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INVESTIGATION ON THE ELECTROMAGNETIC SCATTERING OF PLANE WAVE/GAUSSIAN BEAM BY ADJACENT MULTI-PARTICLES

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Abstract—Based on the equivalence principle and the reciprocity theorem, the multiple scattering up to Nth-order by adjacent multi-particles is considered in this study. It is well known that the first-order solution can easily be obtained by calculating the scattered field from isolated targets when illuminated by a plane wave/Gaussian beam. However, due to the difficulty in formulating the couple scattered field, it is very difficult to find an analytical solution for the higher-order of the scattered field with considering the multiple scattering even for multi-canonical geometries, such as spheres, spheroids, and cubes. In order to overcome this problem, in this present work, the higher-order solutions of electromagnetic scattering for multi-particles are derived by employing the technique based on the reciprocity theorem and the equivalence principle. In specific, using the formulas of the composite scattering field obtained in this work, the bi-static scattering of plane wave/Gaussian beam by adjacent multi-spheres

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is calculated and the results are compared with those obtained from the numerical computations by the Time Domain Integral Equation Method (TDIEM).

1. INTRODUCTION

The study of electromagnetic scattering from discrete particles has been the subject of intensive investigation over the past several decades for its important application in the understanding of remotely sensed data [1–7]. Because of the simplicity of the geometry and the interest in practical applications, scattering of plane wave or Gaussian beam by isolated particles, such as spheres and cylinders, has been well studied both theoretically and experimentally [8–15]. However, when the composite scattered field from the discrete random media is studied, the interactions of the electromagnetic wave between different particles should be taken into account. Due to the important influence on total field, higher-order of the scattered field, i.e., considering the multiple scattering from the multi-particles, has attracted the attention of many researchers. Unfortunately, to obtain the scattering solution up to higher-order, it is necessary to treat electromagnetic interaction between objects that is not only non-plane-wave in character but have non-uniformities in amplitude and phase. For this type of problem, the exact analytical solutions can not be found except in a very small number of cases [15–17]. To overcome these difficulties, a new technique based on the reciprocity theorem is proposed by to evaluate the composite scattered field from two adjacent targets and an approximate solution for the scattered field up to the second-order is obtained [18]. In this paper, we extend the previous research from scattering of the two adjacent 2D targets with plane/Gaussian beam incidence to the case of multi-particles, and an approximate solution up to Nth-order for scattered field of a plane wave/Gaussian beam from adjacent multi-particles is derived employing the reciprocity theorem [18–22] and the equivalence principle [22]. In the solution, only the previous order scattered field of objects and the equivalent surface electric or/and magnetic current density induced by the incident beam are required.

As an example, in Section 3, the approach proposed in Section 2 is applied to obtain a solution for composite scattered field from adjacent multi-spheres. In Section 4, the bi-static scattering is discussed and the results are compared with numerical computations by employing the Time Domain Integral Equation Method.

2. SCATTERING OF A PLANE WAVE/GAUSSIAN BEAM BY ADJACENT PARTICLES

When we are interested in a limited region of space, it is well known that all uninteresting regions outside this space can be replaced by using equivalent sources, which include equivalent electric current and/or magnetic current [22, 23]. In general, when the composite scattered field from discrete particles is studied, the field outside the scatterers is of interest. Thus, the scatterers can be replaced by employing equivalent surface electric current and/or surface magnetic current. In this section, based on the equivalence principle and the reciprocity theorem, a solution for composite scattering of Gaussian beam by discrete particles is derived.

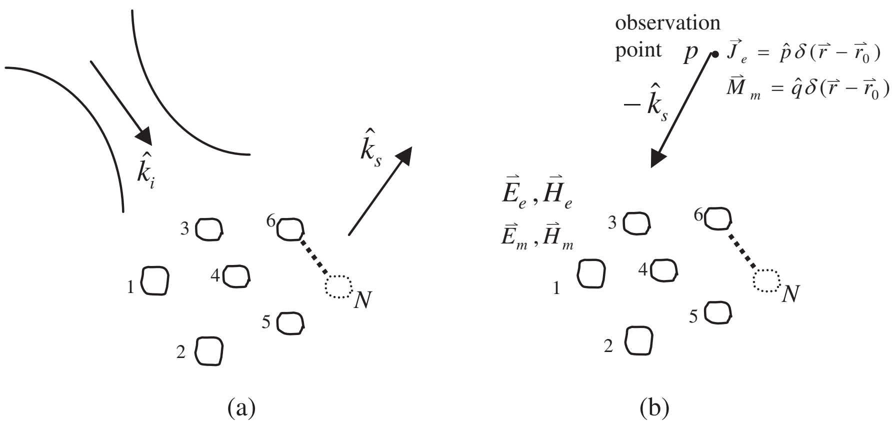


Figure 1. Geometry of the scattering problem.

As is shown in Fig. 1(a), N particles, which are randomly distributed, are illuminated by a plane wave/Gaussian beam. $\vec{E}_1^i$ and $\vec{H}_1^i$ denote the electric and magnetic field of the incident field, respectively. Without loss of the generality, suppose the incident wave would induce an equivalent electric current density $\vec{J}_n$ and an equivalent magnetic current density $\vec{M}_n$ on the surface of particle n in the absence of the other particles. Firstly, considering the equivalent electric current density $\vec{J}_n$ as the primary source, the electric and magnetic fields produced by $\vec{J}_n$ in the presence of the other $N-1$ particles are denoted by $\vec{E}_{Jn}$ and $\vec{H}_{Jn}$. On the other hand, considering the equivalent magnetic current density $\vec{M}_n$ as the excitation source, the electric and magnetic fields produced by $\vec{M}_n$ in the presence of the other $N-1$ particles are denoted by $\vec{E}_{Mn}$ and $\vec{H}_{Mn}$, respectively.

Now, let us consider another situation where the source $\vec{J}_n$ and $\vec{M}_n$ are removed and an infinitesimal electric current source $\vec{J}_e = \hat{p}\delta(\vec{r} - \vec{r}_0)$ and an infinitesimal magnetic current source $\vec{M}_m = \hat{q}\delta(\vec{r} - \vec{r}_0)$ are placed at the far-zone observation point P, as shown in Fig. 1(b). Here, the unit polarization vector $\hat{q}(\hat{v}_s$ or $\hat{h}_s)$ and $\hat{p}(\hat{v}_s$ or $\hat{h}_s)$ are related by $\hat{q} = \hat{k}_s \times \hat{p}$, where $\hat{k}_s$ denotes the unit vector of the propagation direction of the scattered field. In the presence of the particles except for the particle n, the electromagnetic fields produced by $\vec{J}_e$ and $\vec{M}_m$ are denoted by $\vec{E}_e$, $\vec{H}_e$ and $\vec{E}_m$, $\vec{H}_m$, respectively. Here, it should be emphasized that the fields $\vec{E}_e$, $\vec{H}_e$, $\vec{E}_m$ and $\vec{H}_m$ not only contain the fields excited by the infinitesimal sources but their scattered fields from the particles except for particle n.

Applying the reaction theorem [22] over the entire space, which results in

$$\begin{aligned}
&\int\limits_{S_n} \Big[\vec{J}_n \cdot \left(\vec{E}_e + \vec{E}_m\right) - \vec{J}_e \cdot \left(\vec{E}_{Jn} + \vec{E}_{Mn}\right) \\
&\quad + \vec{M}_m \cdot \left(\vec{H}_{Jn} + \vec{H}_{Mn}\right) - \vec{M}_n \cdot \left(\vec{H}_e + \vec{H}_m\right)\Big]\, dS \\
&= \oiint\limits_{s_\infty} \Big[\left(\vec{E}_{Jn} + \vec{E}_{Mn}\right) \times \left(\vec{H}_e + \vec{H}_m\right) \\
&\quad - \left(\vec{E}_e + \vec{E}_m\right) \times \left(\vec{H}_{Jn} + \vec{H}_{Mn}\right)\Big] \cdot d\vec{S} \\
&\quad + \sum_{l=1, l\neq n}^{N} \oiint\limits_{s_l} \Big[\left(\vec{E}_{Jn} + \vec{E}_{Mn}\right) \times \left(\vec{H}_e + \vec{H}_m\right) \\
&\quad - \left(\vec{E}_e + \vec{E}_m\right) \times \left(\vec{H}_{Jn} + \vec{H}_{Mn}\right)\Big] \cdot d\vec{S}
\end{aligned} \tag{1}$$

where S_∞ represents a closed sphere at infinity. At an infinite distance away from the source, the following two relations exist

$$\left(\vec{E}_{Jn} + \vec{E}_{Mn}\right) = -Z_0 \hat{n} \times \left(\vec{H}_{Jn} + \vec{H}_{Mn}\right) \tag{2}$$

$$\left(\vec{E}_e + \vec{E}_m\right) = -Z_0 \hat{n} \times \left(\vec{H}_e + \vec{H}_m\right) \tag{3}$$

