Progress In Electromagnetics Research M, Vol. 7, 15–28, 2009

A PRACTICAL METHOD FOR RANGE MIGRATION COMPENSATION IN CHIRP RADAR

J. Li, H. Liu, and Z. He

Department of Electronic Engineering
University of Electronic Science and Technology of China
Jianshe North Road, Chengdu 610054, China

Abstract—The echo signal characteristic of a chirp pulse train from a moving target is analyzed. It is pointed out that the range migration is caused by the migration exponential item (MEI) and can be compensated by removing the MEI. On this basis, we proposed a new practical compensation method with very low computation burden through shifting control on the matching weight coefficients. The parasitical sidelobes due to the quantization effect can be restrained effectively by storing multi-groups of weight. Simulation results proved the availability of the method.

1. INTRODUCTION

Range migration is an unavoidable problem in the imaging of moving targets or long time coherent integration. Target motion will influence the target detection in surveillance radar or the focus of imaging in SAR or ISAR [1–3]. Many researches on motion compensation containing range migration compensation have been done [3–9]. Typical migration compensation methods include range-bin realignment [4], envelope interpolation algorithm [9], Keystone transform [6, 7], time-frequency analysis [8], etc. The computation burden and the processing complexity are the problems facing these methods when used in practice. This paper aims at only range migration problem and investigats a practical compensation method with low additional computation burden.

The echo signal characteristic of a chirp pulse train from a moving target is analyzed at first. It is point out that the range migration is caused by the migration exponential item (MEI) of the

Corresponding author: J. Li (lijun_sc@hotmail.com).

echo signal and can be compensated by removing the MEI before the processing of match filtering and coherent integration. On this basis, an implementary method by shifting control on the frequency domain weight coefficients of the digital match filter is put forward. This method combines the compensation processing with the match filtering, so it greatly reduces the additional computation burden. Due to the quantification effect of digital processing, this method will produce parasitic sidelobes in the output, as the processing results of range-bin realignment and envelope interpolation algorithms. By storing multi-groups of weight coefficients, the parasitic sidelobes can be depressed effectively.

Our discussion is based on the hypotheses: (1) The velocity of the target is constant during the coherent processing interval (CPI); (2) The range migration in a CPI is larger than the width of the pulse compression output, but far smaller than the width of the transmitted chirp pulse.

2. SIGNAL MODEL AND CHARACTERISTIC ANALYSIS

The effect on the radar electromagnetic wave due to the motion of a target presents the echo signal time scale change (time stretching or shrinking). Assume that the transmitted signal is $s(t)$, then the echo from a moving target with velocity radial velocity v can be expressed as [10]:

$$s_r(t) = \eta s\big(\kappa(t - t_0)\big) \tag{1}$$

where η is the amplitude of the echo signal (hereinafter assume $\eta = 1$); $t_0 = 2R_0/(c+v)$ is the delay of the wavefront; c is the velocity of light; and R_0 is the original distance of the target.

$$\kappa = (c + v)/(c - v) \tag{2}$$

is the compressing coefficient.

When transmitting a chirp pulse train, $s(t)$ can be expressed by

$$s(t) = \sum_{m=0}^{M-1} u(t - mT)e^{j2\pi f_c t} \tag{3}$$

here, M is the amount of pulse burst; T denotes the PRI; f_c represents the carrier frequency; and

$$u(t) = rect\left(\frac{t}{T_P}\right) e^{j\frac{1}{2}\mu t^2} \tag{4}$$

is the chirp signal; μ is modulation rate; and T_p denotes the width of the pulse. And

$$rect\left(\frac{t}{T_p}\right)=\begin{cases}1 & 0\leq t\leq T_p\\ 0 & \text{else}\end{cases} \tag{5}$$

According to (1) and (3), the received signal can be expressed as:

$$s_r(t)=\sum_{m=0}^{M-1}u(\kappa(t-t_0)-mT)e^{j2\pi f_c(\kappa(t-t_0))} \tag{6}$$

After mixing with the local oscillator signal, the output base band signal is:

$$s_{rI}(t)=\sum_{m=0}^{M-1}u(\kappa(t-t_0)-mT)e^{j2\pi f_d t}e^{-j\phi} \tag{7}$$

where $f_d=2vf_c/(c-v)$ is the Doppler frequency, and $\phi=2\pi f_c\kappa t_0$ is the initial phase of the echo signal.

