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FRACTIONAL DUAL INTERFACE IN CHIRAL NIHILITY MEDIUM

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Abstract—A perfect electric conductor placed in chiral nihility medium and excited by a uniform plane wave has been considered as an original problem. Using fractional operators, solutions to Maxwell equations which may be regarded as intermediate step between the original solution and dual to the original solution are determined. Each fractional operator is composed of fractional curl operator. As more than one dual to original solution exists, so in each case, corresponding impedance of the fractional dual reflector has been determined.

1. INTRODUCTION

Lakhtakia introduced term nihility for such medium, whose $\epsilon = 0$, $\mu = 0$ [1]. Tretyakov etȧl., extended the nihility concept to chiral medium and introduced the concept of chiral nihility [2]. Chiral nihility is a medium with following properties of constitutive parameters at certain frequency [2]

$$\epsilon = 0, \qquad \mu = 0, \qquad \kappa \neq 0$$

where κ is chirality parameter of the medium. Thus the resulting constitutive relations for isotropic chiral nihility medium reduce to [2–4]

$$\mathbf{D} = -j\kappa\sqrt{\epsilon_0\mu_0}\,\mathbf{H}$$
$$\mathbf{B} = j\kappa\sqrt{\epsilon_0\mu_0}\,\mathbf{E}$$

Taking time dependence as $\exp(j\omega t)$, Maxwell equations for chiral nihility medium can be written as

$$\nabla \times \mathbf{E} = k_0\kappa\mathbf{E} \tag{1a}$$
$$\nabla \times \mathbf{H} = k_0\kappa\mathbf{H} \tag{1b}$$

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where $k_0 = \omega/c$ is the wavenumber in vacuum. In present discussion, the impedance of chiral nihility medium is considered close to the impedance of free space.

$$\eta = \eta_0 \lim_{\epsilon \to 0, \mu \to 0} \sqrt{\frac{\mu}{\epsilon}}$$

This means impedance of chiral nihility medium is close to impedance of free space η_0.

According to the duality principle for ordinary isotropic medium, if $(\mathbf{E}, \eta_0 \mathbf{H})$ is one solution to Maxwell equations then $(\eta_0 \mathbf{H}, -\mathbf{E})$ is another solution to Maxwell equations. Solution $(\eta_0 \mathbf{H}, -\mathbf{E})$ is termed as dual solution to the original solution $(\mathbf{E}, \eta_0 \mathbf{H})$, because Maxwell equations remain unchanged for both solutions. It is obvious from Equations (1) that if $(\mathbf{E}, \eta \mathbf{H})$ is one solution to Maxwell equations then other solutions for which Maxwell equations remain unchanged are $(\mp \eta \mathbf{H}, \pm \mathbf{E})$, $(-\mathbf{E}, -\eta \mathbf{H})$, and $(\pm \eta \mathbf{H}, \pm \mathbf{E})$. Set of solutions which may be discerned as dual solutions to the original solution $(\mathbf{E}, \eta \mathbf{H})$ is $\{(\eta \mathbf{H}, -\mathbf{E}), (-\eta \mathbf{H}, \mathbf{E}), (-\mathbf{E}, -\eta \mathbf{H})\}$ [5, 6].

For an isotropic and homogeneous medium, the solutions which may be regarded as intermediate step between the original and dual to the original solutions may be obtained using the following relations [7]

$$\mathbf{E}_{fd} = \frac{1}{(jk_0)^\alpha} (\nabla \times)^\alpha \mathbf{E}$$
$$\eta_0 \mathbf{H}_{fd} = \frac{1}{(jk_0)^\alpha} (\nabla \times)^\alpha \eta_0 \mathbf{H}$$

where $(\nabla \times)^\alpha$ means fractional curl operator. Mathematical recipe to fractionalize a linear operator is given in [4]. $k_0 = \omega\sqrt{\mu_0 \epsilon_0}$ is the wavenumber and η_0 is impedance of the medium. It may be noted that fd means fractional dual solutions. It is obvious from above set of equations that for $\alpha = 0$,

$$\mathbf{E}_{fd} = \mathbf{E}, \qquad \eta_0 \mathbf{H}_{fd} = \eta_0 \mathbf{H}$$

and for $\alpha = 1$

$$\mathbf{E}_{fd} = \eta_0 \mathbf{H}, \qquad \eta_0 \mathbf{H}_{fd} = -\mathbf{E}$$

