

## DESIGN OF A BROADBAND TRANSITION USING THE CONSTANT IMPEDANCE STRUCTURE APPROACH

P. Fuerholz and A. Murk

Institute of Applied Physics  
Sidlerstrasse 5, Berne 3012, Switzerland

**Abstract**—Transitions from circular waveguides to rectangular waveguides are used in many situations. One particular case is between the feed of a circular corrugated horn antenna and following rectangular waveguide structures. Since the field patterns are not the same on both sides the conversion from rectangular to circular waveguide always results in a certain amount of power reflected back in one waveguide. Much effort has been put in designing special converters which reduce this effect. For many designs, reflection is reduced by introducing a certain number of waveguide steps. By adjusting the distance between these steps, one can get destructive interference of the returned signal at specific frequencies. In this paper, an alternative approach, the constant impedance structure (CIS) has been chosen. This eliminates the need to design a waveguide converter for minimum return loss at discrete frequencies. The transition obtained by this approach is compared to a transition based on linear surface interpolation.

### 1. INTRODUCTION

Waveguide transitions are used where a transition from circular to rectangular waveguide elements is needed. Although microwave circuits are normally built using rectangular waveguide, for propagation to free space circular is more favorable. Therefore, a conversion from one waveguide type to the other needs to be made. The easiest way to do this is simply to connect a rectangular and a circular waveguide. For such a simple transition, the return loss is at  $-10$  dB. This is much too high for many applications. An alternative approach is to conically drill the circular waveguide into the rectangular one. Such a transition

---

Corresponding author: P. Fuerholz (fuerholz@iap.unibe.ch).

is very easy to manufacture. However, the only parameter affecting the return loss is the length. In order to perform well in terms of return loss, the waveguide converter needs to be several wavelengths long. A alternative approach is to use a linear interpolation conversion from rectangular to circular waveguide, this conversion is referred to as lofted conversion in this paper. This design is more difficult to realize than the drilled conversion. On the other hand, in terms of electrical performance, this design exceeds the drilled conversion. A very promising design for a waveguide converter places several pieces of constant cross-section between the circular and rectangular waveguide [1, 2]. At each junction, some power is returned. By adjusting the length of the waveguide pieces, one can have partial destructive interference of the returned signal between two adjacent waveguide junctions at discrete frequencies.

In this paper, a different approach is considered. Instead of using a sequence of waveguides with constant cross-section, we distort the cross-section shape of the transition to minimize return loss. More precisely, we force the transition to have constant impedance for the first propagating mode above cutoff along the waveguide. Application of this approach is presented. The design technique is presented in Part 2 of this paper while Part 3 shows the results of a numerical study varying the total length of the transition. The computation of the return loss and the modal analysis is done for the drilled transition, the optimized drilled transition and the lofted transition.

## 2. THE EQUIVALENT IMPEDANCE STRUCTURE APPROACH

From circuit theory, it is well known that an transversal impedance jump in a transmission line results in power reflected backward. The relation between the return loss  $\rho$ , and the transversal impedance of two transmission line segments, denoted as  $Z_1$  and  $Z_2$  is

$$\rho = \frac{Z_1 - Z_2}{Z_1 + Z_2}. \quad (1)$$

This relation is only valid for TEM mode. In our case, we use either TE or TM-modes carrying waveguides. Therefore, the transmission line theory needs to be extended in order to be correct in this case. Assuming a single propagating waveguide mode in both waveguides at the junction, the return loss is expressed as

$$\rho_{wg} = \frac{Z_1 - q^2 Z_2}{Z_1 + q^2 Z_2} \quad (2)$$

using mode-matching theory [3]. A waveguide mode is defined as an eigenvalue/eigenfunction combination of

$$\nabla \times \nabla \times \vec{E} - k^2 \epsilon_r \vec{E} = 0 \quad (3)$$

with Dirichlet boundary conditions,  $\vec{E}$  as the electric field (eigenfunction) as  $k^2$  the squared radial propagation constant (eigenvalue), and  $\epsilon_r$  denoting the relative electric permittivity on the problem domain. The eigenfunctions are subject to the orthogonality relation

$$\delta_{mo} \delta_{np} = \iint_{\Omega_1} \vec{E}_{mn} \vec{E}_{op} \quad (4)$$

with  $\delta_{mo}$ ,  $\delta_{np}$  representing the Kroenecker delta function and  $\Omega_1$  signifying the surface on which (3) is defined.  $q$  denotes the overlap integral of the two field configurations

$$q = \iint_{\Omega} \vec{E}_{mn} \vec{E}_{op} \quad (5)$$

where  $\Omega$  is the surface enclosed by the smaller waveguide along the junction. As the previous sentence suggests, Equation (2) is only valid if the border of the surface of the larger waveguide at the junction completely surrounds the border of the smaller waveguide.