Denote t as:

$$t=\hat{t}+mT \tag{8}$$

where $\hat{t}$ is the "fast time" in a PRI, and m represents the "slow time" for different period. For simplification, we supposed that $t_0\in[0,T]$. Then the base band signal $s_{rI}(t)$ can be expressed as a bidimensional form:

$$s_{rI}(\hat{t},m)=u\left[\kappa(\hat{t}-t_0)+(\kappa-1)mT\right]e^{j2\pi f_d(\hat{t}+mT)}e^{-j\phi} \tag{9}$$

Define migration factor κ_v:

$$\kappa_v=\kappa-1=\frac{2v}{c-v} \tag{10}$$

κ_v represents the relative amount of migration between adjacent PRIs.

According to (4) and (10), (9) can be expanded as:

$$\begin{aligned}s_{rI}(\hat{t},m)=&\,rect\left(\frac{\kappa(\hat{t}-t_0)+\kappa_v mT}{T_P}\right)\cdot e^{j\frac{1}{2}\mu(\kappa(\hat{t}-t_0)+\kappa_v mT)^2}\\ &\cdot e^{j2\pi f_d(\hat{t}+mT)}e^{-j\phi}\\ =&\,rect\left(\frac{\kappa(\hat{t}-t_0)+\kappa_v mT}{T_P}\right)\cdot e^{j\frac{1}{2}\mu(\kappa(\hat{t}-t_0))^2}e^{j2\pi f_d\hat{t}}\\ &\cdot e^{j\frac{1}{2}\mu[(\kappa_v mT)^2-2\kappa t_0\kappa_v mT]}e^{j2\pi f_d mT}e^{j\mu\kappa\hat{t}\kappa_v mT}e^{-j\phi}\end{aligned} \tag{11}$$

Define:

$$U_1 = e^{j\frac{1}{2}\mu(\kappa(\hat{t}-t_0))^2} e^{j2\pi f_d \hat{t}} \tag{12}$$

$$U_2 = e^{j\frac{1}{2}\mu(\kappa_v mT)^2} e^{-j\mu t_0 \kappa\kappa_v mT} e^{j2\pi f_d mT} \tag{13}$$

$$U_3 = e^{j\mu\kappa\hat{t}\kappa_v mT} \tag{14}$$

The total migration in a CPI is $\kappa_v MT = 2vMT/(c-v)$, which is far smaller than the width T_p of the chirp pulse before compressed, and so

$$rect\left(\frac{\kappa(\hat{t}-t_0)+\kappa_v mT}{T_p}\right) \approx rect\left(\frac{\hat{t}-t_0}{T_p}\right) \tag{15}$$

Then the base band signal can be expressed as:

$$s_{rI}(\hat{t},m) = rect\left(\frac{\hat{t}-t_0}{T_P}\right) U_1 U_2 U_3 e^{-j\phi} \tag{16}$$

3. CHARACTERISTIC ANALYSIS OF THE BASEBAND SIGNAL AND THE MEI

Obviously, $U_1(\hat{t})$ is the common item of single chirp pulse echo from a moving target, which can be processed by ordinary match filtering.

$U_2(m)$ is a function of slow time m. It can be written as

$$U_2(m) = U_{21}(m)U_{22}(m)U_{23}(m) \tag{17}$$

where, $U_{23}(m) = e^{j2\pi f_d mT}$ is the normal element caused by Doppler frequency, and $U_{21}(m) = e^{j\frac{1}{2}\mu(\kappa_v mT)^2}$ is an extra chirp component in the slow time, which will widen the Doppler bandwidth slightly. $U_{22}(m) = e^{-j\mu t_0 \kappa\kappa_v mT}$ is a sine wave, whose frequency is determined by t_0, and U_{22} may change the Doppler frequency of the target, but it does not influence the coherent integration.

Notice that U_1 and U_2 have nothing to do with the range migration, and $U_3 = e^{j\mu\kappa\hat{t}\kappa_v mT}$ is related not only to the fast time $\hat{t}$, but also to the slow time m. Thereby, U_3 is the determinant of range migration and coherent integration loss. So we call U_3 migration exponential item (MEI).

