \exp\left(jk_{2z}d\right) \end{bmatrix} \tag{18}
$$

or

$$
\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = [\Delta]\,[R_2]\,[\Delta] \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{19}
$$

where

$$[\Delta] = \begin{bmatrix} \exp(-jk_{1z}d) & 0 \\ 0 & \exp(-jk_{2z}d) \end{bmatrix} \tag{20}$$

and

$$[R_2] = \begin{bmatrix} R_{211} & R_{212} \\ R_{221} & R_{222} \end{bmatrix} \tag{21}$$

where

$$R_{211} = \frac{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} - \sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} + \sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}$$

$$R_{212} = \frac{2j\eta\sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} + \sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}$$

$$R_{221} = \frac{-2j\frac{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha}}{\eta}}{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} + \sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}$$

$$R_{222} = \frac{\sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha} - \sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha}}{\sqrt{1-\left(\frac{k_0}{k_1}\right)^2 \sin^2\alpha} + \sqrt{1-\left(\frac{k_0}{k_2}\right)^2 \sin^2\alpha}}$$

From Eq. (17) and Eq. (19), we have

$$\begin{bmatrix} B_\perp \\ B_\parallel \end{bmatrix} = \left([r] + [T]\left([\Delta][R_2][\Delta] - [R]\right)^{-1}[t]\right)\begin{bmatrix} A_\perp \\ A_\parallel \end{bmatrix} \tag{22}$$

Using these reflection coefficients, the initial amplitude and initial phase are calculated for paraboloidal reflector coated with chiral medium in the next section.

4. GEOMETRIC OPTICS FIELD FOR THE PARABOLOIDAL REFLECTED COATED WITH CHIRAL MEDIUM

Consider the reflection of a plane wave traveling along positive z-axis, incident on a paraboloidal reflector as shown in Figure 2. The equation

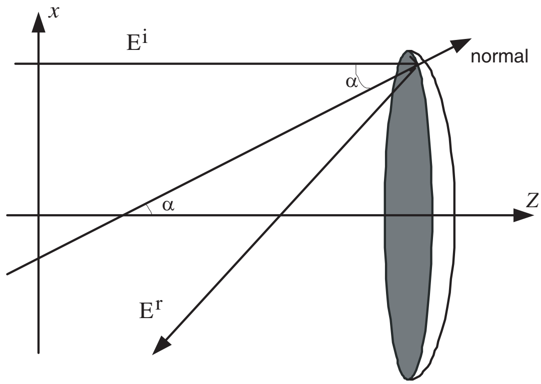

Figure 2. Paraboloidal reflector with chiral layer define by $\zeta = g(\xi, \eta)$.

of the surface of the paraboloidal reflector is given by

$$\zeta = f(\xi, \eta) = f - \frac{\rho^2}{4f} = f - \frac{\xi^2 + \eta^2}{4f}$$

where, (ξ, η, ζ) are the Cartesian coordinates of the point on the paraboloidal reflector, f is the focal length of the paraboloidal reflector and $\rho^2 = \xi^2 + \eta^2$. The incident wave traveling along z-axis is expressed as

$$\mathbf{E}_i = \mathbf{a}_x \exp(-jk_o z) \tag{23}$$

this wave makes an angle α with unit surface normal

$$\mathbf{a}_n = \sin\alpha\cos\gamma\mathbf{a}_x + \sin\alpha\sin\gamma\mathbf{a}_y + \cos\alpha\mathbf{a}_z \tag{24}$$

where, α and γ are given as

$$\sin\alpha = \frac{\rho}{\sqrt{\rho^2 + 4f^2}}, \quad \cos\alpha = \frac{2f}{\sqrt{\rho^2 + 4f^2}}, \quad \tan\gamma = \frac{\eta}{\xi} \tag{25}$$

The wave reflected from the reflector coated with chiral medium is given by

$$\mathbf{E}_r = \mathbf{E}_{ro} \exp\{jk_o(x \sin 2\alpha \cos\gamma + y \sin 2\alpha \sin\gamma + z \cos 2\alpha)\} \tag{26}$$

The initial value of reflected wave may be obtained by Snell's law of reflection as

$$\mathbf{E}_r = -\mathbf{E}_i + 2(\mathbf{E}_i \cdot \mathbf{a}_n)\mathbf{a}_n \tag{27}$$

and its rectangular components can be represented as

$$E_{xo} = B_\perp \sin^2\gamma - B_{||}\cos^2\gamma\cos 2\alpha \tag{28}$$
$$E_{yo} = -\cos\gamma\sin\gamma(B_{||}\cos 2\alpha + B_\perp) \tag{29}$$
$$E_{zo} = B_{||}\sin 2\alpha\cos\gamma \tag{30}$$