The return loss of the waveguide transition vanishes completely when the field coupling value  $q$  is equal to one, and when the impedance is equal in both waveguide pieces. Because of the differing field configurations, the field coupling from a circular to rectangular waveguide mode is less than one. We do not have control over this parameter. However, the impedance along the transition can be controlled by varying the shape of the waveguide transition.

## 2.1. Design of a CIS-based Transition

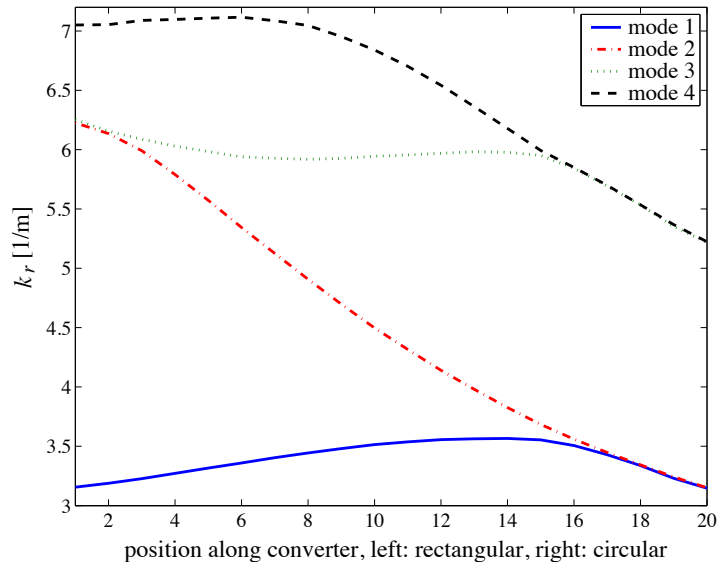
We start by designing a waveguide transition based on a rectangular waveguide with a conical opening into the circular waveguide. Let the cutoff frequency of the waveguide on both sides be  $f_{\text{Cutoff}}$ . From basic waveguide theory, the width of the rectangular waveguide, denoted as  $a$ , is

$$a = \frac{c}{2f_{\text{Cutoff}}} \quad (6)$$

with  $c$  being the speed of light. Because we want single-modedness of the largest possible bandwidth, we choose the rectangular waveguide height  $b$  to be  $a/2$ . For the circular wave radius  $r$ , we obtain

$$r = \frac{\chi c}{f_{\text{Cutoff}} 2\pi} \quad (7)$$

where  $\chi$  is the first zero of the derivative of the Bessel function of the first kind of order 1. We proceed by computing the radial wavenumbers of the TE waveguide modes for many cross-sections at different axial positions representing different cross-section shapes. This has been done numerically using finite elements modelling (FEM). Since by design the cutoff frequency of the lowest-order mode is the same for the rectangular and the circular waveguide, the radial propagation constant for the lowest-order mode is also equal at the left and right end of the transition (see Fig. 1). The index shown on the  $x$ -axis of Figs. 1–3 represents the cross-section shape of the transition with 1 being a rectangular cross-section and 20 being a circular cross-section. However, in the middle of the transition it increases. This implies that we have return loss because of varying impedance in the transition. Additionally, the effective cutoff frequency of the transition increases since the mode is below cutoff at the center of the transition for the lower part of the frequency spectrum. The cutoff frequency is mainly affected by the size of the waveguide, so we have to increase the size of the waveguide systematically to correct for the increase of radial wavenumber along the converter. For this, the radial waveguide number curve for the lowest order mode is interpreted as an increase factor for the width of the rectangular waveguide, while retaining the