When tracking slow targets with narrowband signal, we have $\kappa_v \approx 0$, so $U_3 \approx 1$, $U_{21} \approx 1$, $U_{22} \approx 1$, and

$$s_{rI}(\hat{t},m) \approx rect\left(\frac{\hat{t}-t_0}{T_P}\right) e^{j\frac{1}{2}\mu(\kappa(\hat{t}-t_0))^2} e^{j2\pi f_d \hat{t}} e^{j2\pi f_d mT} e^{-j\phi} \tag{18}$$

The processing in the fast and slow times are mutually independent. The ordinary method is match filtering in the fast time and coherent integration using FFT algorithm in the slow time.

As the signal bandwidth is much wider and the target velocity is much higher, the migration factor κ_v becomes greater, and the range migration across pulse repetition period can not be ignored. The loss of gain in coherent integration arising from MEI must be taken into account.

Since U_3 is the determinant of range migration, we can eliminate range migration by remove the MEI. And this can be achieved by multiplying $s_{rI}(\hat{t}, m)$ with $e^{-j\mu\kappa\hat{t}\kappa_v mT}$.

$$s_{rIout}(\hat{t}, m) = s_{rI}(\hat{t}, m)e^{-j\mu\kappa\hat{t}\kappa_v mT} \tag{19}$$

The above operation results in the change of Doppler frequency shift in each PRI, which then leads to the complementary migration of the signal envelope. The new signal matrix $s_{rIout}(\hat{t}, m)$ then can be processed with the ordinary method.

4. RANGE MIGRATION COMPENSATION BASED ON SHIFTING THE MATCHING WEIGHT COEFFICIENTS

As shown above, the range migration can be compensated by removing the MEI. This method is essentially based on the Doppler-range relationship of chirp signal, and it does not produce any parasitic sidelobes in the output as long as the compensation is accurate.

In this section, we will introduce a new practical method with low additional computation burden and low processing complexity.

The matched filtering of chirp signal can be completed in the frequency domain by means of Fast Fourier Transform (FFT) [11], and the implementary scheme is shown in Fig. 1. The matching weight is the spectrum of the local chirp signal, which can be calculated beforehand by FFT and stored in the storage of the processing system.

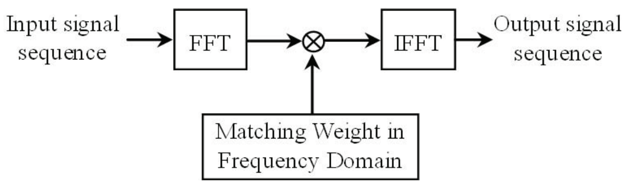

Figure 1. Digital match filtering performed in frequency domain.

The matching weight coefficients can be expressed as

$$U(k)=DFT(u^*(-n))=\sum_{n=0}^{N-1}u^*(-n)e^{-j\frac{2\pi}{N}nk} \quad k=0,1,\cdots,N-1 \tag{20}$$

where * denotes conjugation operation; $u^*(-n) = u^*(-nT_s)$ is the samples of local chirp signal $u^*(-t)$ with sampling period T_s; and N is the length of the sequence.

4.1. Description of the Method Based on Shifting the Matching Weight Coefficients

In order to compensate the range migration between different pulse repetition periods, different matching signals should be used:

$$v(t) = u(t)e^{-j2\pi f_b(m)\hat{t}} \tag{21}$$

where $f_b(m)$ is the introduced frequency component in different periods for revising the envelope position of the matched filtering output.

The output signal of match filtering can be expressed as:

$$\begin{aligned} y(\hat{t},m) &= s_{rI}(\hat{t},m) \otimes v^*(-t) \\ &= \left\{u^*(-t) \otimes rect\left(\frac{\hat{t}-t_0}{T_P}\right) \cdot U_1 \cdot U_3 \cdot e^{-j2\pi f_b(m)\hat{t}}\right\} \cdot U_2 \cdot e^{-j\phi} \\ &= \left\{u^*(-t) \otimes \left[rect\left(\frac{\hat{t}-t_0}{T_P}\right)\right.\right. \\ &\quad \left.\left. \cdot e^{j\frac{1}{2}\mu(\kappa(\hat{t}-t_0))^2} \cdot e^{j2\pi\hat{t}(f_d+\frac{1}{2\pi}\mu\kappa\kappa_v mT - f_b(m))}\right]\right\} \cdot U_2 \cdot e^{-j\phi} \end{aligned} \tag{22}$$

where $\otimes$ denotes convolution integration.