The reflected wave vector is found using the relation $\mathbf{p}^r = \mathbf{p}^i - 2(\mathbf{p}^i \cdot \mathbf{a}_n)\mathbf{a}_n$, which is derived from Snell's law, and is given as

$$\mathbf{p}^r = -2\sin 2\alpha\cos\gamma\mathbf{a}_x - 2\sin 2\alpha\sin\gamma\mathbf{a}_y - \cos 2\alpha\mathbf{a}_z \tag{31}$$

The Jacobian of transformation from the Cartesian to the ray coordinates is given by $J(t) = D(t)/D(0)$, where

$$\begin{aligned} D(t) &= \frac{\partial(x,y,z)}{\partial(\xi,\eta,t)} \\ &= \begin{vmatrix} 1+\frac{\partial p_x}{\partial\xi}t & \frac{\partial p_y}{\partial\xi}t & \frac{\partial\varsigma}{\partial\xi}+\frac{\partial p_z}{\partial\xi}t \\ \frac{\partial p_x}{\partial\eta}t & 1+\frac{\partial p_y}{\partial\eta}t & \frac{\partial\varsigma}{\partial\eta}+\frac{\partial p_z}{\partial\eta}t \\ p_x & p_y & p_z \end{vmatrix} \end{aligned} \tag{32}$$

$$D(t) = Ut^2 + Vt + W \tag{33}$$
$$J(t) = \frac{U}{W}t^2 + \frac{V}{W}t + 1 \tag{34}$$

solving the above determinant we get the values of

$$U = -\frac{\cos^4\alpha}{f^2}, \quad V = 2\frac{\cos^2\alpha}{f}, \quad W = -1$$

thus Jacobian ($J(t)$) becomes

$$J(t) = \frac{\cos^4\alpha}{f^2} - 2\frac{\cos^2\alpha}{f} + 1 = \left(\frac{\cos^2\alpha}{f}t - 1\right)^2 \tag{35}$$

The phase function s can be written as

$$s = \zeta + t \tag{36}$$

The GO field is given by

$$\mathbf{E}_r(r) = \mathbf{E}_{ro}\left(\frac{\cos^2\alpha}{f}t - 1\right)^{-1}\exp\{-jk_o(\zeta + t)\} \tag{37}$$

The GO field can also be written in their rectangular components as follows

$$\begin{aligned} E_{rx}(r) &= \left(B_\perp \sin^2\gamma - B_\| \cos^2\gamma \cos 2\alpha\right)\left(\frac{\cos^2\alpha}{f}t-1\right)^{-1} \\ &\quad \times \exp\{-jk_o(\zeta+t)\} \end{aligned} \tag{38}$$

$$\begin{aligned} E_{ry}(r) &= -\cos\gamma\sin\gamma\left(B_\|\cos 2\alpha + B_\perp\right)\left(\frac{\cos^2\alpha}{f}t-1\right)^{-1} \\ &\quad \times \exp\{-jk_o(\zeta+t)\} \end{aligned} \tag{39}$$

$$E_{rz}(r) = B_\| \sin 2\alpha \cos\gamma \left(\frac{\cos^2\alpha}{f}t-1\right)^{-1} \times \exp\{-jk_o(\zeta+t)\} \tag{40}$$

The focal point equation is obtained by putting the Jacobian equal to zero, the following equation is obtained

$$f = t\cos^2\alpha \tag{41}$$

At the point satisfying Eq. (41), the GO field becomes infinite. So to find finite field around this point, we use Maslov's method. To evaluate field by Eq. (9), for which Eq. (10) is calculated, as follows

$$J(t)\frac{\partial(p_x,p_y)}{\partial(x,y)} = \frac{1}{D(0)}\begin{vmatrix} \frac{\partial p_x}{\partial\xi} & \frac{\partial p_y}{\partial\xi} & 0 \\ \frac{\partial p_x}{\partial\eta} & \frac{\partial p_y}{\partial\eta} & 0 \\ 0 & 0 & \frac{\partial z}{\partial t} \end{vmatrix} = \frac{\cos^4\alpha\cos^2 2\alpha}{f^2} \tag{42}$$

the phase function in Eq. (9) can be calculated as

$$s(p_x,p_y) = \zeta + \frac{z-\zeta}{p_z} - (\xi + p_x t)p_x - (\eta + p_y t)p_y + xp_x + yp_y$$

by putting $\zeta = f\cos 2\alpha/\cos^2\alpha$, $\eta = 2f\tan\alpha\sin\gamma$ and $\xi = 2f\tan\alpha\cos\gamma$ the phase function for these rays are

$$s(p_x,p_y) = 2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha \tag{43}$$

by putting Eq. (42) and (43) in Eq. (9), the field expression which is valid around the focal point of the paraboloidal reflector coated with chiral medium is given by

$$\begin{aligned} \mathbf{E}_r(r) &= \frac{k_0}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}_{ro}\left(\frac{\cos^4\alpha\cos^2 2\alpha}{f^2}\right)^{\frac{-1}{2}}\exp\{-jk_o(2f \\ &\quad -x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha)\}dp_x dp_y \end{aligned} \tag{44}$$