By substituting Eq. (2) and Eq. (3) into Eq. (1), the integral over S_∞ on the right-hand side vanishes. Based on the equivalence principle and the extinction theorem [22], also the surface integral over the surface of the $N-1$ particles vanishes since $\hat{n} \times \vec{E} = \hat{n} \times \vec{H} = 0$. Thus, Eq. (1) can be written as

$$\int\limits_{S_n} \left[\vec{J}_n \cdot \left(\vec{E}_e + \vec{E}_m\right) - \vec{J}_e \cdot \left(\vec{E}_{Jn} + \vec{E}_{Mn}\right) \right.$$
$$\left. + \vec{M}_m \cdot \left(\vec{H}_{Jn} + \vec{H}_{Mn}\right) - \vec{M}_n \cdot \left(\vec{H}_e + \vec{H}_m\right)\right] dS = 0 \qquad (4)$$

Suppose the particles are all perfectly electric conducting scatterers, as well known, the surface magnetic current density is $\vec{M}_n = 0$, and based on the reciprocity theorem, the elementary magnetic current source $\vec{M}_m$ equal to zero, too. Then, Eq. (4) can be simplified as follows

$$\int\limits_{S_n} \left(\vec{J}_n \cdot \vec{E}_e - \vec{J}_e \cdot \vec{E}_{Jn}\right) dS = 0 \qquad (5)$$

Suppose N perfectly magnetic particles are considered, the surface electric current $\vec{J}_n = 0$, and based on the reciprocity theorem, the elementary electric current source $\vec{J}_e$ equal to zero, too. Then, Eq. (4) can be written as

$$\int\limits_{S_n} \left(\vec{M}_m \cdot \vec{H}_{Mn} - \vec{M}_n \cdot \vec{H}_m\right) dS = 0 \qquad (6)$$

In the case of the scatterers are all dielectric, using Eq. (4)–Eq. (6), the following three equations are obtained

$$\int\limits_{S_n} \left(\vec{J}_n \cdot \vec{E}_e - \vec{J}_e \cdot \vec{E}_{Jn}\right) dS = 0 \qquad (7)$$

$$\int\limits_{S_n} \left(\vec{M}_m \cdot \vec{H}_{Mn} - \vec{M}_n \cdot \vec{H}_m\right) dS = 0 \qquad (8)$$

$$\int\limits_{S_n} \left(\vec{J}_n \cdot \vec{E}_m - \vec{J}_e \cdot \vec{E}_{Mn} + \vec{M}_m \cdot \vec{H}_{Jn} - \vec{M}_n \cdot \vec{H}_e\right) dS = 0 \qquad (9)$$

Because $\vec{J}_e$ and $\vec{M}_m$ are both infinitesimal sources, Eq. (7)–Eq. (8) can also be written as the following forms

$$\hat{p}\cdot\vec{E}_{Jn}=\int\limits_{S_n}\vec{J}_n\cdot\vec{E}_e dS \tag{10}$$

$$\hat{q}\cdot\vec{H}_{Mn}=\int\limits_{S_n}\vec{M}_n\cdot\vec{H}_m dS \tag{11}$$

$$\hat{p}\cdot\vec{E}_{Mn}-\hat{q}\cdot\vec{H}_{Jn}=\int\limits_{S_n}\left(\vec{J}_n\cdot\vec{E}_m-\vec{M}_n\cdot\vec{H}_e\right)dS \tag{12}$$

Then, using Eq. (10)–Eq. (11), the first-order scattered field from particle n and the secondary scattered field, i.e., rescattered field from the other $N-1$ particles when illuminated by the first-order scattered field of particle n, can be evaluated.

In Eqs. (10) and (11), if the multiple scattered field up to $N-1$ order scattered fields of $\vec{E}_{ed}$ and $\vec{H}_{md}$ from the particles except for the particle n are all taken into account, we should note that $\vec{E}_e$ and $\vec{H}_m$ can be obtained as

$$\begin{aligned}\vec{E}_e=&\ \vec{E}_{ed}+\sum_{l_1=1,l_1\neq n}^{N}\vec{E}_{eds}^{(1)l_1}+\sum_{l_1=1,l_1\neq n}^{N}\ \sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{E}_{eds}^{(2)l_1l_2}+\ldots\\&+\sum_{l_1=1,l_1\neq n}^{N}\ \sum_{l_2=1,l_2\neq n,l_1}^{N}\cdots\sum_{l_{N-1}=1,l_{N-1}\neq n,l_{N-2}}^{N}\vec{E}_{eds}^{(N-1)l_1l_2\ldots l_{N-1}}\end{aligned} \tag{13}$$

$$\begin{aligned}\vec{H}_m=&\ \vec{H}_{md}+\sum_{l_1=1,l_1\neq n}^{N}\vec{H}_{mds}^{(1)l_1}+\sum_{l_1=1,l_1\neq n}^{N}\ \sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{H}_{mds}^{(2)l_1l_2}+\ldots\\&+\sum_{l_1=1,l_1\neq n}^{N}\ \sum_{l_2=1,l_2\neq n,l_1}^{N}\cdots\sum_{l_{N-1}=1,l_{N-1}\neq n,l_{N-2}}^{N}\vec{H}_{mds}^{(N-1)l_1l_2\ldots l_{N-1}}\end{aligned} \tag{14}$$

In Eqs. (13) and (14), the multiple scattered field of $\vec{E}_{ed}$ and $\vec{H}_{md}$ by the particle n does not be considered. If these scattered fields are also taken into account, Eqs. (13) and (14) should be rewritten as the following expressions, i.e.,

$$\begin{aligned}\vec{E}_e &= \vec{E}_{ed} + \sum_{l_1=1, l_1\neq n}^{N} \vec{E}_{eds}^{(1)l_1} + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq n, l_1}^{N} \vec{E}_{eds}^{(2)l_1 l_2} + \ldots \\ &\quad + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq l_1}^{N} \ldots \sum_{l_N=1, l_N\neq n, l_{N-1}}^{N} \vec{E}_{eds}^{(N)l_1 l_2 \ldots l_N} \end{aligned} \tag{15}$$

$$\begin{aligned}\vec{H}_m &= \vec{H}_{md} + \sum_{l_1=1, l_1\neq n}^{N} \vec{H}_{mds}^{(1)l_1} + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq n, l_1}^{N} \vec{H}_{mds}^{(2)l_1 l_2} + \ldots \\ &\quad + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq l_1}^{N} \ldots \sum_{l_N=1, l_N\neq n, l_{N-1}}^{N} \vec{H}_{mds}^{(N)l_1 l_2 \ldots l_N} \end{aligned} \tag{16}$$

Substituting Eqs. (15) and (16) into Eqs. (10) and (11), the two following equations are obtained

$$\begin{aligned}\hat{p}\cdot\vec{E}_{Jn} &= \int\limits_{S_n} \vec{J}_n \cdot \left[\vec{E}_{ed} + \sum_{l_1=1, l_1\neq n}^{N} \vec{E}_{eds}^{(1)l_1} + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq n, l_1}^{N} \vec{E}_{eds}^{(2)l_1 l_2} + \ldots \right. \\ &\quad \left. + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq l_1}^{N} \ldots \sum_{l_N=1, l_N\neq n, l_{N-1}}^{N} \vec{E}_{eds}^{(N)l_1 l_2 \ldots l_N} \right] dS \\ &= \int\limits_{S_n} \vec{J}_n \cdot \vec{E}_{ed} dS + \int\limits_{S_n} \vec{J}_n \cdot \sum_{l_1=1, l_1\neq n}^{N} \vec{E}_{eds}^{(1)l_1} dS \\ &\quad + \int\limits_{S_n} \vec{J}_n \cdot \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq n, l_1}^{N} \vec{E}_{eds}^{(2)l_1 l_2} dS + \ldots \\ &\quad + \int\limits_{S_n} \vec{J}_n \cdot \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq l_1}^{N} \ldots \sum_{l_N=1, l_N\neq n, l_{N-1}}^{N} \vec{E}_{eds}^{(N)l_1 l_2 \ldots l_N} dS \\ &= \hat{p}\cdot \left[\vec{E}_{Jn}^{(0)} + \sum_{l_1=1, l_1\neq n}^{N} \vec{E}_{Jn}^{(1)l_1} + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq n, l_1}^{N} \vec{E}_{Jn}^{(2)l_1 l_2} + \ldots \right. \\ &\quad \left. + \sum_{l_1=1}^{N} \sum_{l_2=1, l_2\neq l_1}^{N} \ldots \sum_{l_N=1, l_N\neq n, l_{N-1}}^{N} \vec{E}_{Jn}^{(N)l_1 l_2 \ldots l_N} \right] \end{aligned} \tag{17}$$

$$\begin{aligned}\hat{q}\cdot\vec{H}_{Mn}=&\int_{S_n}\vec{M}_n\cdot\left[\vec{H}_{md}+\sum_{l_1=1,l_1\neq n}^{N}\vec{H}_{mds}^{(1)l_1}+\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{H}_{mds}^{(2)l_1l_2}+\ldots\right.\\&\left.+\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq l_1}^{N}\ldots\sum_{l_N=1,l_N\neq n,l_{N-1}}^{N}\vec{H}_{mds}^{(N)l_1l_2\ldots l_N}\right]dS\\=&\int_{S_n}\vec{M}_n\cdot\vec{H}_{md}dS+\int_{S_n}\vec{M}_n\cdot\sum_{l_1=1,l_1\neq n}^{N}\vec{H}_{mds}^{(1)l_1}dS\\&+\int_{S_n}\vec{M}_n\cdot\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{H}_{mds}^{(2)l_1l_2}dS+\ldots\\&+\int_{S_n}\vec{M}_n\cdot\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq l_1}^{N}\ldots\sum_{l_N=1,l_N\neq n,l_{N-1}}^{N}\vec{H}_{mds}^{(N)l_1l_2\ldots l_N}dS\\=&\,\hat{q}\cdot\left[\vec{H}_{Mn}^{(0)}+\sum_{l_1=1,l_1\neq n}^{N}\vec{H}_{Mn}^{(1)l_1}+\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{H}_{Mn}^{(2)l_1l_2}+\ldots\right.\\&\left.+\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq l_1}^{N}\ldots\sum_{l_N=1,l_N\neq n,l_{N-1}}^{N}\vec{H}_{Mn}^{(N)l_1l_2\ldots l_N}\right]\end{aligned}\tag{18}$$