Obviously, the condition under which the envelope position of $y(\hat{t},m)$ is independent of m is

$$f_b(m) = \frac{1}{2\pi}m\mu\kappa\kappa_v T + f_{d0} \tag{23}$$

here, $\frac{1}{2\pi}m\mu\kappa\kappa_v T$ is exactly the frequency item of the MEI. f_{d0} is the initial Doppler frequency, which could be determined by apriori information about the target velocity. f_{d0} has no direct effect on the envelope migration, but it will influence the estimation precision of target distance.

It is impossible to storage all potential weight coefficients corresponding to different $f_b(m)$. If we quantize $f_b(m)$ with $1/NT_s$, then it can be written as:

$$f_b(m) = \frac{k_b}{NT_s} \quad \text{or} \quad k_b = \frac{f_b(m)}{NT_s} \quad k_b = 0, 1, \ldots, M-1 \tag{24}$$

Formula (24) is an approximate expression of the quantized result. And then $v(t)$ can be expressed as:

$$v(t) = u(t)e^{-j\frac{2\pi}{NT_s}k_b t} \tag{25}$$

According to (20), we have

$$V(k) = DFT(v^*(-n)) = \sum_{n=0}^{N-1} u^*(-n)e^{-j\frac{2\pi}{N}n(k+k_b)} = U(k+k_b) \tag{26}$$

Therefore, $V(k)$ can be obtained by shifting $U(k)$, and doesn't need to be stored in addition. So we get the method of range migration compensation by shifting the matching weight coefficients, as shown in Fig. 2.

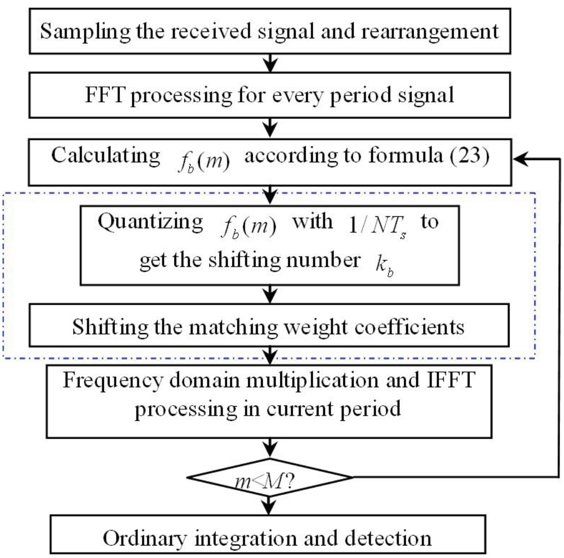

Figure 2. Long time coherent integration with migration compensation by shifting the matching weight coefficients.

This method only appends a few division and shifting operations to the ordinary matching processing, so the additional computation burden is very limited.

4.2. Parasitic Sidelobes and Restraining Method

There exists quantization error when using $1/NT_s$ to quantize $f_b(m)$. It results in parasitic sidelobes produced in the Doppler domain. This is because the envelopes of the matched filtering output are aligned approximatively, and periodic amplitude modulation to the output in the slow time domain is produced. The position and amplitude of the parasitic sidelobes are related to the velocity of the target. This kind of parasitic sidelobe also exists in the range-bin realignment algorithm and the envelope interpolation algorithm.

The effectual method to restrain the parasitic sidelobe is to reduce the quantization error. When the sequence length and the sampling period are determined, we can store multi-groups of weight to reduce the quantization error of $f_b(m)$. Suppose there are L groups of weight, the lth group of weight coefficients can be wrote as:

$$V_l(k) = \sum_{n=0}^{N-1} v^*(-n) e^{-j\frac{2\pi}{N}n(k+l/L)} \quad l = 0, 1, \ldots, L-1 \tag{27}$$

Thus, the quantized interval can be turned into $1/LNT_s$, which is much smaller than the frequency interval of the matching weight coefficients.

Therefore, we should modify the processing flow of long time coherent integration shown in Fig. 2 by replacing the portion in dashdotted line with the following framework.

Compared with the ordinary algorithm, the above algorithm in Fig. 3 needs much more storage space, which is not the vital bottleneck in signal processing because of the hypergrowth of the digital technique. In practical use, it can get a balance between the hardware expense and the processing performance.

