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A NEW METHOD FOR DETERMINING THE SCATTERING OF LINEAR POLARIZED ELEMENT ARRAYS

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Abstract—A new method for analyzing the RCS of array antennas is presented in this paper. In the proposed method, the total RCS of the array can be simply decomposed to array RCS factor and element RCS factor. By this decomposition, the effects of the array distribution and antenna elements on scattering can be clearly exhibited. Thus, the analyzing of scattering characteristic of array antennas becomes easier. Moreover, proposed method has good compatibility for calculating the RCS of the array antennas with the same array distribution for different element.

1. INTRODUCTION

Scattering from antennas has been the subject of study since 1950's. Dicke investigated the properties of antenna scattering with the intent of determining the antenna parameters [1]. Extensive work has been done with regard to dipole scattering and the effect of the terminal-load impedance [2, 3]. The radar cross section (RCS) of horns [4, 5], reflector antennas [5, 6], microstrip patches [7, 8], and arrays [9, 10] have also been studied.

Recently, Bartsevich introduced a new method of antenna RCS calculation for multi-element wire antenna array, which combines thin wire integral equation method with complex object RCS calculation method [11]. He et al. analyzed the RCS of an 2×2 array by exploiting surrounding fractal UC-EBG ground plane [12], Alves e Ricardo introduce a simplified technique to estimate the monostatic RCS of planar array antennas [13], and extensive work has been done with regard to cylindrical cavities and microstrip patch arrays [15].

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In most of above works, there are limits in analyzing the scattering characteristic of element and array. For example, in an array antenna's scattering analyzing, these reference methods only calculate the whole RCS but can not exhibit the effects of the antenna element and array distribution on the scattering, respectively. Moreover, these methods are complicated and time consuming in calculating the scattering of arrays.

Base on these reasons discussed above, we developed a simply method that the RCS of array antenna can be calculated by the multiplication of array scattering factor and element scattering factor. By this decomposition, the effects of antenna element and array distribution on the RCS are clearly exhibited. From this exhibition, the analyzing of scattering of array antenna can be more simply and efficiently.

2. THEORY OF ANTENNA RADAR CROSS SECTION

The basic equation of antenna scattering has been presented by several authors [2, 3]. It gives the total scattered field for a linearly polarized antenna when the antenna port is terminated with a load Z_L

$$\vec{E}_a^s(Z_L) = \vec{E}_s(Z_L^*) + \left[\frac{i\eta_0}{4\lambda R_a}\vec{h}\left(\vec{h}\cdot\vec{E}^i\right)\frac{e^{-jkr}}{r}\right]\Gamma_0 \qquad (1)$$

Where $Z_a = R_a + jX_a$ is the radiation impedance; $\vec{h}$ is the element effective height; $\vec{E}^i$ is incident wave; and $\Gamma_0 = \frac{Z_L - Z_a^*}{Z_L + Z_a^*}$. Other quantities in Equation (1) are the wavelength λ, the impedance of free space, $\eta_0 = 377\,\text{ohms}$, the free-space wave number, $k_0 = 2\pi/\lambda$ and the distance from the target to the observation point r. Traditionally, the antenna scattered field has been decomposed into two components called the structural and antenna (radiation) modes [11]. In Equation (l), $\vec{E}_s$ is the structural mode; the second term on the right-hand side is the antenna mode. When the load impedance is the complex conjugate of the radiation impedance, Z_L is called a conjugate-matched load.

3. FORMULATIONS OF CALCULATING ARRAY RCS

A plane array, whose elements are x-polarized, is shown in Fig. 1. The array has M and N elements along X and Y axes, respectively, in where M and N are integers. Assuming a unit magnitude and θ-polarized incident plane wave impinges on the array, and then the

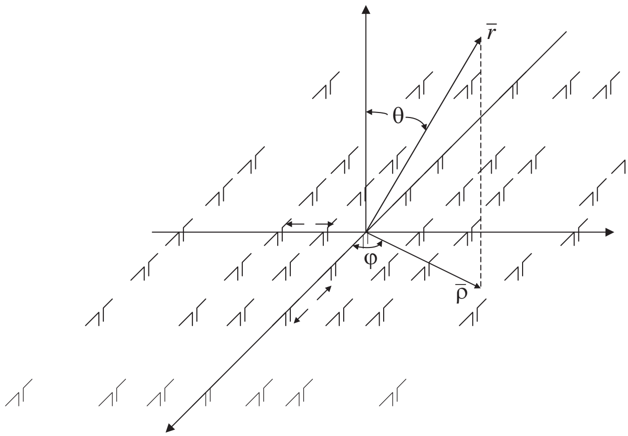

Figure 1. A dipole plane array antenna with $m \times n$ elements.