where, $\vec{E}_{ed}$ is the direct electric field generated by $\vec{J}_e=\hat{p}\delta(\vec{r}-\vec{r}_0)$ and $\vec{H}_{md}$ is the direct magnetic field excited by $\vec{M}_m=\hat{q}\delta(\vec{r}-\vec{r}_0)$. $\vec{E}_{eds}^{(i)l_1l_2\ldots}$ and $\vec{H}_{mds}^{(i)l_1l_2\ldots}$ ($i=1,\,2,\,3,\,\ldots,\,N$) are the multiple scattered fields from the particles when illuminated by $\vec{E}_{ed}$ and $\vec{H}_{md}$, respectively.

In the following discussions, it should be noted that the particle n is an arbitrary one among all the particles. Therefore, the scattered field from the other particles can be calculated by using the similar way. Thus, the composite scattered field of Gaussian beam by the particles is obtained as

$$\vec{E}^{s}=\sum_{n=1}^{N}\left(\vec{E}_{Jn}+\vec{E}_{Mn}\right)\tag{19}$$

In Eq. (19), $\vec{E}_{Jn}$ can be evaluated by Eq. (17). Meanwhile, using the relation $\vec{E}_{Mn} = -Z_0 \hat{k}_s \times \vec{H}_{Mn}$, $\vec{E}_{Mn}$ is obtained by solving Eq. (18).

3. ELECTROMAGNETIC SCATTERING OF GAUSSIAN BEAM BY ADJACENT MULTI-SPHERES

In the previous section, the expressions, which can be applied to evaluate the scattered field from any adjacent particles with known geometries and dielectric properties, are derived. As an example of the application of the new approach, in this section an approximate analytical solution is derived for N multi-layered spheres. As shown in Fig. 2, the assumption is made that the multi-layered spheres lie in the far-field region of each other, i.e., the following conditions should be satisfied

$$\begin{cases} 2a_{l_1}^2/\lambda < \tilde{r}_{l_1 l_2} \\ 2a_{l_2}^2/\lambda < \tilde{r}_{l_1 l_2} \end{cases} \text{here } l_1, l_2 \in [1, N] \text{ and } l_1 \neq l_2 \tag{20}$$

where λ denotes the wavelength of the incident wave, $\vec{\tilde{r}}_{l_1 l_2} = \vec{r}_{l_2} - \vec{r}_{l_1}$ and $\tilde{r}_{l_1 l_2} = |\vec{r}_{l_2} - \vec{r}_{l_1}|$ is the distance between sphere centers of sphere l_1 and l_2, a_{l_1} and a_{l_2} are the radii of sphere l_1 and l_2, respectively. Suppose

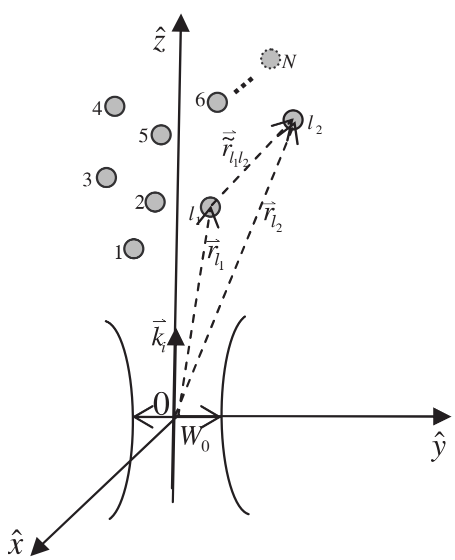


Figure 2. Geometry and coordinates of the scattering model.

a Gaussian beam, which propagates along the positive z-direction, is incident upon the multi-layered spheres. As is apparent in Fig. 2, the incident Gaussian beam has its focal point at origin point and W_0 denotes its beam-waist radius. Neglect the time factor $\exp(-i\omega t)$, the spatial distribution of the amplitude of the electric component $E_{\hat{\eta}}$ in the $z = 0$ plane is given by [11]

$$E_{\hat{\eta}}(x, y, 0) = E_0 \exp\left[-\frac{(x^2 + y^2)}{W_0^2}\right] \tag{21}$$

where E_0 denotes the amplitude of the electric component at the center of the beam and, in the following, E_0 is set to be 1. The polarization $\hat{\eta}$ may be chosen to be either $\hat{x}$ (TM) or $\hat{y}$ (TE). In the next step, the approximate analytical solutions of the scattered field by the multi-layered spheres are derived.

3.1. Soulutions to the First-order Scattered Field

When the observation point $\vec{r}_0$ lies in the far-field zone, the expressions of $\vec{E}_{ed}$ and $\vec{H}_{md}$ along $-\hat{k}_s$ are given by [17, 23]

$$\vec{E}_{ed}(r_0) = \frac{-ik_0 Z_0}{4\pi r_0} \exp(ik_0 r_0) \exp\left(-i\vec{k}_s \cdot \vec{r}\right) \hat{k}_s \times \hat{k}_s \times \hat{p} \tag{22}$$

$$\vec{H}_{md}(r_0) = \frac{-ik_0 Y_0}{4\pi r_0} \exp(ik_0 r_0) \exp\left(-i\vec{k}_s \cdot \vec{r}\right) \hat{k}_s \times \hat{k}_s \times \hat{q} \tag{23}$$

where $\hat{k}_s$ denotes the propagation direction of the scattered field, k_0 is the wave number of the incident wave in free space, $Z_0 = 1/Y_0$ characteristic impedance.

Invoking the equivalent principle, suppose the equivalent electric and magnetic current density on the surface of sphere n are denoted by $\vec{J}_n$ and $\vec{M}_n$, respectively. Using Eq. (17) and Eq. (18), the first-order scattered field of sphere n can be expressed as

$$\hat{p} \cdot \vec{E}_{Jn}^{(0)} = \int_{S_n} \vec{J}_n \cdot \vec{E}_{ed} dS \tag{24}$$

$$\hat{q} \cdot \vec{H}_{Mn}^{(0)} = \int_{S_n} \vec{M}_n \cdot \vec{H}_{md} dS \tag{25}$$

Substituting Eq. (22) and Eq. (23) into Eq. (24) and Eq. (25), using Stratton-Thu formulation [22], the first-order scattered field from sphere #1 when illuminated by the incident Gaussian beam is given by

$$\begin{aligned}\hat{p}\cdot\vec{E}_n^{(0)} &= \hat{p}\cdot\left(\vec{E}_{Jn}^{(0)}+\vec{E}_{Mn}^{(0)}\right)=-\frac{e^{ik_0r_0}}{4\pi k_0r_0}\exp\left(-i\vec{k}_s\cdot\vec{r}_n\right)k_0^2\hat{p}\\ &\quad\cdot\left[iZ_0\hat{k}_s\times\hat{k}_s\times\int\limits_{s_n}\vec{J}_n\exp\left(-i\vec{k}_s\cdot\vec{r}'\right)ds'\right.\\ &\quad\left.-i\hat{k}_s\times\int\limits_{s_n}\vec{M}_n\exp\left(-i\vec{k}_s\cdot\vec{r}'\right)ds'\right]\\ &= -\frac{e^{ik_0r_0}}{r_0}\exp\left(-i\vec{k}_s\cdot\vec{r}_n+i\vec{k}_i\cdot\vec{r}_n\right)\hat{p}\cdot\bar{S}_n^g\left(\hat{k}_i,\hat{k}_s\right)\end{aligned} \tag{26}$$

In Eq. (26), the relation $\vec{E}_{Mn}^{(0)}=-Z_0\hat{k}_s\times\vec{H}_{Mn}^{(0)}$ is used. $\vec{r}_n$ is the position vector of the sphere center of sphere n. $\bar{S}_n^g(\hat{k}_i,\hat{k}_s)$, the bi-static scattered electric field amplitude vector of sphere n when it is illuminated by the Gaussian beam, can be expressed as

$$\bar{S}_n^g\left(\hat{k}_i,\hat{k}_s\right)=\frac{i}{k}\left[SS_{1n}(\theta,\varphi)\hat{\varphi}+SS_{2n}(\theta,\varphi)\hat{\theta}\right] \tag{27}$$