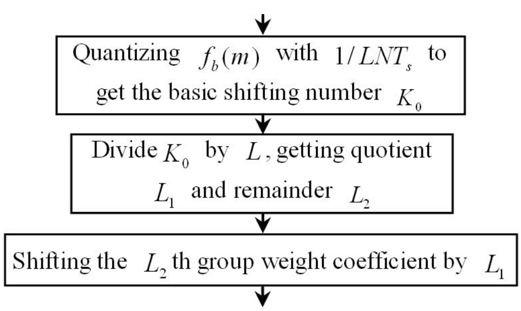

Figure 3. Choosing and shifting control of the matching weight coefficients when storing multi-groups of weight.

5. SIMULATION RESULTS

The simulation conditions are given as follows: the chirp burst is comprised of 100 pulses with period $PRI = 1\,\text{ms}$; the pulse width is $T_p = 20\,\mu\text{s}$; and the signal bandwidth is 20 MHz. The target velocity is 640 m/s, and the sampling period of the receiver is $0.025\,\mu\text{s}$.

Figure 4 shows the range migration phenomenon of the echo signals in a CPI. It is the amplitude contour map of the planar signal which is the output of the match filtering in the fast time without any compensation. It can be seen that the migration span of the envelope within a CPI reaches about 8 resolution cells. If the signal is coherent integrated directly in the slow time, great loss will be produced as shown in Fig. 5.

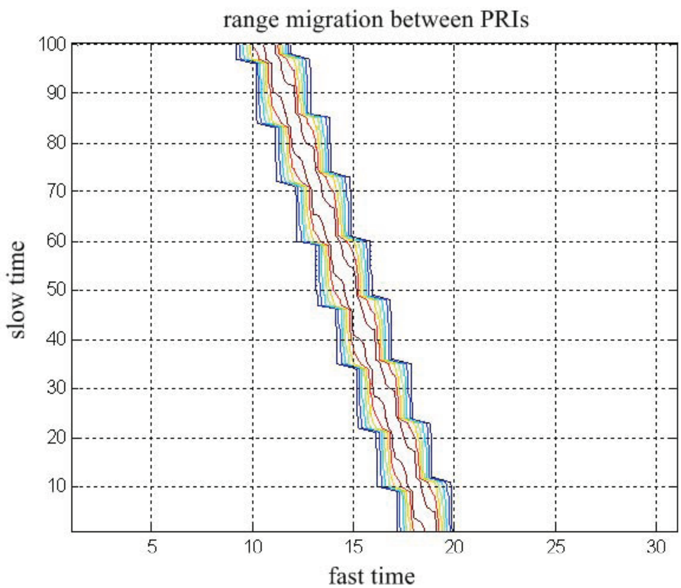

Figure 4. Range migration between PRIs without compensation.

Figure 5 is the integration result observed in the Doppler dimension. Due to the range-Doppler coupling relationship of the chirp signal, the Doppler migration arises simultaneously with the range migration. In order to reveal the parasitical sidelobe phenomenon of compensation algorithms, the following figures only present the integration result in the Doppler dimension, namely ambiguity velocity dimension.

Figures 6 and 7 give the integration results of the range-bin realignment algorithm and the envelope interpolation algorithm

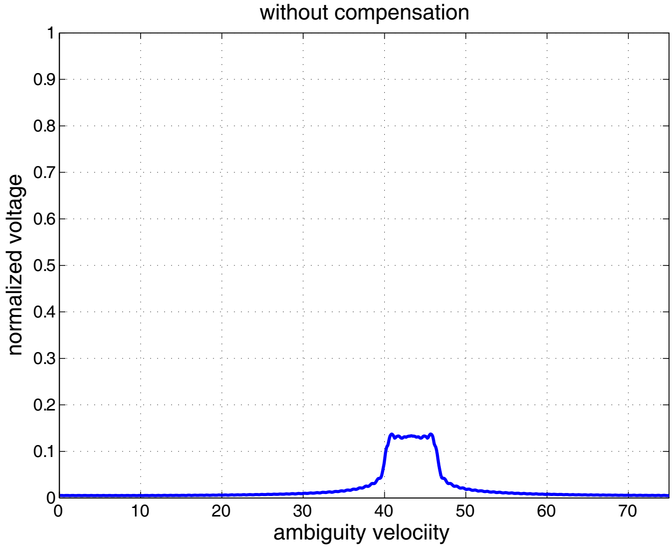

Figure 5. The integratoin output without compensation.

