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UNIFORM ANALYTIC SCATTERED FIELDS OF A PEC PLATE ILLUMINATED BY A VECTOR PARAXIAL GAUSSIAN BEAM

J. Hillairet and J. Sokoloff †

Université de Toulouse
UPS, INPT; LAPLACE
118 route de Narbonne, F-31062 Toulouse cedex 9, France
CNRS; LAPLACE; F-31062 Toulouse, France

S. Bolioli

ONERA Centre de Toulouse
Electromagnetic and Radar Department
31055 Toulouse, France

Abstract—A uniform asymptotic expression is developed for calculating the fields scattered by a perfect electrically conducting plate illuminated by a vectorial gaussian beam. This expression has been obtained under the physical optics approximation using the saddle point method. Some numerical applications are presented and compared with some reference methods such as a MoM. A brief parameter study of this solution is presented.

1. INTRODUCTION

Uniform asymptotic expression is developed in this paper for calculating the fields scattered by a Perfect Electrically Conducting (PEC) plate illuminated by a Gaussian Beam (GB). This expression has been obtained under the Physical Optics approximation (PO). This work is part of a general Gaussian Beams tracking method which is used for calculate interactions of electromagnetic waves with complex 3D objects.

Ray tracing and ray techniques have been widely used in electromagnetics engineering for scattering problems. However, some

Corresponding author: J. Hillairet (julien.hillairet@cea.fr).
† J. Hillairet and J. Sokoloff are also with CNRS, LAPLACE, F-31062 Toulouse, France.

difficulties may arise with some complex situations, such as caustics. Moreover, the number of rays and the computation time may increase for huge complex cases.

In order to avoid these problems, some recent approaches use different basis functions involving Gaussian Beams for tracking fields in complex environments [1–4]. In comparison to classical ray techniques, the GB set enables one to avoid caustic problems and reduces the number of launched elements. Moreover, asymptotic methods allow to express propagation and transformation of the GBs by interfaces in closed form expressions. Gaussian Beams Tracking (GBT) methods are based on the ability to expand a source field known on a surface into a summation of Gaussian beams [5, 6]. During the last years, GBT has been successfully applied to the computation of the fields scattered by mono and multilayer dielectric radomes [7–11] or other complex 3D objects [12, 13].

In a general GBT problem, some GB may impact a finite surface. In this case, one must take into account the diffraction effects due to the edges. In this paper, we develop an analytical expression to evaluate the first order diffraction effects as previously done for reflectors by [1]. Evaluating the PO integral consists in the derivation of a double integral over the illuminated surface of an object. Reduction of surface integrals to contour integrals has been proposed in order to reduce computation time [14]. However, due to its relative complexity, this method is restricted to simple illuminations such as plane or spherical waves. More recently, this approach has been used with a vector complex source point which is a generalisation of a classical source point to complex space coordinates. The paraxial approximation corresponds to the fields of a Gaussian Beam [15]. However, the resulting line integral has to be numerically evaluated.

When the integration domain has some rotational symmetry properties and using appropriate changes of integration variables, one can derive an analytic result with the use of classical asymptotic expansions for single integral [1, 16, 17]. In the present situation of a rectangular surface, this approach can not be directly used.

In the present paper, we extend the stationary phase method for a double integral as presented in [18] for a spherical wave illumination to the case of an incident vector paraxial gaussian beam. The stationary phase method is an asymptotic approximation which may stand for high frequency double bounded oscillating integrals [19, 20]. In this approach, the principal contributions to the double integral come from points called critical points. Their locations depend on the incident field and on the observation points. These points may be located inside, outside or on the boundaries of the integration surface. However, some authors have advised about the difficulty of finding uniform asymptotic evaluation [21]. One has often to switch between non-uniform and uniform expressions as the critical points get coupled [18], inducing some discontinuities.

Moreover, in the case of GB illumination, these stationary points are complex. The stationary phase method is physically meaningful when the critical points are reals. Physical interpretation of complex critical points is not easy unless one uses the saddle point method. Unfortunately, the saddle point method is more difficult to apply rigorously to multidimensional bounded integral than the stationary phase method [22]. So, in order to derive an analytical expression of the PO integral with the saddle point method, we proceed by splitting the integration domain. This zoning permits one to deal only with canonical integrals, for which uniform asymptotic expansions exist [23–25].