For the case of TM polarization, i.e., the electric field of the Gaussian beam is polarized to the x-axis, in Eq. (27), $SS_{1n}(\theta,\varphi)$ and $SS_{2n}(\theta,\varphi)$can be written as [10]

$$\begin{aligned}SS_{1n}(\theta,\varphi) &= \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{2u+1}{u(u+1)}\left[iva_{uv}\pi_u^{|v|}(\cos\theta)\right.\\ &\quad\left.-b_{uv}\tau_u^{|v|}(\cos\theta)\right]\exp(iv\varphi)\end{aligned} \tag{27a}$$

$$\begin{aligned}SS_{2n}(\theta,\varphi) &= \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{2u+1}{u(u+1)}\left[a_{uv}\tau_u^{|v|}(\cos\theta)\right.\\ &\quad\left.+ivb_{uv}\pi_u^{|v|}(\cos\theta)\right]\exp(iv\varphi)\end{aligned} \tag{27b}$$

For the case of TE polarization, i.e., the electric field of the Gaussian beam is polarized to the y-axis, the following two expressions is obtained

$$SS_{1n}(\theta,\varphi) = \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{2u+1}{u(u+1)}\left[iva_{uv}\pi_u^{|v|}(\cos\theta)-b_{uv}\tau_u^{|v|}(\cos\theta)\right]\exp\left[iv\left(\varphi-\frac{\pi}{2}\right)\right] \quad (27c)$$

$$SS_{2n}(\theta,\varphi) = \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{2u+1}{u(u+1)}\left[a_{uv}\tau_u^{|v|}(\cos\theta)+ivb_{uv}\pi_u^{|v|}(\cos\theta)\right]\exp\left[iv\left(\varphi-\frac{\pi}{2}\right)\right] \quad (27d)$$

In Eqs. (27a)–(27d), $a_{uv} = g_{u,\mathrm{TM}}^{v}a_u^p$ and $b_{uv} = g_{u,\mathrm{TE}}^{v}b_u^p$, where $g_{u,\mathrm{TM}}^{v}$ and $g_{u,\mathrm{TE}}^{v}$ are the beam-shape coefficients appearing in Generalized Lorenz-Mie Theory (GLMT) [12, 24]. And the parameters a_u^p and b_u^p are the scattering coefficients for the plane wave [10].

Then, the first-order scattered field of the Gaussian beam by the multi-layered spheres can be written as

$$\hat{p}\cdot\vec{E}^{(0)} = \hat{p}\cdot\sum_{n=1}^{N}\vec{E}_n^{(0)} \quad (28)$$

where $\vec{E}_n^{(0)}(\vec{r})$ is expressed as Eq. (26).

3.2. Soulutions to the Second-order Scattered Field

In this part, the solutions for the second-order scattered field are derived.

In Eqs. (17) and (18), the second-order scattered field from sphere l_1 when illuminated by the first-order scattered field from sphere n can be written as

$$\hat{p}\cdot\vec{E}_{Jn}^{(1)} = \int_{S_n}\vec{J}_n\cdot\sum_{l_1=1,l_1\neq n}^{N}\vec{E}_{eds}^{(1)l_1}dS = \sum_{l_1=1,l_1\neq n}^{N}\int_{S_n}\vec{J}_n\cdot\vec{E}_{eds}^{(1)l_1}dS \quad (29)$$

$$\hat{q}\cdot\vec{H}_{Mn}^{(1)} = \int_{S_n}\vec{M}_n\cdot\sum_{l_1=1,l_1\neq n}^{N}\vec{H}_{mds}^{(1)l_1}dS = \sum_{l_1=1,l_1\neq n}^{N}\int_{S_n}\vec{M}_n\cdot\vec{H}_{mds}^{(1)l_1}dS \quad (30)$$

In Eqs. (29) and (30), $\vec{E}_{eds}^{(1)l_1}$ is the scattered field of $\vec{E}_{ed}$ by sphere l_1 and $\vec{H}_{mds}^{(1)l_1}$ is the scattered field of $\vec{H}_{md}$ by sphere l_1. Under the approximation that $\vec{E}_{ed}$ and $\vec{H}_{md}$ are considered as plane wave propagating along the $-\hat{k}_s$ direction, the expressions for $\vec{E}^{(1)l_1}_{eds}$ and $\vec{H}^{(1)l_1}_{mds}$ can be written as

$$\vec{E}^{(1)l_1}_{eds} = \frac{-ik_0 Z_0}{4\pi}\frac{e^{ik_0 r_0}}{r_0} e^{-ik_0 \hat{k}_s\cdot\vec{r}_{l_1}} \frac{e^{ik_0 r'}}{r'} \bar{S}^{l_1}_{eds}\left(-\hat{k}_s, \hat{r}'\right) \tag{31}$$

$$\vec{H}^{(1)l_1}_{mds} = \frac{-ik_0 Y_0}{4\pi}\frac{e^{ik_0 r_0}}{r_0} e^{-ik_0 \hat{k}_s\cdot\vec{r}_{l_1}} \frac{e^{ik_0 r'}}{r'} \bar{S}^{l_1}_{mds}\left(-\hat{k}_s, \hat{r}'\right) \tag{32}$$

where $\vec{r}_{l_1}$ is the position vector of the sphere l_1, $\bar{S}^{l_1}_{eds}(-\hat{k}_s, \hat{r}')$ and $\bar{S}^{l_1}_{mds}(-\hat{k}_s, \hat{r}')$ denote the electric and magnetic field bi-static scattering amplitude vector of multi-layered sphere l_1 when it is illuminated by a plane wave [25, 26]. r' denotes the distance between the sphere center of sphere l_1 and the point at $\vec{r}$, that is $r' = |\vec{r} - \vec{r}_{l_1}|$ and $\hat{r}' = (\vec{r} - \vec{r}_{l_1})/|\vec{r} - \vec{r}_{l_1}|$. Then Eq. (29) and Eq. (30) can be written as

$$\begin{aligned}\hat{p}\cdot\vec{E}^{(1)}_{Jn}\left(\vec{r}\right) &= \frac{-ik_0 Z_0}{4\pi}\frac{e^{ik_0 r_0}}{r_0}\sum_{l_1=1, l_1\neq n}^{N} e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}} \\ &\qquad \int\limits_{S_n} \vec{J}_n \cdot \frac{e^{ik_0 r'}}{r'} \bar{S}^{l_1}_{eds}\left(-\hat{k}_s, \hat{r}'\right) dS \end{aligned} \tag{33}$$

$$\begin{aligned}\hat{q}\cdot\vec{H}^{(1)}_{Mn} &= \frac{-ik_0 Y_0}{4\pi}\frac{e^{ik_0 r_0}}{r_0}\sum_{l_1=1, l_1\neq n}^{N} e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}} \\ &\qquad \int\limits_{S_n} \vec{M}_n \cdot \frac{e^{ik_0 r'}}{r'} \bar{S}^{l_1}_{mds}\left(-\hat{k}_s, \hat{r}'\right) dS \end{aligned} \tag{34}$$

Keeping in mind the conditions on the dimensions of, and the distance between, the spheres as specified in Eq. (20), Eq. (33) and Eq. (34) can be evaluated analytically. In Eq. (33), Eq. (34), noting that $\vec{r}$ is the position of the point on the surface of sphere n, then $|\vec{r} - \vec{r}_{l_1}| \approx |\vec{r}_n - \vec{r}_{l_1}|$, thus

$$\hat{r}' = \frac{\vec{r} - \vec{r}_{l_1}}{\left|\vec{r} - \vec{r}_{l_1}\right|} \approx \frac{\vec{r}_n - \vec{r}_{l_1}}{\left|\vec{r}_n - \vec{r}_{l_1}\right|} = \hat{\vec{r}}_{nl_1} \tag{35}$$

Under this approximation, in Eq. (33) and Eq. (34), $\bar{S}^{l_1}_{eds}(-\hat{k}_s, \hat{r}')$ and $\bar{S}^{l_1}_{mds}(-\hat{k}_s, \hat{r}')$ are not functions of the integration variables.

Therefore, these two equations can be rewritten as

$$\hat{p}\cdot\vec{E}^{(1)}_{Jn}=\frac{-ik_0Z_0}{4\pi}\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\cdot\int_{S_n}\vec{J}_n\frac{e^{ik_0r'}}{r'}dS$$

$$=\frac{ik_0Z_0}{4\pi}\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\cdot\left[\hat{\tilde{r}}_{l_1n}\times\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\vec{J}_n\frac{e^{ik_0r'}}{r'}dS\right]\quad(36)$$

$$\hat{q}\cdot\vec{H}^{(1)}_{Mn}=\frac{-ik_0Y_0}{4\pi}\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{mds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\cdot\int_{S_n}\vec{M}_n\frac{e^{ik_0r'}}{r'}dS$$

$$=\frac{-ik_0Y_0}{4\pi}\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\left[-\hat{\tilde{r}}_{l_1n}\times\bar{S}^{l_1}_{mds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\right]\cdot\int_{S_n}\vec{M}_n\frac{e^{ik_0r'}}{r'}dS$$

$$=\frac{-ik_0Y_0}{4\pi}\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\cdot\left[\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\vec{M}_n\frac{e^{ik_0r'}}{r'}dS\right]\quad(37)$$