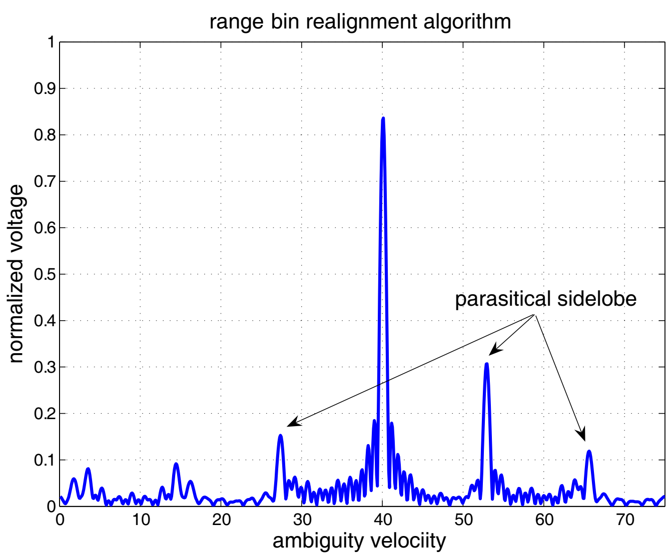

Figure 6. The integratoin output of range-bin realignment algorithm.

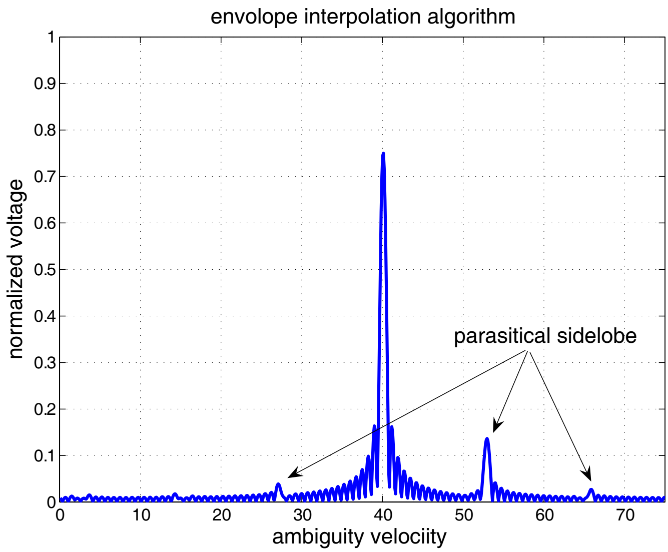

Figure 7. The integratoin outputs of envelope interpolation algorithm.

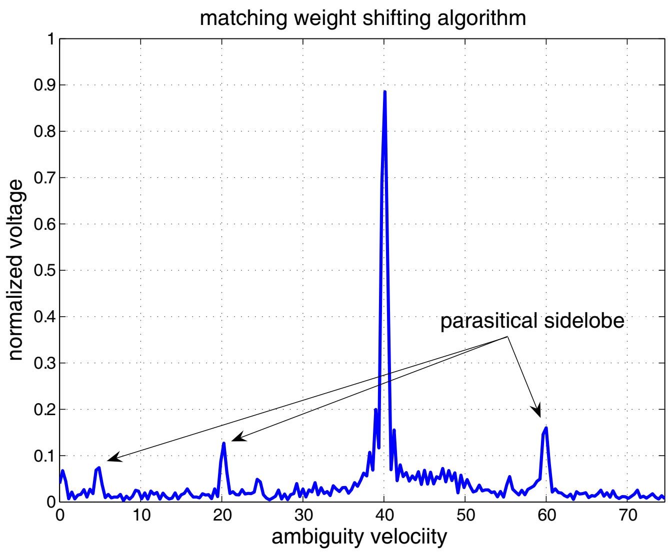

Figure 8. The integratoin output of the matching weight shifting algorithm (using single group of weight).