The paper is organized as follows: In Section 2 the incident and scattered fields by GB are expressed under a canonical form, in Section 3 the mathematical derivation of the uniform expansion is described. Section 4 presents numerical results. Appendix reports some mathematical derivations. An $\exp[+j\omega t]$ time dependance convention is assumed and suppressed throughout this paper.

2. PO INTEGRAL

The general geometry of the problem is illustrated on Fig. 1. $\mathbf{r}(x, y, z)$ is the location of a point in a cartesian coordinate system

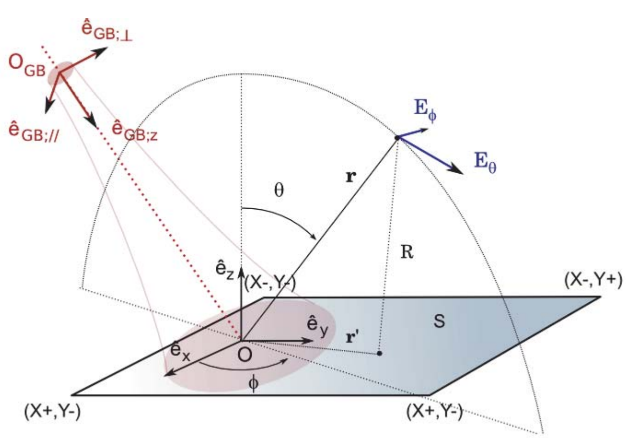

Figure 1. General configuration.

$(O,\, \hat{\mathbf{e}}_x,\, \hat{\mathbf{e}}_y,\, \hat{\mathbf{e}}_z)$, where $\hat{\mathbf{e}}_z$ is normal to a PEC plate. We denote by $(O_{\mathrm{GB}}, \hat{\mathbf{e}}_{\mathrm{GB};\|}, \hat{\mathbf{e}}_{\mathrm{GB};\perp}, \hat{\mathbf{e}}_{\mathrm{GB};z})$ the local coordinate system of an incident gaussian beam (GB) which is illuminating the plate. The superscript notation $\mathbf{r}^{(\mathrm{GB})}\left(x^{(\mathrm{GB})}, y^{(\mathrm{GB})}, z^{(\mathrm{GB})}\right)$ expresses $\mathbf{r}$ into the local scheme of the incident GB. To simplify the derivation without loosing the generality, the origin O is also located at the intersection of the direction $\hat{\mathbf{e}}_{\mathrm{GB};z}$ with the plane $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y)$. The incident electric and magnetic fields on P are expressed by:

$$\mathbf{E}_i(\mathbf{r}) = \mathbf{A}_E(\mathbf{r})u(\mathbf{r}) \tag{1}$$

$$\mathbf{H}_i(\mathbf{r}) = \frac{1}{Z_0}\mathbf{A}_H(\mathbf{r})u(\mathbf{r}) \tag{2}$$

with

$$u(\mathbf{r}) = \left(\frac{\det \mathbb{Q}\left(z^{(\mathrm{GB})}\right)}{\det \mathbb{Q}(0)}\right)^{\frac{1}{2}} \cdot \exp\left[-\frac{jk}{2}\begin{pmatrix}x^{(\mathrm{GB})}\\ y^{(\mathrm{GB})}\end{pmatrix} \mathbb{Q}\left(z^{(\mathrm{GB})}\right)\begin{pmatrix}x^{(\mathrm{GB})}\\ y^{(\mathrm{GB})}\end{pmatrix} - jkz^{(\mathrm{GB})}\right] \tag{3}$$

where the superscript ^{t} denotes the transpose of a vector. k is the wave-number and Z_0 the free space wave impedance. $\mathbb{Q}$ is the complex curvature matrix of the incident beam, which elements q_{ij} are defined with a negative imaginary part [26, 27]. $\mathbf{A}_E$ and $\mathbf{A}_H$ vectors denote electric and magnetic fields amplitude. They are expressed by [7]:

