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UNCERTAINTY ANALYSIS OF THE RATIONAL FUNCTION MODEL USED IN THE COMPLEX PERMITTIVITY MEASUREMENT OF BIOLOGICAL TISSUES USING PMCT PROBES WITHIN A WIDE MICROWAVE FREQUENCY BAND

Z. Wang, W. Che †, and L. Zhou

Department of Electrical Engineering
Nanjing University of Science and Technology
China

Abstract—The PMCT probe, saying, one kind of open-ended coaxial probe adopted widely in microwave coagulation therapy of cancer, has been used to measure the complex permittivity of freshly excised specimens of normal animal tissues. The RFM model for PMCT probe is developed to extract the anticipant permittivity of specimens under test. In addition, the effects of several factors on the measurement results have been considered and discussed, including different temperature and reference materials, as well as the sampling frequency range and intervals of the rational function model. All the experiments have been conducted at the microwave frequency range from 450 MHz to 14.5 GHz.

1. INTRODUCTION

Over the half past century, many researches have been carried out in the microwave measurement of complex permittivity [1, 2]. Now the combination of the transmission line method and advanced computational data processing techniques are widely used in the medical diagnoses and detect [3–5]. The basic ideas lie in that, the complex reflection coefficient at the connector plane is usually recorded and post-processed, and the complex permittivity can thus be calculated from the reflection coefficient at the probe aperture using standard inverse techniques. However, many previous works have

Corresponding author: W. Che (yeeren_che@yahoo.com.cn).
† Also with Institute for High Frequency Engineering, Technische Universitat Munchen, Germany.

indicated, the physical damage of aperture during the handling of the probe, and the imperfect connection of the probe and the specimen, would cause measurement errors, especially for most of the lossy biological tissues with highly sensitive permittivity to the environment such as the measuring frequency and ambient temperature etc [6]. Therefore, the rational function model (RFM) has been proposed to compensate for the propagation effects of the probe and correct the errors caused by the inaccurate connection between the probe and the specimen, as well as some other system uncertainties [7–9]. However, we may point out, according to the theoretical analysis of the admittance model [10], the RFM is an approximate model which ignores the effects of the higher order modes generated in higher frequency range. In this way, it should be very interesting and meaningful to investigate the effects on measurement errors caused by the intervals of the sampling frequency intervals, the frequency range and the model variations.

In our work, the percutaneous microwave coagulation therapy (PMCT) probe has been employed in the complex permittivity measurement of biological tissues [11–14]. This kind of probe is actually one monopole radiator used widely in the microwave coagulation therapy of cancer treatment. The RFM model has been developed for the PMCT probe, the complex permittivity of the specimens under test can thus be extracted from the measured reflection coefficients. In addition, some uncertainties of the rational function model in the measurements have been considered and discussed.

2. THEORETICAL BACKGROUND

2.1. Rational Function Model

In 1994, the rational function model of the rectangular waveguide was firstly given as it described the relationship between the terminal admittance and the reflection coefficient. The measurement work was carried out and the RFM of the coaxial line was in succession approximately proposed, the uncertainty of this method was also discussed [8]. In this work, it has been employed as the theoretical approach to extract complex permittivity of biological tissues. In case of connection with a homogeneous isotropic half space, the normalized aperture admittance of a 50 ohm Teflon-filled open-ended coaxial probe

can be expressed as:

$$Y = \frac{\sum_{n=1}^{8}\sum_{p=1}^{6}\hat{\alpha}_{np}(\sqrt{\varepsilon_r})^p(sa)^n}{1+\sum_{m=1}^{8}\sum_{q=0}^{6}\hat{\beta}_{mq}(\sqrt{\varepsilon_r})^q(sa)^m} \tag{1}$$

where ε_r is the permittivity of the half space, s is the complex frequency, a is the radius of inner conductor of the line, while $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ are the model coefficients (as shown in Table 1). The theoretical principle of permittivity measurement of the specimen under test from rational function model is schematically illustrated in Fig. 1.