Using Eq. (36) and Eq. (37), the secondary scattered field $\hat{p}\cdot\vec{E}^{(1)}(\vec{r})$ from sphere l_1 is given as

$$\hat{p}\cdot\vec{E}^{(1)}_{n}=\hat{p}\cdot\left(\vec{E}^{(1)}_{Jn}+\vec{E}^{(1)}_{Mn}\right)=k_0^2\frac{e^{ik_0r_0}}{r_0}\left\{\sum_{l_1=1,l_1\neq n}^{N}e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\right.$$

$$\left.\cdot\left[\hat{\tilde{r}}_{l_1n}\times\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\frac{iZ_0}{4k_0\pi}\frac{e^{ik_0r'}}{r'}\vec{J}_ndS-Z_0\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\frac{iY_0}{4k_0\pi}\frac{e^{ik_0r'}}{r'}\vec{M}_ndS\right]\right\}$$

$$=\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}\left\{e^{-ik_0\hat{k}_s\cdot\vec{r}_{l_1}}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\right.$$

$$\left.\cdot k_0^2\left[\hat{\tilde{r}}_{l_1n}\times\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\frac{iZ_0}{4k_0\pi}\frac{e^{ik_0r'}}{r'}\vec{J}_ndS-Z_0\hat{\tilde{r}}_{l_1n}\times\int_{S_n}\frac{iY_0}{4k_0\pi}\frac{e^{ik_0r'}}{r'}\vec{M}_ndS\right]\right\}$$

$$=\frac{e^{ik_0r_0}}{r_0}\sum_{l_1=1,l_1\neq n}^{N}\left\{e^{-ik_0\left(\hat{k}_s\cdot\vec{r}_{l_1}-\hat{k}_i\cdot\vec{r}_n\right)}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\cdot\frac{e^{ik_0\left|\vec{\tilde{r}}_{l_1n}\right|}}{\left|\vec{\tilde{r}}_{l_1n}\right|}\bar{S}^{ng}_{eds}\left(\hat{k}_i,\hat{\tilde{r}}_{l_1n}\right)\right\}\quad(38)$$

where $\bar{S}^{l_1}_{eds}(-\hat{k}_s,\hat{\tilde{r}}_{nl_1})$ is the bi-static scattered electric field amplitude vector of the sphere l_1 when it is illuminated by a plane wave and $\bar{S}^{ng}_{eds}(\hat{k}_i,\hat{\tilde{r}}_{l_1n})$ is the scattered electric field amplitude vector of sphere n when the incident wave is a Gaussian beam. Meanwhile, in Eq. (38), the relation $\vec{E}^{(1)}_{Mn}=-Z_0\hat{k}_s\times\vec{H}^{(1)}_{Mn}$ and Stratton-Thu formulation are employed. Because the sphere n is an arbitrary one among all the spheres, the secondary scattered fields of the Gaussian beam by all the multi-layered spheres can be written as

$$\begin{aligned}\hat{p}\cdot\vec{E}^{(1)} &= \frac{e^{ik_0 r_0}}{r_0}\sum_{n=1}^{N}\sum_{l_1=1,l_1\neq n}^{N}\left[e^{-\left(\hat{k}_s\cdot\vec{r}_{l_1}-\hat{k}_i\cdot\vec{r}_n\right)}\bar{S}^{l_1}_{eds}\left(-\hat{k}_s,\hat{\tilde{r}}_{nl_1}\right)\right.\\ &\quad\left.\cdot\frac{e^{ik_0\left|\vec{\tilde{r}}_{l_1n}\right|}}{\left|\vec{\tilde{r}}_{l_1n}\right|}\bar{S}^{ng}_{eds}\left(\hat{k}_i,\hat{\tilde{r}}_{l_1n}\right)\right]\end{aligned}\tag{39}$$

3.3. Soutions to the Third- and Higher-order of the Scattered Field

In this part, the solutions for the third- and higher-order of the scattered field are derived. First, we derive the solutions for the third-order scattered field.

Suppose multi-layered sphere n, l_1 and l_2 are arbitrary three scatterers among spheres, in order to calculate the third-order scattered field, i.e., the rescattered field of secondary scattered field from sphere l_1. To deal this problem, sphere n can be considered as one scatterer and the spherer-sphere pair, which is composed of sphere l_1 and l_2, as the other one. Because the first-order and the secondary scattered field has been solved in 3.1 and 3.2, as given in Eq. (17) and Eq. (18), this problem can be reduced to calculate the multiple scattered fields $\vec{E}^{(2)l_1l_2}_{eds}$ and $\vec{H}^{(2)l_1l_2}_{mds}$. In Eq. (17) and Eq. (18), the third-order scattered field can be written as

$$\begin{aligned}\hat{p}\cdot\vec{E}^{(2)}_{Jn} &= \int_{S_n}\vec{J}_n\cdot\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{E}^{(2)l_1l_2}_{eds}dS\\ &= \sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\int_{S_n}\vec{J}_n\cdot\vec{E}^{(2)l_1l_2}_{eds}dS\end{aligned}\tag{40}$$

$$\begin{aligned}\hat{q}\cdot\vec{H}^{(2)}_{Mn} &= \int_{S_n}\vec{M}_n\cdot\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\vec{H}^{(2)l_1l_2}_{mds}dS\\ &= \sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\int_{S_n}\vec{M}_n\cdot\vec{H}^{(2)l_1l_2}_{mds}dS\end{aligned}\tag{41}$$

Suppose an infinitesimal electric current source $\vec{J}_{en} = \hat{\xi}\delta(\vec{r} - \vec{r}_n)$ and an infinitesimal magnetic current source $\vec{M}_{mn} = \hat{\zeta}\delta(\vec{r} - \vec{r}_n)$ are placed at the point of sphere center of sphere n, here, the unit polarization vector $\hat{\xi}$ and $\hat{\zeta}$ are related by $\hat{\zeta} = \hat{k}_{2s} \times \hat{\xi}$, denotes the unit vector of the propagation direction of the excited field by $\vec{J}_{en} = \hat{\xi}\delta(\vec{r} - \vec{r}_n)$. Because the dimension of the spheres is very small relative to the distance between the spheres, the electric field produced by $\vec{J}_{en}$ and the magnetic field produced by $\vec{M}_{mn}$ at the point of sphere l_2 can be written as

$$\vec{E}_{edn} = -\frac{ik_0 Z_0}{4\pi\left|\vec{r}_{l_2} - \vec{r}_n\right|} e^{-ik_0 \hat{r}_{l_2 n}\cdot\vec{r}_n} \exp\left[ik_0 \hat{r}_{l_2 n}\cdot\vec{r}\right] \hat{r}_{l_2 n}\times\hat{r}_{l_2 n}\times\hat{\xi} \quad (42)$$

$$\vec{H}_{edn} = -\frac{ik_0 Y_0}{4\pi\left|\vec{r}_{l_2} - \vec{r}_n\right|} e^{-ik_0 \hat{r}_{l_2 n}\cdot\vec{r}_n} \exp\left[ik_0 \hat{r}_{l_2 n}\cdot\vec{r}\right] \hat{r}_{l_2 n}\times\hat{r}_{l_2 n}\times\hat{\zeta} \quad (43)$$

where, $\hat{r}_{l_2 n} = \frac{(\vec{r}_{l_2} - \vec{r}_n)}{|\vec{r}_{l_2} - \vec{r}_n|}$.

Considering the conditions on the dimensions of, and the distance between the spheres as specified in Eq. (20), the scattered fields of $\vec{E}_{edn}$ and $\vec{H}_{edn}$ by sphere l_2 are obtained as

$$\vec{E}^{(1)l_2}_{edn} = -\frac{ik_0 Z_0}{4\pi\left|\vec{r}_{l_2} - \vec{r}_n\right|} e^{ik_0\left|\vec{r}_{l_2} - \vec{r}_n\right|} \frac{e^{ik_0 r'}}{r'} \bar{S}_{el_2}\left(\hat{r}_{l_2 n}, \hat{r}'\right) \quad (44)$$

$$\vec{H}^{(1)l_2}_{edn} = -\frac{ik_0 Y_0}{4\pi\left|\vec{r}_{l_2} - \vec{r}_n\right|} e^{ik_0\left|\vec{r}_{l_2} - \vec{r}_n\right|} \frac{e^{ik_0 r'}}{r'} \bar{S}_{ml_2}\left(\hat{r}_{l_2 n}, \hat{r}'\right) \quad (45)$$

Suppose the field $\vec{E}_{ed}$ and $\vec{H}_{md}$ would induce an equivalent electric current density $\vec{J}_{edl_1}$ and an equivalent magnetic current density $\vec{M}_{mdl_1}$ on the surface of sphere l_1 in the absence of the other two scatterers. Then, based on the reciprocity theorem, the fields $\vec{E}^{(2)l_1 l_2}_{eds}$ and $\vec{H}^{(2)l_1 l_2}_{mds}$ can be calculated by using the following two equations

$$\begin{aligned}\vec{E}^{(2)l_1 l_2}_{eds} = & -\frac{ik_0 Z_0}{4\pi\left|\vec{r}_{l_2} - \vec{r}_n\right|} e^{ik_0\left|\vec{r}_{l_2} - \vec{r}_n\right|} \frac{-ik_0 Z_0}{4\pi r_0} \exp(ik_0 r_0) \\ & \exp\left(-i\vec{k}_s\cdot\vec{r}_{l_1}\right) \int_{S_{l_1}} \frac{e^{ik_0 r'}}{r'} \bar{S}_{el_2}\left(\hat{r}_{l_2 n}, \hat{r}'\right)\cdot\vec{J}_{edl_1} dS \end{aligned} \quad (46)$$

$$\vec{H}^{(2)l_1l_2}_{mds} = -\frac{ik_0Y_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\frac{-ik_0Y_0}{4\pi r_0}\exp\left(ik_0r_0\right)$$
$$\exp\left(-i\vec{k}_s\cdot\vec{r}_{l_1}\right)\int\limits_{S_{l_1}}\frac{e^{ik_0r'}}{r'}\bar{S}_{ml_2}\left(\hat{r}_{l_2n},\hat{r}'\right)\cdot\vec{J}_{mdl_1}dS \qquad (47)$$