$$\mathbf{A}_E(\mathbf{r}) = a_{\|}\hat{\mathbf{e}}_{\mathrm{GB};\|} + a_{\perp}\hat{\mathbf{e}}_{\mathrm{GB};\perp} + a_{E;z}(\mathbf{r})\hat{\mathbf{e}}_{\mathrm{GB};z} \tag{4}$$

$$\mathbf{A}_H(\mathbf{r}) = -a_{\perp}\hat{\mathbf{e}}_{\mathrm{GB};\|} + a_{\|}\hat{\mathbf{e}}_{\mathrm{GB};\perp} + a_{H;z}(\mathbf{r})\hat{\mathbf{e}}_{\mathrm{GB};z} \tag{5}$$

where

$$a_{E;z}(\mathbf{r}) = -a_{\|}\left(q_{11}x^{\mathrm{GB}} + q_{12}y^{\mathrm{GB}}\right) - a_{\perp}\left(q_{12}x^{\mathrm{GB}} + q_{22}y^{\mathrm{GB}}\right) \tag{6}$$

$$a_{H;z}(\mathbf{r}) = a_{\|}\left(q_{12}x^{\mathrm{GB}} + q_{22}y^{\mathrm{GB}}\right) - a_{\perp}\left(q_{11}x^{\mathrm{GB}} + q_{12}y^{\mathrm{GB}}\right) \tag{7}$$

Coefficients $a_{\|}$ and $a_{\perp}$ are the eigenvalues of the Gaussian beams expansion [5, 9].

The scattered electric field is calculated using the Kottler current integral formulation at large distance [28]:

$$\mathbf{E}_s(\mathbf{r}) = \frac{jkZ_0}{4\pi\sqrt{\varepsilon_r}}\iint_S \hat{\mathbf{R}} \times \hat{\mathbf{R}} \times \hat{\mathbf{J}}(\mathbf{r}')\frac{e^{-jkR}}{R}dx'dy' \tag{8}$$

where S denotes the surface of the plate, $\mathbf{r}'(x',\, y',\, z' = 0)$ represents the coordinates of any point on S. As usual, R denotes the distance between the observation point $\mathbf{r}$ and the point $\mathbf{r}'$ on the integration domain. $\mathbf{R}$ corresponds to the vector $\mathbf{r}-\mathbf{r}'$ and $\hat{\mathbf{R}}$ is the unit vector defined by $\mathbf{R}/R$. The electric current $\mathbf{J}(\mathbf{r}')$ is given within the PO approximation by

$$\mathbf{J}(\mathbf{r}') \approx 2\hat{\mathbf{e}}_z \times \mathbf{H}_i(\mathbf{r}') \tag{9}$$

The transformation between the local coordinates associated to the incident GB and the absolute coordinates is:

$$\mathbf{r}^{(\mathrm{GB})} = [m_{ij}](\mathbf{r}-\mathbf{O}_{\mathrm{GB}}) \tag{10}$$

where the transformation matrix $[m_{ij}]$ is known. Using this relation, one can express the incident magnetic field $\mathbf{H}_i(\mathbf{r}')$ in (9) into the absolute coordinates. The distance R can be expanded in a Taylor series about the GB reflection point O in order to obtain a quadratic expression:

$$R \approx r - {}^t\!\begin{pmatrix} x' \\ y' \end{pmatrix}\begin{pmatrix} x/r \\ y/r \end{pmatrix} + \frac{1}{2}{}^t\!\begin{pmatrix} x' \\ y' \end{pmatrix}\mathbb{P}\begin{pmatrix} x' \\ y' \end{pmatrix} \tag{11}$$

where

$$\mathbb{P} = \begin{pmatrix} \frac{1}{r}-\frac{x^2}{r^3} & -\frac{xy}{r^3} \\ -\frac{xy}{r^3} & \frac{1}{r}-\frac{y^2}{r^3} \end{pmatrix} \tag{12}$$