scattering field of the element is given as follows:

$$\vec{E}^{s}_{mn}\left(\vec{k}\right) = \sqrt{Z}\vec{A}_{mn}\left(\vec{k}\right)\frac{e^{-j\vec{k}\cdot\vec{r}_{mn}}}{r_{mn}} \tag{2}$$

where $\vec{A}_{mn}(\vec{k})$ is amplitude vector of the element, and $\vec{r}_{mn}$ represents vector distance from the (m, n) point to the observation point. $\vec{A}_{mn}(\vec{k})$ is determined by:

$$\vec{A}_{mn}\left(\vec{k}\right) = \frac{k}{4\pi j}\int\limits_{s_{mn}}\left(\vec{\vec{I}} - \hat{r}\hat{r}\right)\cdot\vec{J}_{mn}\cdot e^{j\vec{k}\cdot\vec{r}'_{mn}}\cdot ds \tag{3}$$

where S_{mn} is area of the element source. $\vec{J}_{mn} = J_{mn}(1 + \Gamma_{mn}e^{j\varphi_r})\cdot\hat{x}$ and $\vec{\vec{I}}$ are current on the element and a unit vector, respectively. $\vec{r}'$ is element's position vector [16] which calculated by follows:

$$\vec{r}' = \vec{d}_{mn} = mdx\cdot\hat{x} + ndy\cdot\hat{y}$$

when an incident plane wave is homogeneous, and assuming there are no mutual coupling effects between the elements, the induced current density on the elements has equal amplitude: $J_{mn} = J_{00}$ $(m, n = 0, 1, 2, \ldots, M_x, N_y)$, and as Fig. 2 shows, the reflection coefficient of element also has equal amplitude and $\Gamma_{mn}e^{j\varphi_r} = \Gamma_{00}e^{j\varphi_0}$. Hence, $\vec{J}_{mn} = J_{mn}(1 + \Gamma_{mn}e^{j\varphi_r})\cdot\hat{x} = J_{00}(1 + \Gamma_{00}e^{j\varphi_r})\cdot\hat{x} = \vec{J}_{00}$. If the

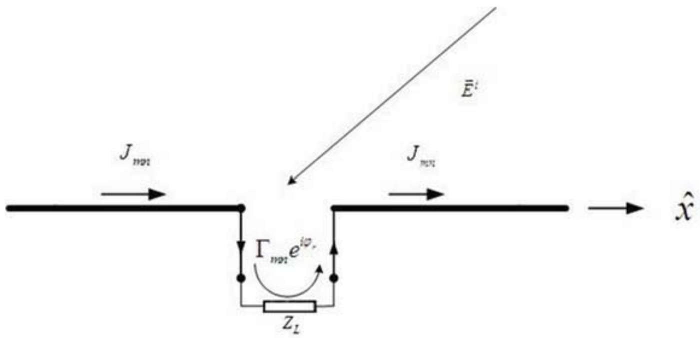

Figure 2. Feed port reflection of element with load Z_L in array.

observation point is far from the source point, $r_{mn} \approx r$. Then, it follows that:

$$\vec{A}_{mn}\left(\vec{k}\right) = \hat{x}\frac{k}{4\pi j}\int_{s_{00}}\left(\vec{\vec{I}} - \hat{r}\hat{r}\right) \cdot J_{00}e^{j\vec{k}\cdot\vec{d}_{mn}} \cdot ds = \vec{A}_{00}\left(\vec{k}\right)e^{j\vec{k}\cdot\vec{d}_{mn}} \tag{4}$$

and

$$\begin{aligned} \vec{E}^s\left(\vec{k}\right)_{mn} &= \sqrt{Z}\vec{A}_{00}\left(\vec{k}\right)\frac{e^{-j\vec{k}\cdot\left(\overleftarrow{r}_{00}-\vec{d}_{mn}\right)}}{r_{00}} \cdot e^{j\vec{k}\cdot\vec{d}_{mn}} \\ &= \vec{E}^s_e\left(\vec{k}\right) \cdot e^{2j\vec{k}\cdot\vec{d}_{mn}} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \vec{k} &= k_0\left[\sin(\theta)\cos(\varphi)\hat{x} + \sin(\theta)\sin(\varphi)\hat{y}\right] \\ 2\vec{k}\cdot\vec{d}_{mn} &= 2k_0\left[\sin(\theta)\cos(\varphi)(m-1)dx + \sin(\theta)\sin(\varphi)(n-1)dy\right] \end{aligned}$$