Conversion from Cartesian coordinates (p_x, p_y) to ray coordinates (ξ, η) is given as

$$\frac{\partial(p_x, p_y)}{\partial(\xi, \eta)} = \frac{\cos^4\alpha\cos 2\alpha}{f^2} \tag{45}$$

Changing from (ξ, η) to angular coordinates (α, γ) by

$$\frac{\partial(\xi, \eta)}{\partial(\alpha, \gamma)} = \frac{4f^2\sin\alpha}{\cos^3\alpha} \tag{46}$$

and using the polar coordinates (r, θ, ϕ) instead of Cartesian coordinates (x, y, z) gives

$$\begin{aligned}\mathbf{E}_r(r) = &\frac{j2k_0 f}{\pi}\int_0^H\int_0^{2\pi}\mathbf{E}_{ro}\tan\alpha\exp\{-jk_o(2f\\&-r\sin\theta\sin 2\alpha\cos(\phi-\gamma)-r\cos\theta\cos 2\alpha)\}d\alpha d\gamma\end{aligned} \tag{47}$$

The upper limit of integration with respect to α is culculated as

$$H = \tan^{-1}(D/2f)$$

where, D is the height of the paraboloidal reflector from horizontal axis. The integration with respect to γ in Eq. (47) can be performed by using the integral form of Bessel function as given by

$$\frac{1}{2\pi j^n}\int_0^{2\pi}\exp(ja\cos(\gamma-\phi))\exp(jn\gamma)d\gamma = J_n(a)$$

where J_n is Bessel function of nth order. The rectangular components of reflected wave is given by

$$\begin{aligned}E_{rx}(r) = &jk_o f\int_0^H 2\tan\alpha\left\{J_0(k_0 r\sin\theta\sin 2\alpha)(B_\perp - B_\parallel\cos 2\alpha)\right.\\&\left.-J_2(k_o r\sin\theta\sin 2\alpha)\cos 2\phi(B_\perp + B_\parallel\cos 2\alpha)\right\}\\&\times\exp\{-jk_o(2f + r\cos\theta\cos 2\alpha)\}d\alpha\end{aligned} \tag{48}$$

$$\begin{aligned}E_{ry}(r) = &-jk_o f\int_0^H 2\tan\alpha\sin 2\phi(B_\perp + B_\parallel\cos 2\alpha)J_2(k_o r\sin\theta\sin 2\alpha)\\&\times\exp\{-jk_o(2f + r\cos\theta\cos 2\alpha)\}d\alpha\end{aligned} \tag{49}$$

$$\begin{aligned}E_{rz}(r) = &k_o f\int_0^H 8\sin^2\alpha\cos\phi B_\parallel J_1(k_o r\sin\theta\sin 2\alpha)\\&\times\exp\{-jk_o(2f + r\cos\theta\cos 2\alpha)\}d\alpha\end{aligned} \tag{50}$$

5. RESULTS AND DISCUSSION

To study the field behavior around the focal region Eq. (47) was solved numerically. In all the simulations $k_o = 1$, $f = 100$, $H = \pi/4$ and $\mu = \mu_o$ were used. The effect of thickness of chiral layer d, chirality parameter β and the relative permittivity ε on the focal region field, simulations were carried out by varying these parameters.

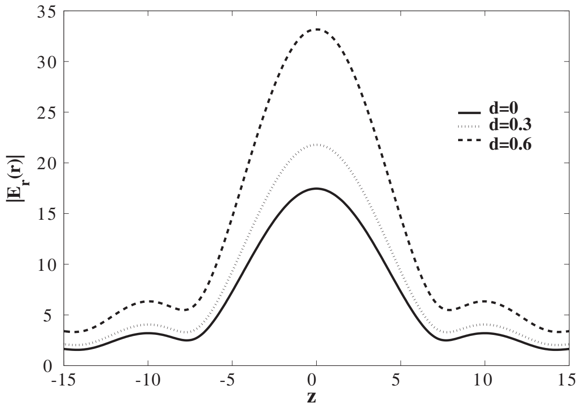

Figure 3. Plot for $|E_r(r)|$ along z-axis, with the dielectric layer of varying thickness d.