Moreover, we assume that the value of the curvature matrix on the surface S is $\mathbb{Q}\left(O^{(\mathrm{GB})}\right)$. Using these assumptions, one can transform the integral (8) to the canonical form:

$$\mathbf{E}_s^{\mathrm{PO}}(\mathbf{r}) = \iint_S \mathbf{f}(x',\, y')e^{-kg(x',\, y')}dx'dy' \tag{13}$$

where

$$\mathbf{f}(x',y') = \frac{jk}{2\pi}\left(\frac{\det\mathbb{Q}(x',y')}{\det\mathbb{Q}(0)}\right)^{\frac{1}{2}} \hat{\mathbf{R}}\times\hat{\mathbf{R}}\times\hat{\mathbf{n}}\times\left[{}^t[m_{ij}]\mathbf{A}_H(x',y')\right] \tag{14}$$

$$\begin{aligned} g(x',y') &= \frac{j}{2}{}^t\!\begin{pmatrix} x' \\ y' \end{pmatrix}\left[\mathbb{P}+{}^t[m_{ij}]\mathbb{Q}(O^{(\mathrm{GB})})[m_{ij}]\right]\begin{pmatrix} x' \\ y' \end{pmatrix} \\ &\quad -j\,{}^t\!\begin{pmatrix} x' \\ y' \end{pmatrix}\mathbf{b}+jc \end{aligned} \tag{15}$$

The coefficients $\mathbf{b}$ and c are defined by

$$\mathbf{b} = \begin{pmatrix} x/r \\ y/r \end{pmatrix} - \begin{pmatrix} m_{31} \\ m_{32} \end{pmatrix} - {}^t[m_{ij}]\mathbb{Q}\left(O^{(\mathrm{GB})}\right)\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{16}$$

$$c = r+\alpha_3+\frac{1}{2}{}^t\!\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}\mathbb{Q}\left(O^{(\mathrm{GB})}\right)\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{17}$$

with

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix} = -[m_{ij}]\begin{pmatrix}x_{O_{\mathrm{GB}}}\\ y_{O_{\mathrm{GB}}}\\ z_{O_{\mathrm{GB}}}\end{pmatrix} \tag{18}$$

The PO canonical integral (13) will next be evaluated asymptotically for large k in next section.

3. UNIFORM ASYMPTOTIC EXPANSION

In this section, we use a uniform asymptotic expansion to derive an analytical expression of the integral (13).

The integration domain S corresponds to a rectangular patch. In order to get well known double canonical integrals, we rewrite the integration domain S as the difference between the whole plane and the complementary domain $\bar{S}$, as illustrated on Fig. 2. So, (13) is expressed as

$$\iint_S(\ldots) = \iint_{\mathbb{R}^2}(\ldots) - \iint_{\bar{S}}(\ldots) \tag{19}$$

The evaluation of an asymptotic expansion on $\iint_{\mathbb{R}^2}$ is well known, specially when the phase term $g(x',\, y')$ is a quadratic function, for which there is only one saddle point. One gets [24]:

$$\begin{aligned}&\iint_{\mathbb{R}^2}\mathbf{f}(x',y')e^{-kg(x',y')}dx'dy'\\ &\approx \frac{2\pi}{k}\Big(-\det(\mathbb{P}+\,{}^t[m_{ij}]\mathbb{Q}\left(O^{(\mathrm{GB})}\right)[m_{ij}]\Big)^{-\frac{1}{2}}\mathbf{f}_s e^{-kg_s}\end{aligned} \tag{20}$$

with

$$\mathbf{f}_s = \mathbf{f}(x'_s,\, y'_s) \tag{21}$$
$$g_s = g(x'_s,\, y'_s) \tag{22}$$

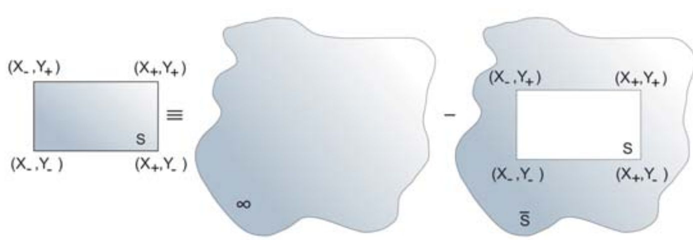

Figure 2. Decomposition of the initial integration domain.