Then the total scattered field of array is obtained by summing over the field scattered by all elements:

$$\begin{aligned} \vec{E}^s\left(\vec{k}\right) &= \sum_{m=0}^{M_x}\sum_{n=0}^{N_y}\vec{E}\left(\vec{k}\right)_{mn} \\ &= \vec{E}^s_e\left(\vec{k}\right)\sum_{m=0}^{M_x}\sum_{n=0}^{N_y}2k_0\left[\sin(\theta)\cos(\varphi)mdx + \sin(\theta)\sin(\varphi)ndy\right] \end{aligned} \tag{6}$$

Then, σ_e and σ_A are defined as the element and array scattering factor respectively.

$$\sigma_e = 4\pi r^2 \cdot \left|\vec{E}_e^s\left(\vec{k}\right)\right|^2 \tag{7}$$

$$\sigma_A = \left|\sum_{m=0}^{M_x}\sum_{n=0}^{N_y} e^{j2k_0[\sin(\theta)\cos(\varphi)mdx+\sin(\theta)\sin(\varphi)ndy]}\right|^2 \tag{8}$$

According the above definition, the total RCS of the array then can be written as follows:

$$\sigma(\theta,\varphi) = \sigma_e(\theta,\varphi)\cdot\sigma_A(\theta,\varphi) \tag{9}$$

By now, the RCS of array antenna has been decomposed to two parts: the element scattering factor and array scattering factor.

4. NUMERICAL RESULTS AND DISCUSSION

Consider a dipole array antenna operating at wavelength λ_0 with the following parameters:

- $M_x = 59$ and $N_y = 59$ (plane array)
- $dx = 0.5\lambda_0$, $dy = 0.5\lambda_0$ $h = (\lambda_0/2)$
- Incident wave frequency equal to the operating frequency of the antenna ($\lambda = \lambda_0$)
- All of the reflection in the feed port of the element is: $\Gamma_{mn} = \Gamma_0 e^{j\varphi_{mn}}$ and $\Gamma_0 = 0.2$

Then, the scattering field of a dipole element is determined as Equation (1),

$$\vec{E}_\theta^s = \hat{x}\cdot\theta\frac{i\eta}{4\lambda R_a}h^2\cos(\theta)\cos(\varphi)\Gamma_0 e^{j\varphi_{mn}}\frac{e^{-jkr}}{r}$$

where, $R_a \approx 24.7(kl)^{2.4}$ [17] is the elements radiation resistance. Using Equations (7) and (8), the element scattering factor is

$$\sigma_e(\theta,\varphi) = 4\pi r^2\left|\frac{\eta}{4\lambda R_a}h^2\cos^2(\theta)\cos^2(\varphi)\Gamma_0\right|^2 \tag{10}$$

and the array scattering factor is:

$$\sigma_A = \left|\sum_{m=0}^{M_x=59}\sum_{n=0}^{N_y=59} e^{j2k_0[\sin(\theta)\cos(\varphi)mdx+\sin(\theta)\sin(\varphi)ndy]}\right|^2 \tag{11}$$

Hence, the RCS of the dipole array is:

$$\sigma(\theta,\varphi) = 4\pi r^2 \left|\frac{\eta_0}{4\lambda R_a} h^2 \cos^2(\theta)\cos^2(\varphi)\Gamma_0\right|^2 \cdot \left|\sum_{m=0}^{M_x=59}\sum_{n=0}^{N_y=59} e^{j2k_0[\sin(\theta)\cos(\varphi)mdx+\sin(\theta)\sin(\varphi)ndy]}\right|^2 \quad (12)$$

The validity of the proposed method is demonstrated by comparing results obtained using Equation (9) to those from a scattering formulation similar to the one described in [18]. From the method in [18], the total RCS of an array is:

$$\sigma(\theta,\varphi) = \frac{4\pi A_e^2}{\lambda^2}\left|\sum_{m=0}^{M_x}\sum_{n=0}^{N_y}\Gamma_{mn(\theta,\varphi)} e^{j\vec{k}\cdot\vec{d}_{mn}}\right|^2 \quad (13)$$

where $A_e \approx \cos(\theta)dxdy$.