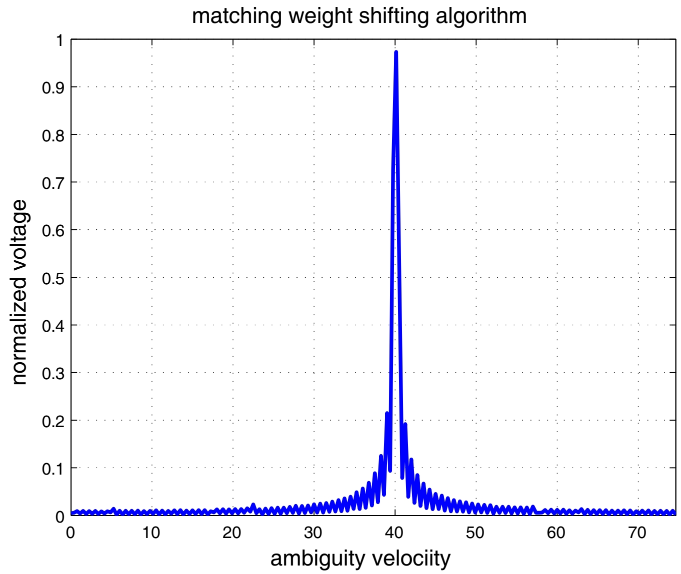

Figure 9. The integratoin output of the matching weight shifting algorithm (using 16 groups of weights).

respectively. The integration performance was improved markedly, but unexpected parasitical sidelobes were introduced in the output, which might cause false alarm or sheltering small targets.

Figures 8 and 9 show the results of the matching weight shifting algorithm. It can be seen, by using multi-groups of weight, that the parasitical sidelobes can be restrained effectively, and the integration loss can be further reduced.

6. CONCLUSION

This paper discusses the range migration compensation problem in chirp radar while knowing the apriori information about the target velocity. It is pointed out that the range migration is relative with the migration exponential item (MEI), and it could be eliminated by removing the MEI of the echo signals. A more practical method combining the digital match filtering with compensation processing is put forward. This method compensates the envelope migration by shifting the matching weight coefficients. Through storing multi-groups of weight, the parasitical sidelobes due to the quantization of the spectrum could be restrained effectively and the improvement effect of coherent integration could be further increased.

Actually, it is not strictly required that the exact velocity information should be known and the velocity should be constant, because the method has good Doppler resilience, that is to say, when there exists some error about the velocity for compensation, proper integration result can be obtained. When without any priori knowledge of the target velocity, several velocity channels (or Doppler channels) could be set to cover the whole possible scope of the velocity, and each channel compensates the migration according to different velocity.

The proposed algorithm is a method with very low additional computation burden and complexity, which is especially applicable in the coherent integration and detection of high velocity targets in ordinary chirp radar or imaging of moving targets in the wideband radar.

REFERENCES

1. Koo, V. C., Y. K. Chan, and H. T. Chuah, "Multiple phase difference method for real-time SAR autofocus," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 3, 375–388, 2006.
2. Lim, T. S., V. C. Koo, H. T. Ewe, and H. T. Chuah, "A SAR autofocus algorithm based on particle swarm optimization," *Progress In Electromagnetics Research B*, Vol. 1, 159–176, 2008.
3. Park, S. H., H. T. Kim, and K. T. Kim, "Stepped-frequency ISAR motion compensation using particle swarm optimization with an island model," *Progress In Electromagnetics Research*, PIER 85, 25–37, 2008.
4. Chen, C. C. and H. C. Andrews, "Targets motion induced radar imaging," *IEEE Trans. on Aerosp. Electron. Syst.*, Vol. 16, No. 1, 2–14, 1980.
5. Delisle, G. Y. and H. Wu, "Moving target imaging and trajectory computation using ISAR," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 30, No. 3, 887–899, 1994.
6. Perry, R. P., R. C. Dipietro, and R. L. Fante, "SAR imaging of moving targets," *IEEE Trans. on Aerosp. Electron. Syst.*, Vol. 35, No. 1, 188–200, 1999.
7. Perry, R. P., R. C. Dipietro, and R. L. Fante, "Coherent integration with range migration using keystone formatting," *2007 IEEE Radar Conference*, 863–868, 2007.
8. Wang, J., S. H. Zhang, and Z. Bao, "On motion compensation for weak radar reflected signal detection," *6th International Conference on Signal Processing*, Vol. 2, 1445–1448, 2002.

9. Chen, Y. Z., Y. F. Zhu, H. Z. Zhao, and Q. Fu, "Detection algorithm research of high velocity moving target based on the envelope interpolation," *Signal Processing (China)*, Vol. 20, No. 4, 387–390, 2004.
10. Ding, L. F. and F. L. Geng, *Radar Principle*, Xidian University Publishing House, Xi'an, China, 2002.
11. Skolnik, M. I., *Radar Handbook*, McGraw-Hill, New York, 1990.