Keeping in mind the conditions on the dimensions of, and the distance between the spheres as specified in Eq. (20), Eqs. (46) and (47) can be evaluated analytically. In Eq. (46), Eq. (47), noting that $\vec{r}'$ is the position of the point on the surface of sphere l_1, then $|\vec{r}'-\vec{r}_{l_2}|\approx|\vec{r}_{l_1}-\vec{r}_{l_2}|$, thus

$$\hat{r}'=\frac{\vec{r}'-\vec{r}_{l_2}}{\left|\vec{r}'-\vec{r}_{l_2}\right|}\approx\frac{\vec{r}_{l_1}-\vec{r}_{l_2}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}=\hat{\hat{r}}_{l_1l_2} \qquad (48)$$

Under this approximation, in Eq. (46) and Eq. (47), $\bar{S}_{el_2}(\hat{r}_{l_2n},\hat{r}')$ and $\bar{S}_{ml_2}(\hat{r}_{l_2n},\hat{r}')$ are not functions of the integration variables. Therefore, these two equations can be rewritten as

$$\begin{aligned}\hat{\xi}\cdot\vec{E}^{(2)l_1l_2}_{eds} &= -\frac{ik_0Z_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\\ &\quad\frac{-ik_0Z_0}{4\pi r_0}\exp(ik_0r_0)\bar{S}_{el_2}(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2})\cdot\int\limits_{S_{l_1}}\frac{e^{ik_0r'}}{r'}\vec{J}_{edl_1}dS\\ &= -\frac{ik_0Z_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\\ &\quad\frac{\exp(ik_0r_0)}{r_0}\bar{S}_{el_2}\left(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2}\right)\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1}\right)\end{aligned} \qquad (49)$$

$$\begin{aligned}\hat{\zeta}\cdot\vec{H}^{(2)l_1l_2}_{mds} &= -\frac{ik_0Y_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\\ &\quad\frac{-ik_0Y_0}{4\pi r_0}\exp(ik_0r_0)\bar{S}_{ml_2}\left(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2}\right)\cdot\int\limits_{S_{l_1}}\frac{e^{ik_0r'}}{r'}\vec{J}_{mdl_1}dS\\ &= -\frac{ik_0Y_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\\ &\quad\frac{\exp(ik_0r_0)}{r_0}\bar{S}_{ml_2}\left(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2}\right)\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{ml_1}\left(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1}\right)\end{aligned} \qquad (50)$$

Then, substituting Eq. (49) and Eq. (50) into Eq. (40) and Eq. (41), the following two equations are obtained

$$\begin{aligned}\hat{p}\cdot\vec{E}^{(2)}_{Jn} &= \frac{\exp(ik_0r_0)}{r_0}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\right.\\ &\quad\cdot\left\{\bar{S}_{el_2}\left(\hat{r}_{l_2n},\hat{\bar{r}}_{l_1l_2}\right)\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1}\right)\right\}\\ &\quad\left.\cdot\int_{S_n}\vec{J}_n\cdot\hat{\xi}\frac{-ik_0Z_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}dS\right\}\end{aligned}\qquad(51)$$

$$\begin{aligned}\hat{q}\cdot\vec{H}^{(2)}_{Mn} &= \frac{\exp(ik_0r_0)}{r_0}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\right.\\ &\quad\cdot\left\{\bar{S}_{ml_2}\left(\hat{r}_{l_2n},\hat{\bar{r}}_{l_1l_2}\right)\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{ml_1}\left(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1}\right)\right\}\\ &\quad\left.\cdot\int_{S_n}\vec{M}_n\cdot\hat{\zeta}\frac{-ik_0Y_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}dS\right\}\end{aligned}\qquad(52)$$

In Eq. (51) and Eq. (52), noting that $\bar{S}_{el_2}(\hat{r}_{l_2n},\hat{\bar{r}}_{l_1l_2})\cdot\bar{S}_{el_1}(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1}) = \bar{S}_{ml_2}(\hat{r}_{l_2n},\hat{\bar{r}}_{l_1l_2})\cdot\bar{S}_{ml_1}(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1})$, the third-order scattered field from multi-layered sphere l_1 is obtained as

$$\begin{aligned}\hat{p}\cdot\vec{E}^{(2)}_{n} &= \hat{p}\cdot\left(\vec{E}^{(2)}_{Jn}+\vec{E}^{(2)}_{Mn}\right)\\ &= \frac{\exp(ik_0r_0)}{r_0}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\right.\\ &\quad\cdot\left\{\bar{S}_{el_2}\left(\hat{r}_{l_2n},\hat{\bar{r}}_{l_1l_2}\right)\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1}\right)\right\}\\ &\quad\hat{\xi}\cdot\left[\int_{S_n}\vec{J}_n\frac{-ik_0Z_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}dS-Z_0\hat{r}_{l_2n}\right.\\ &\quad\left.\left.\times\int_{S_n}\vec{M}_n\frac{-ik_0Y_0}{4\pi\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}dS\right]\right\}\end{aligned}$$

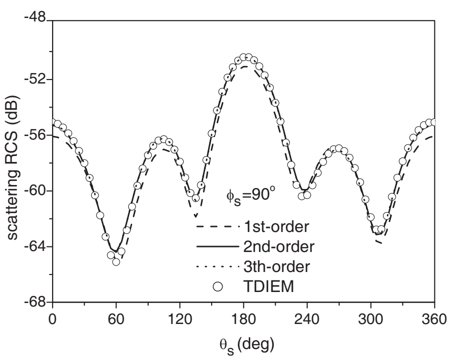
$$
\begin{aligned}
&= \frac{\exp(ik_0 r_0)}{r_0}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}+i\vec{k}_i\cdot\vec{r}_n\right]\right.\\
&\quad\cdot\left\{\bar{S}_{el_2}(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2})\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1}\right)\right\}\\
&\quad\left.\frac{e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}}{\left|\vec{r}_{l_2}-\vec{r}_n\right|}e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}\hat{\xi}\cdot\bar{S}^g_n\left(\hat{k}_i,\hat{\hat{r}}_{l_2n}\right)\right\}
\end{aligned}
\tag{53}
$$

where $\bar{S}_{el_1}(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1})$ and $\bar{S}_{el_2}(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2})$ are the bi-static scattered electric field amplitude vectors of the sphere l_1 and l_2 when illuminated by a plane wave and $\bar{S}^g_n(\hat{k}_i,\hat{\hat{r}}_{l_2n})$ is the scattered electric field amplitude vector of sphere n when the incident wave is a Gaussian beam. Because the sphere n is an arbitrary one among all the spheres, the third-order scattered fields of the Gaussian beam by all the multi-layered spheres can be written as

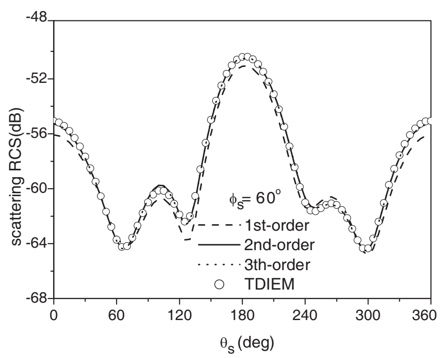
$$
\begin{aligned}
\hat{p}\cdot\vec{E}^{(2)} &= \hat{p}\cdot\left(\vec{E}^{(2)}_{Jn}+\vec{E}^{(2)}_{Mn}\right)\\
&= \frac{\exp(ik_0 r_0)}{r_0}\sum_{n=1}^{N}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[ik_i\cdot\vec{r}_n-i\vec{k}_s\cdot\vec{r}_{l_1}\right]\right.\\
&\quad\cdot\frac{e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}}{\left|\vec{r}_{l_2}-\vec{r}_n\right|}\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\cdot\left[\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1}\right)\cdot\bar{S}_{el_2}\left(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2}\right)\right]\\
&\quad\left.\cdot\left[\hat{h}\cdot\bar{S}^g_n\left(\hat{k}_i,\hat{\hat{r}}_{l_2n}\right)+\hat{v}\cdot\bar{S}^g_n\left(\hat{k}_i,\hat{\hat{r}}_{l_2n}\right)\right]\right\}\\
&= \frac{\exp(ik_0 r_0)}{r_0}\sum_{n=1}^{N}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq n,l_1}^{N}\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}+i\vec{k}_i\cdot\vec{r}_n\right]\right.\\
&\quad\cdot\frac{e^{ik_0\left|\vec{r}_{l_2}-\vec{r}_n\right|}}{\left|\vec{r}_{l_2}-\vec{r}_n\right|}\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\\
&\quad\left.\cdot\left[\bar{S}_{el_1}\left(-\hat{k}_s,\hat{\hat{r}}_{l_2l_1}\right)\cdot\bar{S}_{el_2}\left(\hat{r}_{l_2n},\hat{\hat{r}}_{l_1l_2}\right)\cdot\bar{S}^g_n\left(\hat{k}_i,\hat{\hat{r}}_{l_2n}\right)\right]\right\}
\end{aligned}
\tag{54}
$$