The two solutions are compared in Fig. 3. The highest lobe at $\theta = 0°$ is primarily due to specular scattering from the aperture. The lobes near $\theta = \pm 90°$ is decreased sharply, because of the polarization efficiency [16] $p = \hat{x}\cdot\hat{\theta}$ is quickly coming to zero near the $\theta = \pm 90°$, therefore, the antenna mode scattering field is nearly to zero. In Fig. 3, it can be seen that the two solutions are good agreement with each other, except little amplitude errors. These errors are due to different approximations of the elements scattering field.

The array factor and element factor are compared with the array RCS in Fig. 4. It is shown clearly that, the array factor is a quickly variation function versus incident angle θ, but the element factor is a tardily variation function versus the incident angle θ, which is similar to the array's radiation situation. Additionally, the lobes of the array factor rise near $\pm 90°$ due to the edge diffraction.

Because the shape of the dipole array is square, as Fig. 5 shows, the array factor has four periods with the angle φ changing from -180 degree to 180 degree, which demonstrates that the scattering characteristics of the array depend on the array's distribution.

Near $\varphi = \pm 90°$, the antenna may receive the smallest energy because the polarization efficiency of the dipole p equals to zero, then the element factor and the array RCS is smallest. So, it is clearly illustrated in Fig. 6 that this method is more exact than reference method in $\varphi = \pm 90°$.

An examination of the computed RCS of array reveals several important characteristics:

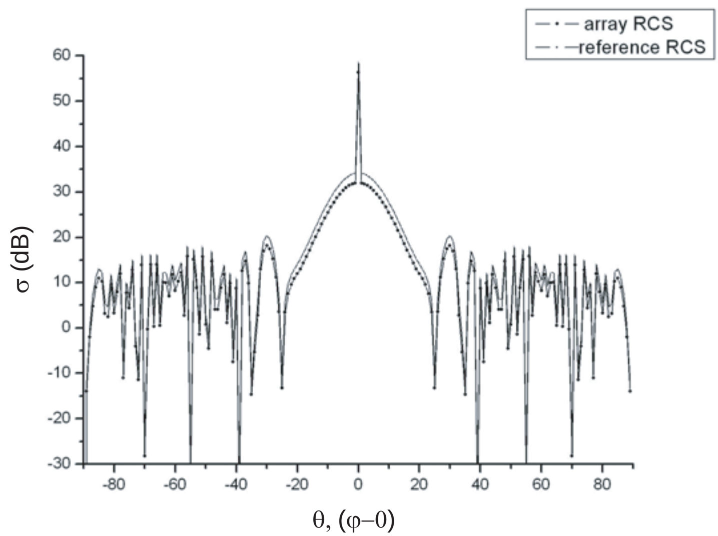

Figure 3. Comparison of the two different solutions for the 60×60 dipole arrays RCS.

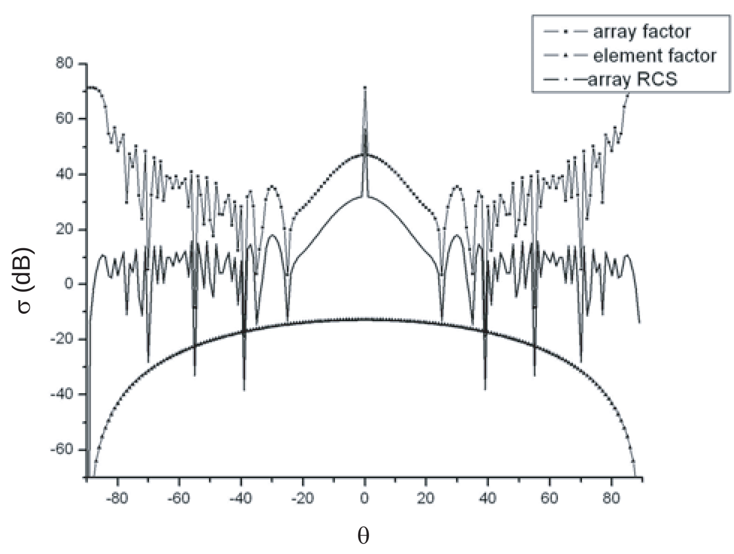

Figure 4. Array factor and the element factor compared with the array RCS.