**Figure 1.** Radial wavenumber of the four lowest order waveguide modes for the drilled transition.

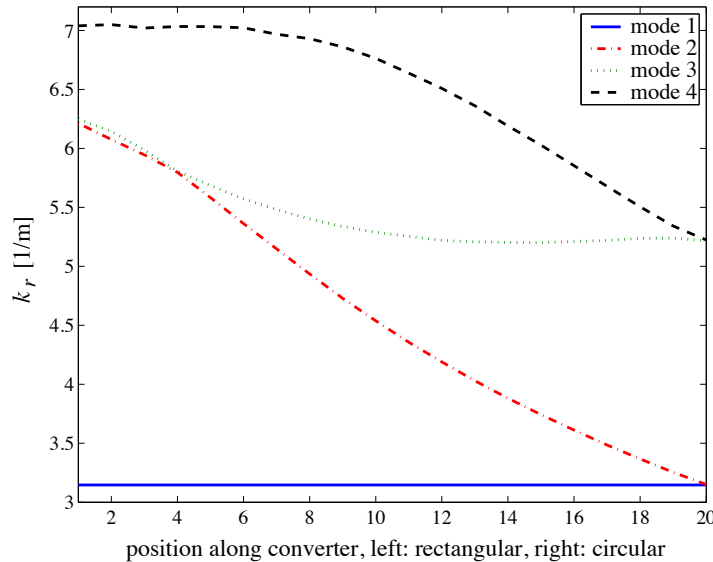
radius of the circular waveguide and the height of the rectangular waveguide at each segment. We choose a linear relation between the increase factor  $If$  and the radial wavenumber  $k_r$

$$If(Pos) = \frac{(k_r(Pos) - k_r(1))}{\max(k_r - k_r(1))} 0.38a \quad (8)$$

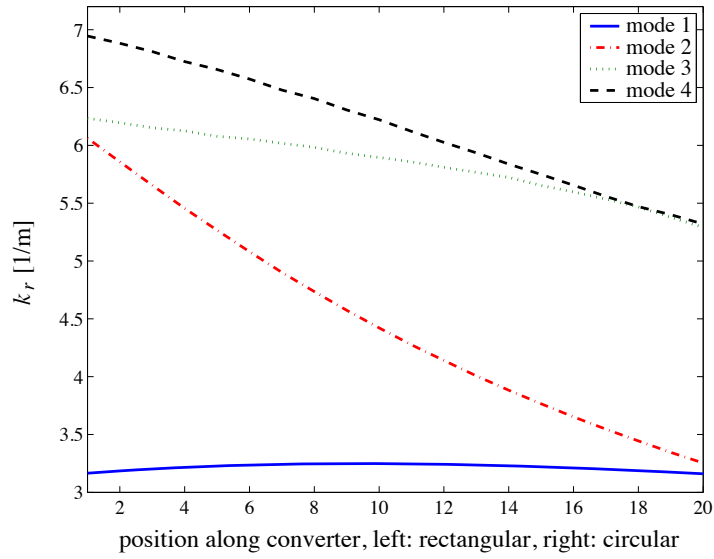
with 0.38 being an arbitrary scaling factor.  $Pos$  denotes an integer index, which marks the positions along the converter. In an iterative loop, the radial wavenumbers are recomputed, the increase factors are computed and the geometry is altered. This iteration has been repeated twenty times. Figure two shows the radial wavenumbers of the four lowest order modes along the transition for the optimized case.

### 3. EXAMPLE CASE AND NUMERICAL ANALYSIS

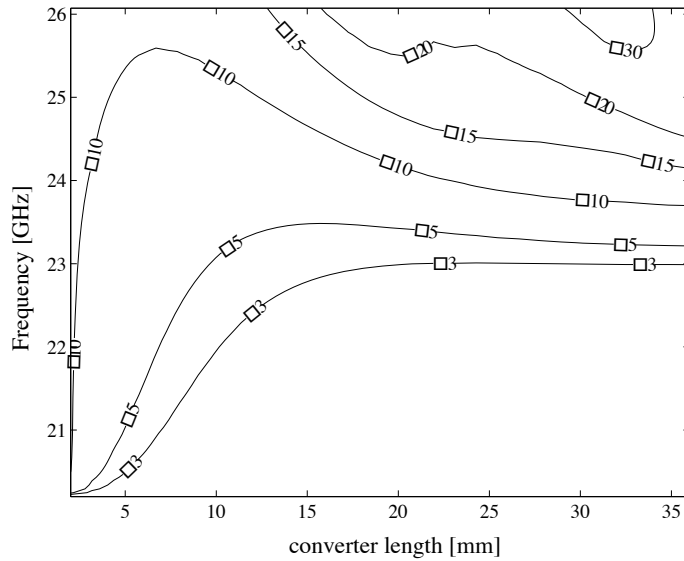
The CIS concept has been applied to a waveguide transition operating at a frequency band around 22 GHz. For the lower cutoff frequency, we choose 20 GHz. Thus we obtain a rectangular waveguide part with a cross-section of  $7.5 \times 3.75$  mm and a circular waveguide part with a radius of 4.4 mm. For the example design the lengths of the straight