The solution which may be regarded as intermediate step between the above two solutions may be obtained by varying parameter α between zero and one. For each value of α, a new solution is obatined. Various contributions related with the research appeared in the published literature. Naqvi and Rizvi [8] determined the sources corresponding to fractional dual solution. Naqvi et al. [9] extended the work [7] and discussed the behavior of fractional dual solution in an unbounded homogeneous chiral medium. Lakhtakia [10] derived theorem which shows that a dyadic operator which commutes with curl operator can be used to find new solutions of the Faraday and Ampere-Maxwell equations. Naqvi and Abbas studied the behavior of complex and higher orders fractional curl operator [11] and determine fractional dual solution for metamaterial having negative permittivity and negative permeability [12]. Fractional transmission lines, fractional waveguides, and fractional cavity resonators were discussed by Naqvi and coworkers [13–25]. Few other interesting contribution are reported in [26, 27].

We have considered a PEC interface placed in chiral nihility medium and excited by a uniform plane wave as our original problem. Our interest is to find fractional dual solution corresponding to our original problem. As there exists more than one dual solutions so each case has been taken into account one by one. In each case, corresponding impedance and characteristics of the fractional dual reflector have been determined.

2. FRACTIONAL DUAL INTERFACE AND CORRESPONDING IMPEDANCE

Consider a planner perfect electric conductor (PEC) interface of infinite extent. Interface is placed at $z = 0$ in chiral nihility medium. It is assumed that PEC interface is excited by a circularly polarized uniform plane wave. Electric and magnetic fields associated with the incident plane wave propagating in nihility medium are given by the following expressions

$$\mathbf{E}^{\rm inc} = e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z) \tag{2a}$$

$$\eta\mathbf{H}^{\rm inc} = je_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z) \tag{2b}$$

Fields reflected by the PEC interface in nihility medium ($z < 0$) are given by

$$\mathbf{E}^{\rm ref} = -e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z) \tag{3a}$$

$$\eta\mathbf{H}^{\rm ref} = je_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z) \tag{3b}$$

Total fields in nihility medium are given by

$$\mathbf{E} = \mathbf{E}^{\rm inc} + \mathbf{E}^{\rm ref} \tag{4a}$$

$$\eta\mathbf{H} = \eta\mathbf{H}^{\rm inc} + \eta\mathbf{H}^{\rm ref} \tag{4b}$$

It may be noted that total electric field is zero everywhere in chiral nihility medium including PEC interface, that is

$$\mathbf{E}^{\text{inc}} + \mathbf{E}^{\text{ref}} = 0$$

Both incident and reflected fields must satisfy Maxwell equations. Our interest is to find field which may be regarded as intermediate step between the original solution and dual to original solution. Our original solution is ($\mathbf{E}$, $\eta\mathbf{H}$) and there exists more than one dual to the original solution in nihility chiral medium. We select each dual to original solution one by one. Operator which connects original solution to a dual solution is different from operator which connects same original solution to other dual solution. Fractionalization of the operator yields solutions which may be regarded as intermediate step between the original solution and dual to original solution.

Let us find fractional dual solution for each case:

Case 1: Operators $\frac{(\nabla\times)^\alpha}{(\mp jk_0\kappa)^\alpha}$ may be used to find fractional dual solution between original solution $(\mathbf{E}, \eta\mathbf{H})$ and dual to original solution $(\eta\mathbf{H}, -\mathbf{E})$, that is

$$\begin{aligned}
\mathbf{E}_{\text{fd}}^{\text{inc}} &= \frac{(\nabla\times)^\alpha}{(-jk_0\kappa)^\alpha}\mathbf{E}^{\text{inc}} = (j)^\alpha e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z) \\
\eta\mathbf{H}_{\text{fd}}^{\text{inc}} &= \frac{(\nabla\times)^\alpha}{(-jk_0\kappa)^\alpha}\eta\mathbf{H}^{\text{inc}} = (j)^\alpha je_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z) \\
\mathbf{E}_{\text{fd}}^{\text{ref}} &= \frac{(\nabla\times)^\alpha}{(jk_0\kappa)^\alpha}\mathbf{E}^{\text{ref}} = -(-j)^\alpha e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z) \\
\eta\mathbf{H}_{\text{fd}}^{\text{ref}} &= \frac{(\nabla\times)^\alpha}{(jk_0\kappa)^\alpha}\eta\mathbf{H}^{\text{ref}} = (-j)^\alpha je_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z)
\end{aligned}$$

and is given by taking linear combination of above results

$$\begin{aligned}
\mathbf{E}_{\text{fd}} &= 2j \sin\left(\frac{\alpha\pi}{2}\right) e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z) \\
\eta\mathbf{H}_{\text{fd}} &= 2j \cos\left(\frac{\alpha\pi}{2}\right) e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \exp(-jk_0\kappa z)
\end{aligned}$$