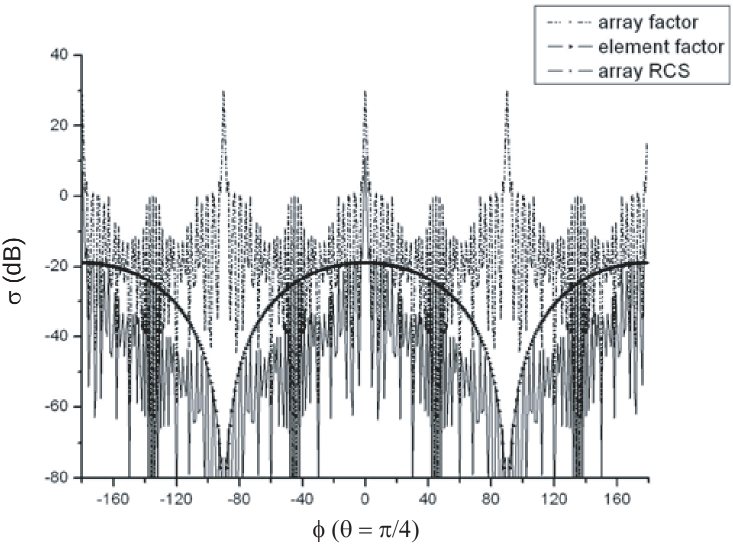

Figure 5. Array factor and the element factor compared with the array RCS with $\theta = \pi/4$.

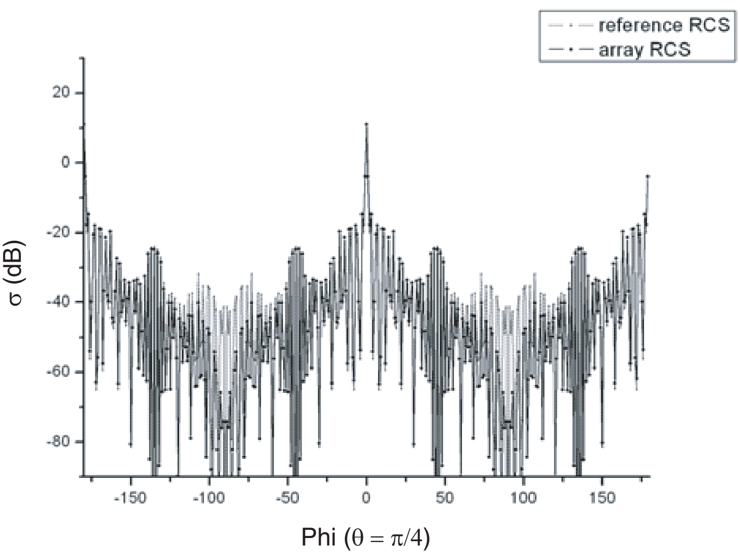

Figure 6. Comparison of the two different solutions for the array RCS with $\theta = \pi/4$.

(1) The level of the lobes of element factor increases with the magnitudes of the reflection coefficients the effective height.

(2) The scattering characteristics of the array depend on the array's distribution, such as the shape of array's aperture, number of elements (m, n), and the distance between elements.

(3) This method shows two main sources of scatter: the edge diffraction; and the array element mismatched impedances (Γ_0). The contribution of both to the array RCS was showing that the edge diffraction effects are dominant in most situations

(4) The results also show possibilities of RCS reduction. For the array factor, the shape of the array distribution can be modified. Instead of a round sharp edge it could be broken onto squared. Also the array characteristics could be optimized to avoid Bragg lobes in frequencies of interest.

5. CONCLUSION

In this paper, it was derived that the RCS of the array antenna can be decomposed to the multiplication of array factor and element factor. In this method, the total RCS of the array can be more simply calculated than others. The RCS of a 60×60 dipole arrays was calculated. From the numerical results, it can be clearly seen that the effects of array distribution and element on the scattering of array antennas. This is most useful for RCS prediction and reduction of the array antennas.

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