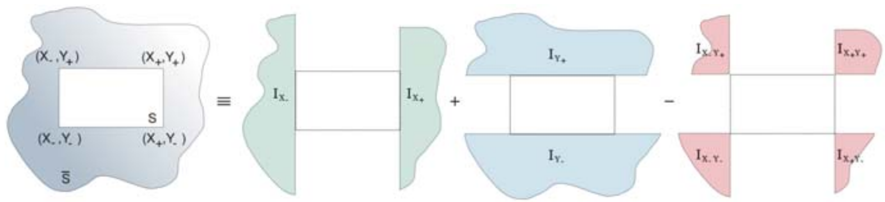

Figure 3. Decomposition of the complementary domain $\bar{S}$ into 8 canonical sub-domains.

where $(x'_s,\, y'_s)$ is the saddle point defined by $\boldsymbol{\nabla} g(x'_s,\, y'_s) = \mathbf{0}$, then:

$$\begin{pmatrix} x'_s \\ y'_s \end{pmatrix} = \left(\mathbb{P} + {}^t[m_{ij}]\mathbb{Q}\left(O^{(\mathrm{GB})}\right)[m_{ij}]\right)^{-1}\mathbf{b} \tag{23}$$

To treat the second double integral over $\bar{S}$, we express $\bar{S}$ as a sum of canonical domains, as illustrated on Fig. 3. So, $\iint_{\bar{S}}$ is expressed as:

$$\begin{aligned} \iint_{\bar{S}} &= I_{X_-} + I_{X_+} + I_{Y_-} + I_{Y_+} \\ &\quad - I_{X_-Y_-} - I_{X_-Y_+} - I_{X_+Y_-} - I_{X_+Y_+} \\ &\equiv \int_{-\infty}^{+\infty}\int_{-\infty}^{X_-} + \int_{-\infty}^{+\infty}\int_{X_+}^{+\infty} + \int_{-\infty}^{Y_-}\int_{-\infty}^{+\infty} + \int_{Y_+}^{+\infty}\int_{-\infty}^{+\infty} \\ &\quad - \int_{-\infty}^{Y_-}\int_{-\infty}^{X_-} - \int_{Y_+}^{+\infty}\int_{-\infty}^{X_-} - \int_{-\infty}^{Y_-}\int_{X_+}^{+\infty} - \int_{Y_+}^{+\infty}\int_{X_+}^{+\infty} \end{aligned} \tag{24}$$

The uniform asymptotic expansion of the first four double integrals is well-known and details of the derivation can be found in [29, §2.3] or [24, §4]. The uniform asymptotic expansion of double integral of type:

$$I_X = \int_{-\infty}^{+\infty}\int_{X}^{+\infty} \mathbf{f}(x',\, y')e^{-kg(x',\, y')}dx'dy' \tag{25}$$

is stated from Ref. [29]. It consists into a double asymptotic expansion, first applied to the integral $\int_{-\infty}^{+\infty}$ then to the integral $\int_{X}^{+\infty}$:

$$I_X \approx e^{-kg(x'_X, y'_X)}\left[\frac{F_X(0)}{\sqrt{k}}\mathcal{Q}\left(\sqrt{k}s_X\right) + \frac{1}{2k}\frac{F_X(s_X) - F_X(0)}{s_X}e^{-ks_X^2}\right] \tag{26}$$

with

$$\frac{\partial g}{\partial y'}(y'_X) = 0 \quad \forall x' \in [X, +\infty] \tag{27}$$

$$s_X = \sqrt{g(X, y'_X) - g(x_s, y'_X)} \tag{28}$$

$$\frac{\partial g(x', y'_X)}{\partial x'}(x'_X) = 0 \tag{29}$$

$$F_X(s_X) = \Phi(X)\frac{s_X}{\frac{\partial g(x', y'_X)}{\partial x'}(X)} \tag{30}$$

$$F_X(0) = \Phi(x'_X)\sqrt{\frac{2}{\frac{\partial^2 g(x', y'_X)}{\partial^2 x'}(x'_X)}} \tag{31}$$

$$\Phi(x) = \sqrt{\frac{2\pi}{k}}\left(\frac{\partial^2 g}{\partial y'^2}(x', y'_X)\right)^{-\frac{1}{2}} \mathbf{f}(x', y'_X) \tag{32}$$

$$\mathcal{Q}(t) = \int_t^\infty e^{-s^2} ds = \frac{\sqrt{\pi}}{2}\mathrm{erfc}(t) \tag{33}$$

Since g (15) is an analytical function, its derivatives can be analytically expressed. They are expressed in closed-form in the previous equations only for the reader's convenience.