Using the similar way, the jth-order scattered field by all the multi-layered spheres can be written as

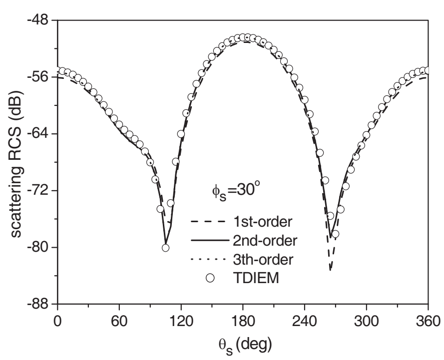
$$\hat{p}\cdot\vec{E}^{(j-1)}=\frac{\exp(ik_0r_0)}{r_0}\sum_{n=1}^{N}\sum_{l_1=1}^{N}\sum_{l_2=1,l_2\neq l_1}^{N}\cdots\sum_{l_{j-1}=1,l_{j-1}\neq n,l_{j-2}}^{N}$$
$$\left\{\exp\left[-i\vec{k}_s\cdot\vec{r}_{l_1}+i\vec{k}_i\cdot\vec{r}_n\right]\cdot\left[\frac{e^{ik_0\left|\vec{r}_{l_{j-1}}-\vec{r}_n\right|}}{\left|\vec{r}_{l_{j-1}}-\vec{r}_n\right|}\cdot\frac{e^{ik_0\left|\vec{r}_{l_{j-2}}-\vec{r}_{l_{j-1}}\right|}}{\left|\vec{r}_{l_{j-2}}-\vec{r}_{l_{j-1}}\right|}\right.\right.$$
$$\left.\cdot\frac{e^{ik_0\left|\vec{r}_{l_{j-3}}-\vec{r}_{l_{j-2}}\right|}}{\left|\vec{r}_{l_{j-3}}-\vec{r}_{l_{j-2}}\right|}\cdot\ldots\cdot\frac{e^{ik_0\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}}{\left|\vec{r}_{l_1}-\vec{r}_{l_2}\right|}\right]$$
$$\cdot\left[\bar{S}_{el_{l_1}}\left(-\hat{k}_s,\hat{\bar{r}}_{l_2l_1}\right)\cdot\bar{S}_{el_{j-1}}\left(\hat{\bar{r}}_{l_{j-1}n},\hat{\bar{r}}_{l_{j-2}l_{j-1}}\right)\right.$$
$$\left.\left.\cdot\bar{S}_{el_{j-2}}\left(\hat{\bar{r}}_{l_{j-2}l_{j-1}},\hat{\bar{r}}_{l_{j-3}l_{j-2}}\right)\cdot\ldots\cdot\bar{S}_{el_2}\left(\hat{\bar{r}}_{l_2l_3},\hat{\bar{r}}_{l_1l_2}\right)\cdot\bar{S}_n^g\left(\hat{k}_i,\hat{\bar{r}}_{l_{j-1}n}\right)\right]\right\}\quad(55)$$



(a)



(b)



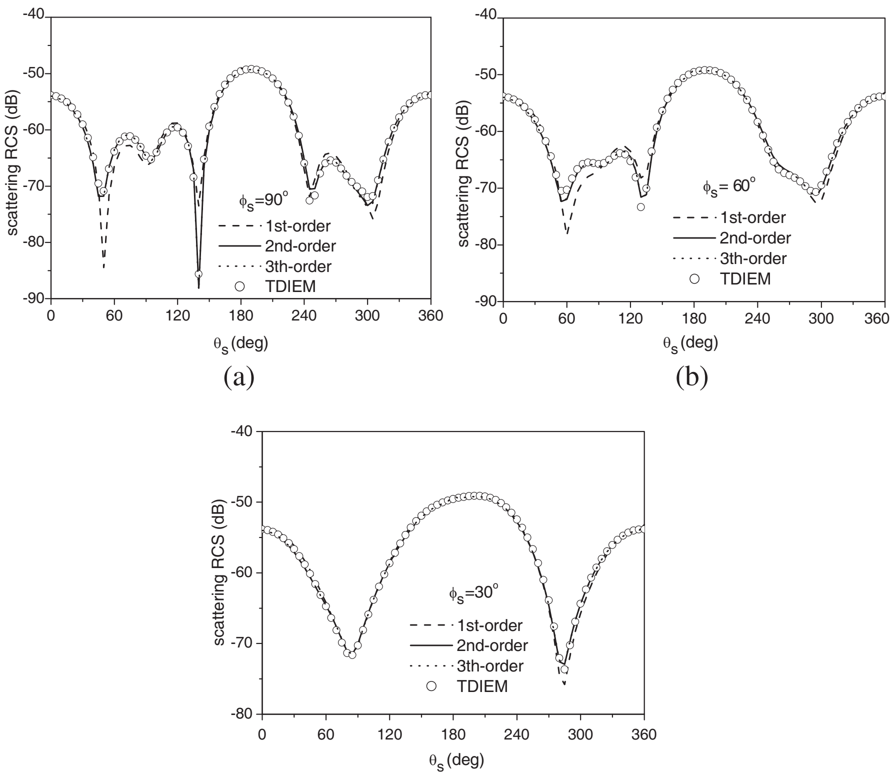
(c)

Figure 3. Bi-static scattering RCS of plane wave from three conducting spheres vs. scattering angles for different azimuthal scattering angles. (a) $\phi_s = 90°$, (b) $\phi_s = 60°$, (c) $\phi_s = 30°$. The position vectors of the three spheres are $\vec{r}_1 = (0\,\text{mm}, 0\,\text{mm}, 0\,\text{mm})$, $\vec{r}_2 = (0\,\text{mm}, 3\,\text{mm}, 0\,\text{mm})$ and $\vec{r}_3 = (0\,\text{mm}, 2\,\text{mm}, 2\,\text{mm})$, respectively.

From the preceding discussions, the scattered field up to N-order can be easily obtained, and its solution can be written as

$$\hat{p} \cdot \vec{E}^{s} = \hat{p} \cdot \left[\vec{E}^{(0)} + \vec{E}^{(1)} + \ldots + \vec{E}^{(N-1)} \right] \tag{56}$$

The composite scattered field of a plane wave by adjacent multi-spheres is obtained by replacing the scattered electric field amplitude vector of sphere n when the incident wave is a Gaussian beam with that when the incident wave is a plane wave in Eq. (28), Eq. (39), Eq. (54) and Eq. (55).



(a) (b) (c)

Figure 4. Bi-static scattering RCS of plane wave from four conducting spheres vs. scattering angles for different azimuthal scattering angles. (a) $\phi_s = 90°$, (b) $\phi_s = 60°$, (c) $\phi_s = 30°$. The position vectors of the four spheres are $\vec{r}_1 = (0\,\text{mm}, 0\,\text{mm}, 0\,\text{mm})$, $\vec{r}_2 = (0\,\text{mm}, 3\,\text{mm}, 0\,\text{mm})$, $\vec{r}_3 = (0\,\text{mm}, 3.5\,\text{mm}, 2.5\,\text{mm})$ and $\vec{r}_4 = (0\,\text{mm}, 0.5\,\text{mm}, 3\,\text{mm})$, respectively.

4. NUMERICAL RESULTS

The Numerical Electromagnetic Code (NEC), which is a computational package based on the Time Domain Integral Equation Method (TDIEM), and only the examples are provided for plane wave case. Therefore, to check the validity of the present method, the bi-static scattering RCS (Radar Cross Section) [26] of a plane wave from adjacent multi-spheres are compared with the data obtained by using NEC. In the following discussions, due to the article size is limited, only TM case is considered and the propagation direction is along the positive z-axis.