Normalized surface impedance of the corresponding fractional dual reflector is

$$\underline{\underline{\mathbf{Z}}} = -\mathbf{j} \tan\left(\frac{\alpha\pi}{\mathbf{2}}\right)(\hat{\mathbf{x}}\hat{\mathbf{y}} - \hat{\mathbf{y}}\hat{\mathbf{x}}) \tag{5}$$

Impedance of the the fractional dual interface is anisotropic and is zero at $\alpha = 0$ while infinite at $\alpha = 1$. This means that for $\alpha = 0$, it seems that as if a PEC interface is placed at $z = 0$ while for $\alpha = 1$, it seems that as if a perfect magnetic conductor (PMC) interface is placed at $z = 0$.

Case 2: Operators $\frac{(\nabla\times)^\alpha}{(\pm jk_0\kappa)^\alpha}$ may be used to find fractional dual solution between $(\mathbf{E}, \eta\mathbf{H})$ and $(-\eta\mathbf{H}, \mathbf{E})$

$$\mathbf{E}_{\rm fd} = -2j\sin\left(\frac{\alpha\pi}{2}\right)e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z)$$
$$\eta\mathbf{H}_{\rm fd} = 2j\cos\left(\frac{\alpha\pi}{2}\right)e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z)$$

Surface impedance of the fractional dual reflector is

$$\underline{\underline{\mathbf{Z}}} = j\tan\left(\frac{\alpha\pi}{2}\right)(\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}}) \tag{6}$$

Impedance of the the interface is anisotropic and is zero at $\alpha = 0$ while infinite at $\alpha = 1$. For $\alpha = 0$, a PEC interface is placed at $z = 0$ while for $\alpha = 1$, a PMC interface is placed at $z = 0$.

Case 3: Operator $\frac{(\nabla\times)^\alpha}{(-k_0\kappa)^\alpha}$ may be used to find fractional dual solution between $(\mathbf{E}, \eta\mathbf{H})$ and $(-\mathbf{E}, -\eta\mathbf{H})$

$$\mathbf{E}_{\rm fd} = -2j\sin(\alpha\pi)\,e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z)$$
$$\eta\mathbf{H}_{\rm fd} = 2j\cos(\alpha\pi)\,e_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})\exp(-jk_0\kappa z)$$

Surface impedance of the fractional dual reflector is

$$\underline{\underline{\mathbf{Z}}} = j\tan(\alpha\pi)(\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}}) \tag{7}$$

Impedance of the the interface is anisotropic and is zero both at $\alpha = 0$ and $\alpha = 1$. It is interesting that both for $\alpha = 0$ and $\alpha = 1$, PEC interface is placed at $z = 0$.

3. CONCLUSIONS

It is noted that for all combinations of original and dual to original solution resulting impedance of fractional dual interface is anisotropic. For sets of solutions $\{(\mathbf{E}, \eta\mathbf{H}), (\eta\mathbf{H}, -\mathbf{E})\}$ and $\{(\mathbf{E}, \eta\mathbf{H}), (-\eta\mathbf{H}, \mathbf{E})\}$ impedance of fractional dual interface becomes zero and infinity for $\alpha = 0$ and $\alpha = 1$ respectively. It means that fractional dual interface behaves as PEC and PMC at $\alpha = 0$ and $\alpha = 1$ respectively. While for combination of solutions $\{(\mathbf{E}, \eta\mathbf{H}), (-\mathbf{E}, -\eta\mathbf{H})\}$, impedance of fractional dual interface yields zero value both for $\alpha = 0$ and $\alpha = 1$. In this case, fractional dual interface behaves as PEC both at $\alpha = 0$ and $\alpha = 1$. These behaviors serve as an interesting feature of nihility medium.

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