One can obtain similar results for I_{X_-}, I_{Y_-} and I_{Y_+}. Four integrals are still remaining. These last integrals have two boundaries, one on each single integral. For the sake of clarity, we proceed in the appendix to the derivation of double integral of type:

$$I_{XY} = \int_Y^\infty \int_X^\infty \mathbf{f}(x', y') e^{-kg(x', y')} dx' dy' \tag{34}$$

Finally, one obtains asymptotic approximation [18].

$$\begin{aligned} I_{XY} \;\approx\; & \frac{\mathbf{f}(X,Y)}{\pi}\left(\frac{\partial^2 g}{\partial x^2}(X,Y)\frac{\partial^2 g}{\partial y^2}(X,Y)\right)^{-\frac{1}{2}} \\ & \cdot e^{-kg(X,Y)}\mathcal{Q}\left(\sqrt{k}s_x\right)e^{ks_X^2}\mathcal{Q}\left(\sqrt{k}s_y\right)e^{ks_Y^2} \end{aligned} \tag{35}$$

with

$$s_X \;=\; \frac{\partial g}{\partial x'}(X,Y)\left(\frac{1}{2\frac{\partial^2 g}{\partial x'^2}}\right)^{\frac{1}{2}} \tag{36}$$

$$s_Y \;=\; \frac{\partial g}{\partial y'}(X,Y)\left(\frac{1}{2\frac{\partial^2 g}{\partial y'^2}}\right)^{\frac{1}{2}} \tag{37}$$

One can obtain similar results for $I_{X_-Y_-}, I_{X_-Y_+}$ and $I_{X_+Y_-}$.

The final asymptotic solution is the sum of the uniform asymptotic expansions, and remains uniform. One does not have to select between different critical point expressions, and then this formulation does not suffer from discontinuities as it may occur when critical points get coupled.

4. NUMERICAL RESULTS

In this section, we compare the scattered field obtained by a numerical evaluation of the PO integral (8), by a MoM method and by our analytic expression. The MoM code is the ONERA ELSEM3D. All the simulations have been performed on a classical personal computer at 3.4 GHz clock. On Fig. 4, an incident TM gaussian beam illuminates a $15\lambda\times 15\lambda$ square plate. The beam incidence angles are $\theta_i = 55°, \phi_i = 0°$ and the distance OO_{GB} is 30λ. This incident beam has been choosen rotationaly symetric, with a waist W_0, by using a diagonal curvature matrix defined by $q_{11} = q_{22} = 2/(jkW_0^2)$ with $kW_0 = 2\pi$. The E_θ component of the electric field is observed for $\theta \in [-90°, 90°]$, $\phi = 0°$, at $r = 1000\lambda$ from the plate.

Asymptotic PO (continuous line) and numerical PO (dash-dotted

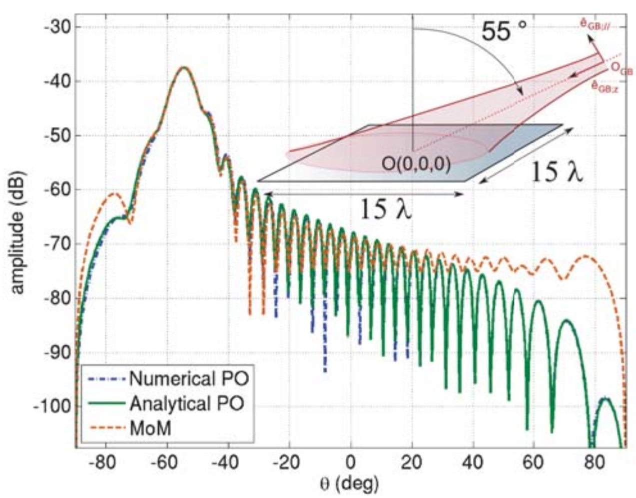

Figure 4. E_θ component at $r = 1000\lambda$ for $\theta \in [-90°, 90°], \phi = 0°, kW_0 = 2\pi$.