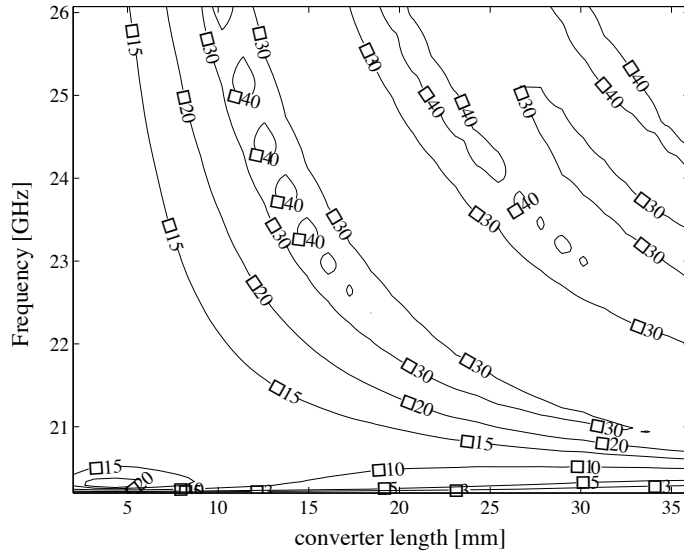
**Figure 2.** Radial wavenumber of the four lowest order waveguide modes for the optimized transition.



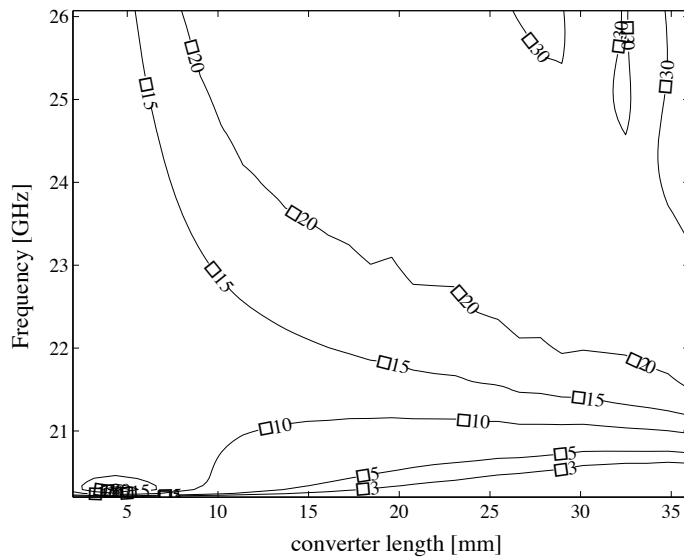
**Figure 3.** Radial wavenumber of the four lowest order waveguide modes for the lofted transition.