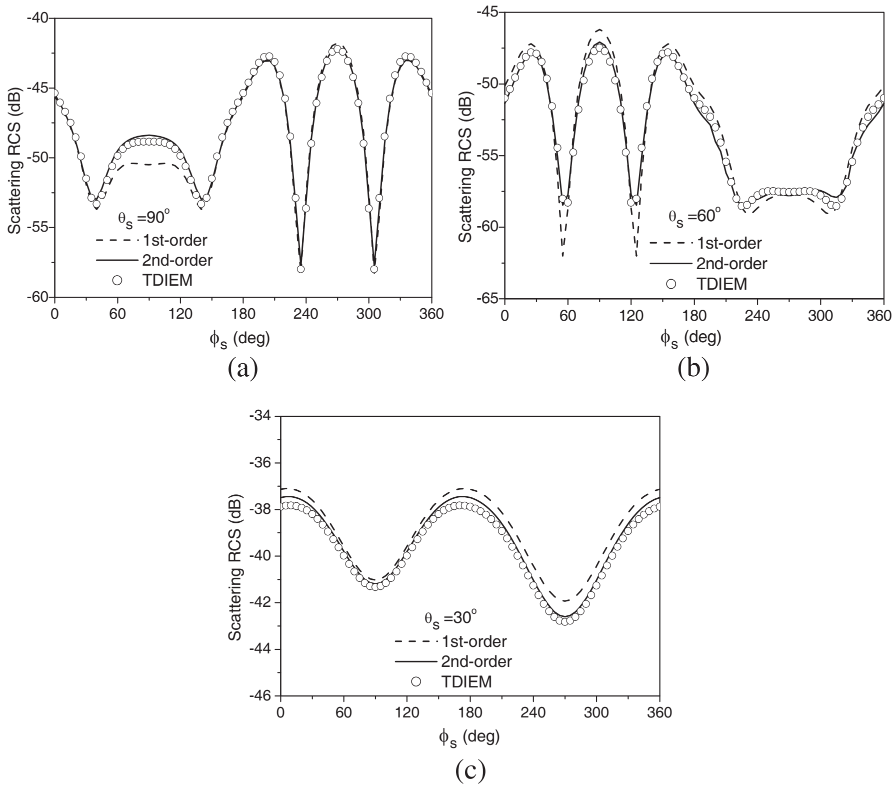


Figure 5. Bi-static scattering RCS of plane wave from four adjacent plasma-coated conducting spheres vs. azimuthal scattering angles for different scattering angles. (a) $\theta_s = 90°$, (b) $\theta_s = 60°$, (c) $\theta_s = 30°$. The position vectors of the four spheres are $\vec{r}_1 = (0\,\mathrm{mm}, 0\,\mathrm{mm}, 0\,\mathrm{mm})$, $\vec{r}_2 = (0\,\mathrm{mm}, 40\,\mathrm{mm}, 0\,\mathrm{mm})$, $\vec{r}_3 = (0\,\mathrm{mm}, 50\,\mathrm{mm}, 40\,\mathrm{mm})$ and $\vec{r}_4 = (0\,\mathrm{mm}, 10\,\mathrm{mm}, 45\,\mathrm{mm})$.

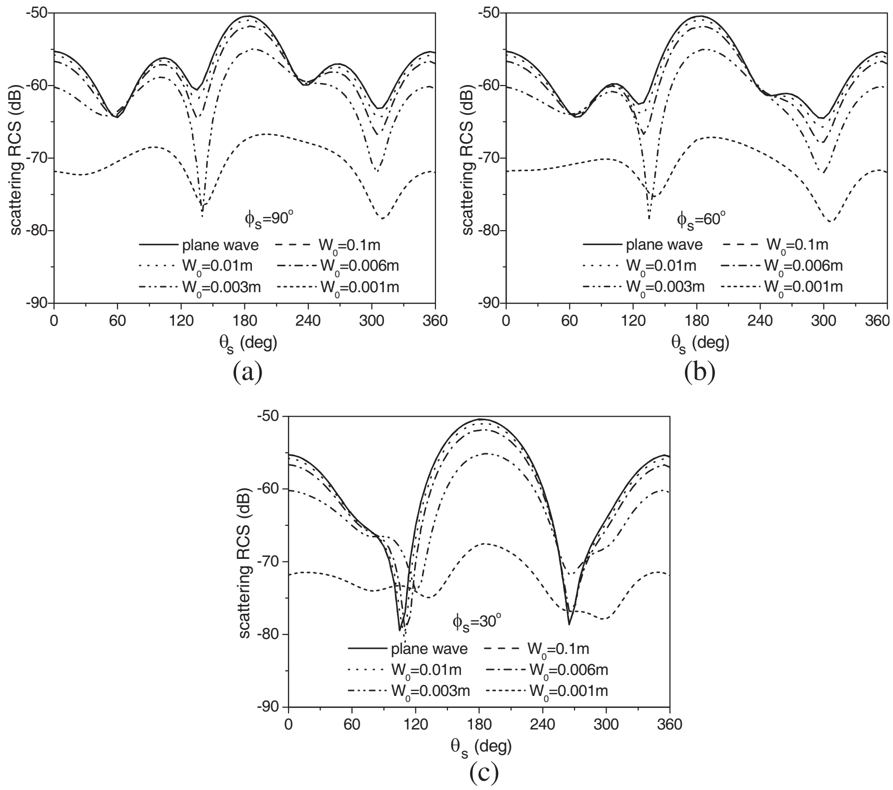


Figure 6. Bi-static scattering RCS of Gaussian beam from three conducting spheres vs. scattering angles for different beam-waist radii. (a) $\phi_s = 90°$, (b) $\phi_s = 60°$, (c) $\phi_s = 30°$.

In Fig. 3 and Fig. 4, the frequency of the excitation wave is 62.5 GHz and the radius of the spheres is 0.5 mm. As is shown in Fig. 3 and Fig. 4, the result up to the second- and third-order provides a reasonable approximation and is in very agreement with the TDIEM data over the angular range. Meanwhile, one can also find that the difference between scattering result up to second- and that up to third-order is very small. Therefore, the third- and the higher-order of the scattered field can be neglected. Then, in the following discussions, only the scattered fields up to second-order are considered.

Figure 5 gives the azimuthal patterns of bi-static scattering RCS for four adjacent plasma-coated conducting spheres for different scattering angle θ_s when the incident field is a plane wave. In Fig. 5, the frequency of the incident wave is 5 GHz and the radius of the

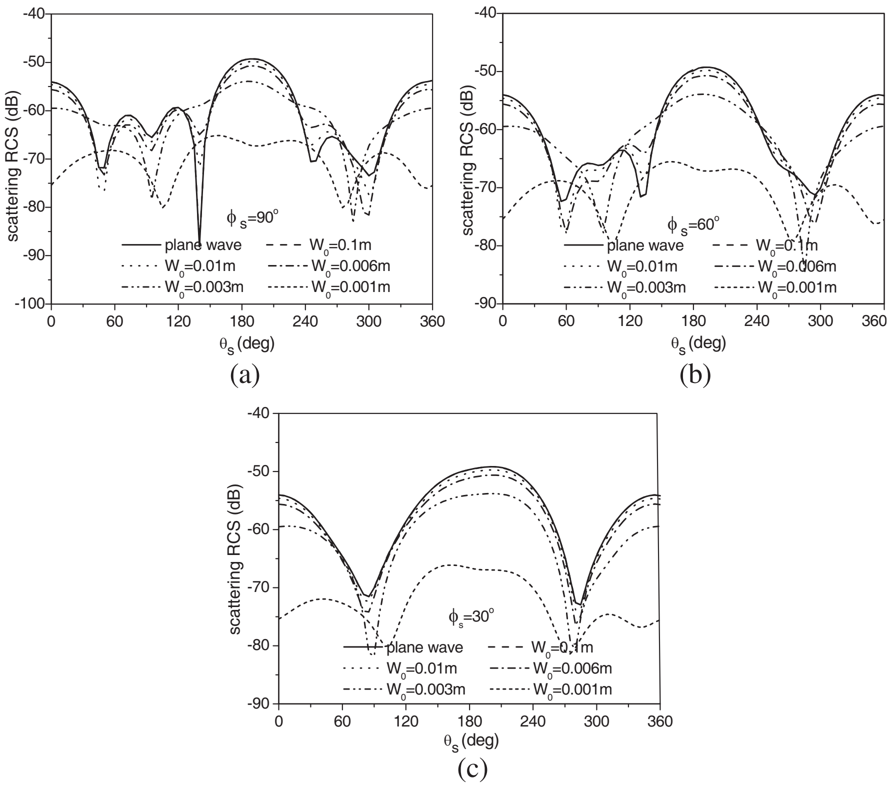


Figure 7. Bi-static scattering RCS of Gaussian beam from four conducting spheres vs. scattering angles for different beam-waist radii. (a) $\phi_s = 90°$, (b) $\phi_s = 60°$, (c) $\phi_s = 30°$.

spheres is 5 mm. The coating thickness is $dr = 1\,\text{mm}$. The collision frequency and the electron density of the plasma are $V_e = 50\,\text{GHz}$ and $N_{e0} = 5.0 \times 10^{17}\,\text{m}^{-3}$, respectively. For the coated spheres case, it can be seen from Fig. 5 that the result up to second-order is in agreement with the Time Domain Integral Equation Method data and provides a reasonable approximation.

It should be pointed out that the validity of our present method has been checked in Fig. 3–Fig. 5, and in the next discussion, emphasis is put on studying the dependence of the bi-static scattering RCS on the bean-waist radii. Fig. 6 and Fig. 7 illustrate the bi-static scattering RCS of Gaussian beam from three- and four-spheres model for different beam-waist radii, and the other parameters in Fig. 6 and Fig. 7 are the same as those in Fig. 3 and Fig. 4, respectively. It is observed clearly that the amplitude of the plane wave incident is larger than that of the Gaussian beam incident, and it is found that the amplitude depends on the beam-waist W_0. As is depicted in Fig. 6 and Fig. 7, the results of Gaussian beam will gradually approach to that of the plane wave case when the beam-waist increases, i.e., as larger the beam waist is, closer result for the Gaussian beam and the plane wave incidence is.

5. CONCLUSIONS

In this research, based on the equivalence principle and the reciprocity theorem, a general approach has been developed for deriving the scattered field up to higher order from discrete particles. The formulation has been applied to obtain approximate analytical solutions up to Nth-order scattered fields of plane wave/Gaussian beam by adjacent multi-spheres. The validity of the present method was verified by comparison with the Time Domain Integral Equation computations, and agreement was obtained. From the comparison, the conclusion that solution up to second-order can give a good agreement is obtained.

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