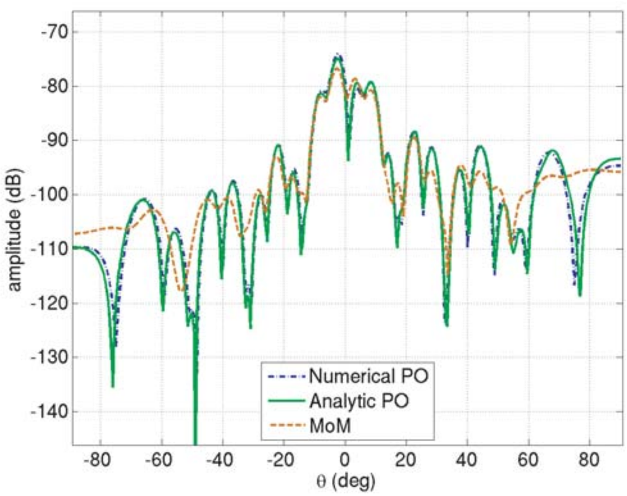

Figure 5. E_ϕ component at $r = 1000\lambda$ for $\theta \in [-90^\circ, 90^\circ], \phi = 112^\circ, kW_0 = 2\pi$.

line) curves are very close. In comparison with the MoM reference curve (dashed line), the PO solutions give correct results only for the first main lobes. While the numerical PO integration and the MoM took respectively 17 s and 21 min to be computed, the asymptotic evaluation was obtained in 0.2 s.

We present on Fig. 5 the scattered electric field in a more general case. The beam incidence angles are $\theta_i = 45^\circ, \phi_i = 0^\circ$ and its local origin O_{GB} is $(50\lambda,\ 0,\ 50\lambda)$. The size of the plate is $20\lambda \times 20\lambda$. The observation of the co-polar component E_ϕ is made for $\theta \in [-90^\circ,\ 90^\circ]$ with $\phi = 112^\circ$ at 1000λ from the plate.

As in the previous case, the analytic PO formulation is still very close to the numerical PO integration. Of course, PO evaluations don't match exactly the MoM result, except for the main lobes.

5. LIMITATIONS OF THE SOLUTION

The derivation of the PO asymptotic solution is mainly based on three hypothesis: A large distance approximation, a constant variation of the curvature matrix on the illuminated surface and the neglected anti diagonal terms in the Hessian matrix of the phase expression in some integrals. This three points are discussed below.

A parameter study of the presented solution in comparison to

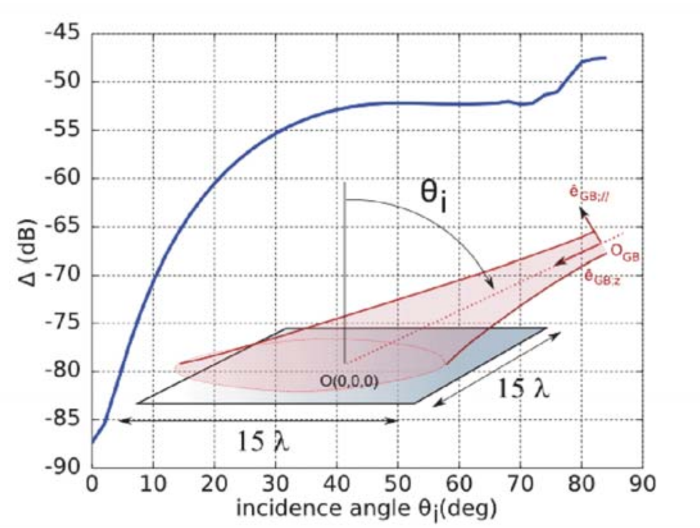

Figure 6. Error Δ (Eq. (38)) between PO numerical integration and PO asymptotic expansion for different beam incident angles θ_i with a $15\lambda \times 15\lambda$ plate. $\|OO_{\mathrm{GB}}\| = 30\lambda$.