**Figure 4.** Return loss versus frequency and transition length for the non-optimized case.

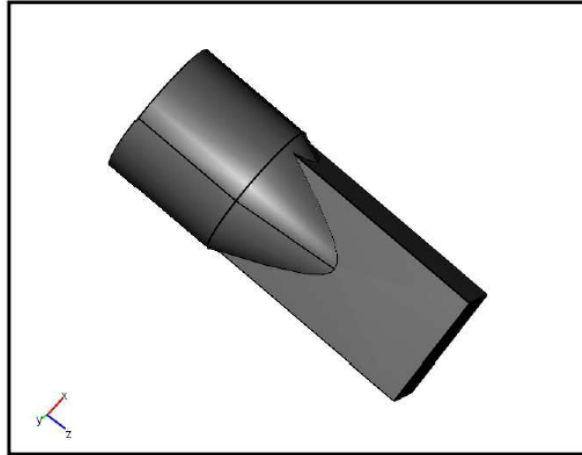


**Figure 5.** Return loss versus frequency and transition length for the optimized case.

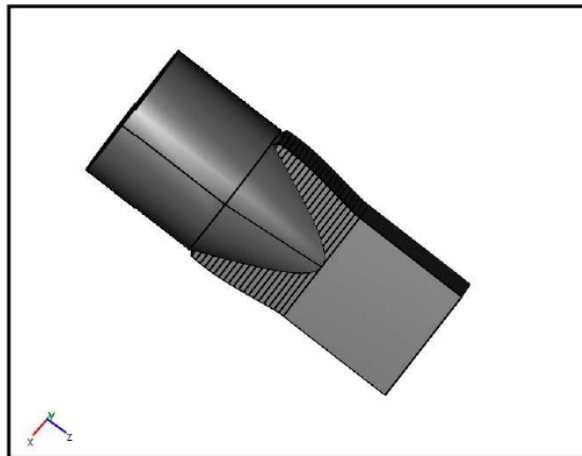


**Figure 6.** Return loss versus frequency and transition length for the lofted transition.

waveguide pieces are 7.5 mm (see Figs. 7–9). The section between the straight waveguide parts is denoted converter section. The single moded frequency band thus ranges from 20 to 26.1 GHz. For this frequency range, the return loss computed for many lengths of the waveguide converter section and for both the non-optimized and the optimized transition. This is accomplished using the 3D RF toolbox from COMSOL multiphysics. In a rough convergence analysis, which consists of variations of the FEM-mesh, the computation uncertainty

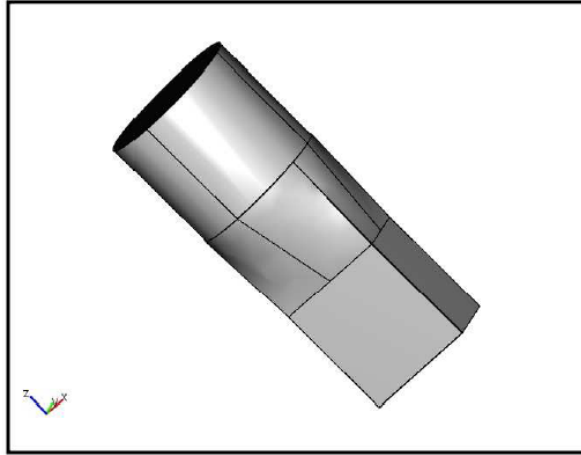


**Figure 7.** Picture of the non-optimized transition.



**Figure 8.** Picture of the optimized transition.





**Figure 9.** Picture of the lofted transition.

is assessed to be 0.5 dB. Fig. 4 shows the return loss versus frequency and length of the transition for the non-optimized case. For very short lengths, the return loss is close to the value for a direct connection between circular and rectangular waveguide. For longer transitions, the return loss decreases at the upper end of the frequency band. At the lower end, we observe an increase in return loss. This is because the radial wavenumber rises at the center of the transition, as pointed out in Part 2 of this paper. For the optimized case, whose return loss versus frequency and length is plotted in Fig. 5, the return loss decreases with increasing length over nearly the entire bandwidth. Furthermore, we can have a return loss below  $-40$  dB at just a few wavelengths. However the return loss curve does not show the deep minimas as present in other designs [1].

The lofted transition shows some increase in radial wavenumber of the lowest order mode. In the non-optimized transition, this results in high return loss values at the lower end of the frequency band. The lofted transition shows a similar return loss pattern with varying frequency and converter length to the optimized case. However, in terms of bandwidth and return loss level the optimized CIS-based converter still outperforms the lofted transition.

#### 4. CONCLUSIONS

In this paper, a frequency independent design technique for broadband rectangular to circular waveguide transitions is presented. This design approach is based on adjustment of the shape of the transition to achieve constant impedance for the lowest-order waveguide mode. Three-dimensional FEM computations confirm the effectiveness of the design approach and the algorithmic implementation. An example case shows that the return loss is in average 10 dB lower for a frequency range of 6 GHz and converter length of 3 to 36 mm when compared to a lofted conversion.

#### ACKNOWLEDGMENT

This work was supported by the Swiss National Science Foundation under Grant 200020-100167 and 200020-19908.

#### REFERENCES

1. Bornemann, J. and M. Mokhtaari, "Initial design and optimization of broad-band and dual-band square-to-circular waveguide transitions," *APMC 2005 Proceedings*, 2005.
2. Holzman, E. L., "A simple circular-to-rectangular waveguide transition," *IEEE Microwave and Wireless Components Letters*, Vol. 15, No. 1, 25–26, January 2005.
3. Wexler, A., "Solution of waveguide discontinuities by modal analysis," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 15, No. 9, 508–517, 1967.