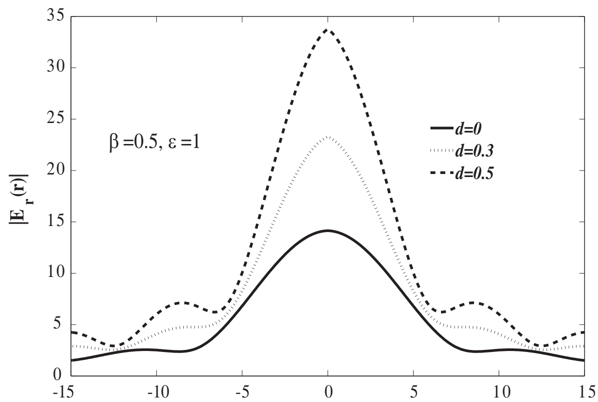

Figure 4. Plot for $|E_r(r)|$ along z-axis, with the dielectric layer of varying thickness d. The impedance of chiral medium is equal to that of free space.

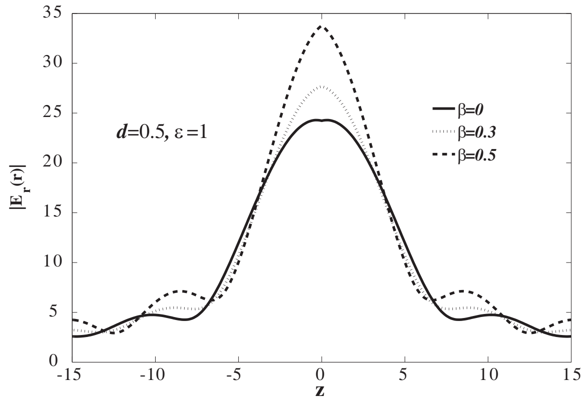

Figure 5. Plot for $|E_r(r)|$ along z-axis, showing the effect of chirality parameter β. The impedance of chiral medium is equal to that of free space.

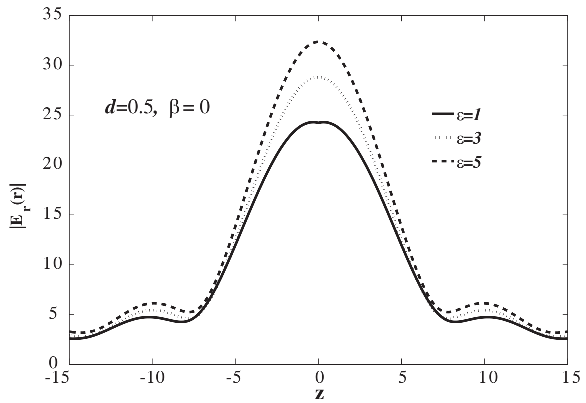

Figure 6. Plot for $|E_r(r)|$ along z-axis, showing the effect of relative permittivity of dielectric layer.

Figure 3 shows the effect of increase in the value of d keeping $\beta = 0$ and $\varepsilon = 3$, in this case the paraboloidal reflector is coated with ordinary dielectric medium. The plots show that by increasing the thickness of coated material the field strength around the focal region increases. Figure 4 shows the effect of increase in the value of d keeping $\beta = 0.5$ and $\varepsilon = 1$, as a consequence the impedance of chiral medium becomes equal to that of free space. As evident from the plots, increase in the value of d increases the field strength around the focal region. Figure 5 shows the effect of increase in the value of β keeping $d = 0.5$ and $\varepsilon = 1$.

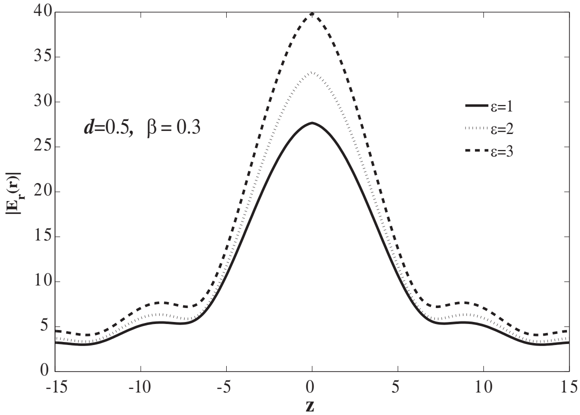

Figure 7. Plot for $|E_r(r)|$ along z-axis, showing the effect of relative permittivity of chiral layer.

This figure shows that increase in the value of β also increases the field strength around the focal region. Figure 6 shows the effect of increase in the value of ε keeping $\beta = 0$ and $d = 0.5$, and the graph shows that by increasing the value of ε, we observe an increase in the field strength around the focal region. Figure 7 shows the effect of changing the value of ε keeping $\beta = 0.3$, $d = 0.5$, the field strength around the focal region again shows an increase with the increase in the value of ε.

6. CONCLUSIONS

The geometrical optics field reflected from a paraboloidal reflector coated with chiral medium was calculated using Maslov's method. The reflected field was analyzed numerically, and the results were discussed. These results show that with increase in the value of chirality parameter, thickness of the chiral coated layer and relative permittivity, the absolute value of the field strength of the paraboloidal reflector around the focal region increases.

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