a numerical PO integral evaluation shows that, as predicted, the observation distance must be at large distance from the illuminated surface. Typically, this distance must be greater than 70λ, in order to get a maximum error Δ inferior to $-70\,\mathrm{dB}$ in normal incidence ($\theta_i = 0°$), where Δ is defined by:

$$\Delta = \frac{\sum \left\| \mathbf{E}_s^{\mathrm{PO;num}} - \mathbf{E}_s^{\mathrm{PO;ana}} \right\|^2}{\sum \left\| \mathbf{E}_s^{\mathrm{PO;num}} \right\|^2} \tag{38}$$

Due to the approximation made on the curvature matrix on the illuminated surface, the analytical expression of the incident field on the integration domain does not exactly corresponds to the exact incident field of a GB. Consequently, the error between numerical and analytic PO as shown on Fig. 6, will increase as the incident field on the surface is rapidly varying. In particular, this is the case when the incident angle of the beam is strong ($\theta_i > 70°$).

However, this limitation has to be considered in relation to the PO approximation. Indeed, we have performed an other parameter study between PO and MoM. As expected, the PO approximation is only valid for the first main lobes of the scattered fields, but also when the incident angle of the beam is smaller to $\theta_i < 70°$. In case of grazing incidence, PO approximation doesn't give anymore the same result for the first main lobes than the one obtained with MoM.

At last, the approximation made into the derivation of integrals of type $\int_Y^\infty \int_X^\infty$ are critical when the total scattered field depends mainly on their contributions. This is the case when the incident field illuminate the plate in the directions which are outside the corners, corresponding to the fourth corner surfaces I_{XY} on Fig. 3. In this case, one can find that the scattered field is low, typically smaller than $-90\,\mathrm{dB}$. In a launching and bouncing algorithm, this kind of contribution may be whether neglected or occulted by some specular or more energizing fields coming from other scatterers.

Finally, in case of an incident field on the plate which does not come from a unique or multiple GB, by example a more complicated field radiated by an antenna, one can expand this incident field as a sum of GB. In this case, each of the GB is very localized on surface and then the asymptotic evaluation gives correct results [13].

6. CONCLUSION

In this paper, we have developed a new uniform analytic formulation in order to evaluate PO fields scattered by a PEC plate illuminated by a vector gaussian beam. Numerical results show that this formulation generally matches a PO numerical integration. This expression can be used in a gaussian beam tracking algorithm, in order to take into account of a first approximation of the diffraction effect by perfect conducting edges.

APPENDIX A. DOUBLE INTEGRAL WITH TWO BOUNDARIES

In this appendix, we derive an asymptotic approximation of a canonical integral I_{XY}:

$$I_{XY} = \int_Y^\infty \int_X^\infty f(x,y) e^{-kg(x,y)} dx dy \tag{A1}$$

where $f(x,y)$ is considered as a slow varying function in comparison to the exp term, k a large parameter and $g(x,y)$ a quadratic function:

$$g(x,y) = \frac{1}{2}\left(a_{11}x^2 + 2a_{12}xy + a_{22}y^2\right) - b_x x - b_y y \tag{A2}$$

The elements a_{ij} and the parameters (b_x, b_y) are complex constants, such as the real part of $g(x,y)$ is positive as $|x|, |y| \to \infty$, in order to ensure the convergence of I_{XY}. It is well known from linear algebra theory that it exists a change of variables which permits one to express $g(x(u,v), y(u,v))$ as a separated functions of (u,v), i.e., $\exp(g(x,y)) = \exp(g_u(u))\exp(g_v(v))$. However, this change of variables makes difficult an asymptotic expansion of the resulting integral, because one has to deal with a non trivial integral, which integrand contains an error function. So, in order to get a close form expression, a first order approximation may consist in neglecting the $a_{12}xy$ term in $g(x,y)$ [18]. By this way, I can be asymptotically approximate as:

$$I_{XY} \approx f(X,Y) \int_X^\infty e^{-kg^{(x)}(x)} dx \int_Y^\infty e^{-kg^{(y)}(y)} dy \tag{A3}$$

An asymptotic expansion of these two integral is well known and one obtains expression (35).

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