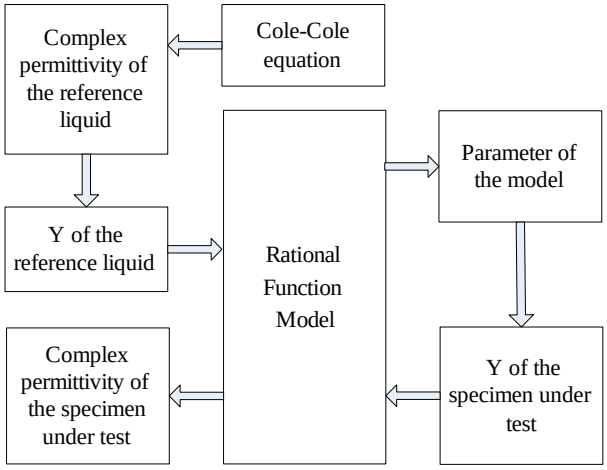

Figure 1. Principle of the permittivity measurement with rational function model.

As we can see in Fig. 1, in order to fix on the RFM parameters as described in the expression of $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ for the PMCT probe, a forward calculation was carried out. The complex permittivity of the reference liquid (distilled water and saline) at different temperature and certain frequency points were calculated through the Cole-Cole equations [11] while the terminal admittance was calculated through measured S-parameters of the reference liquids using a VNA. After substituting the corresponding data of the terminal admittance and the complex permittivity of the reference liquid into (1), the rational function model was established through a group of 104 numbers. At

last, the backward calculation can result in the complex permittivity of the specimens after both of the model parameters and the terminal admittance are ensured.

We may note that the adaptability of the RFM should be investigated before measuring and calculating the complex permittivity of the biological tissues. A group data of $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ from the measurement of the distilled water were calculated and taken as the reference data for further study of the uncertainty items. Noting that, the ambient temperature for testing is 25°C, while the sampling frequencies range from 0.45 GHz to 4.07 GHz. All the variations were expressed through the relative error defined as the ratio of the difference between the data on the test condition and the reference condition over the reference data.

2.2. Cole-Cole Parameters and General Dispersion Equations

The reference data of the calibration liquids are all obtained from the Cole-Cole simplified calculation equations [11], and the general dispersion equation is given bellow:

$$\hat{\varepsilon} = \varepsilon' - j\varepsilon'' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (j\omega\tau)^{1-\alpha}} - j\frac{\sigma}{\omega\varepsilon_0} \tag{2}$$

$$\varepsilon' = \frac{\varepsilon_s - \varepsilon_\infty}{2}\left\{1 - \frac{\sinh[(1-\alpha)\ln\omega\tau]}{\cosh[(1-\alpha)\ln\omega\tau] + \sin\alpha\frac{\pi}{2}}\right\} + \varepsilon_\infty \tag{3}$$

$$\varepsilon'' = \frac{\varepsilon_s - \varepsilon_\infty}{2}\left\{\frac{\cos\alpha\dfrac{\pi}{2}}{\cosh[(1-\alpha)\ln\omega\tau] + \sin\alpha\dfrac{\pi}{2}}\right\} + \frac{\sigma}{\omega\varepsilon_0} \tag{4}$$

where ε_∞ the complex permittivity while the $\omega\tau \gg 1$, ε_s the complex permittivity while the $\omega\tau \ll 1$, τ the time delay of the relax polarization, σ the static conductivity and α distribution parameter are the Cole-Cole parameters calculated from the simplified calculation equations in [11].

Obviously, when the RFM models and the Cole-Cole parameters of the reference liquids are known, the complex permittivity of the biological tissues under test can thus be calculated, the basic procedures are shown in Fig. 2.

Next, the uncertainties of the RFM in the permittivity measurement will be discussed, saying, the effect of several factors on the measurement in cases of different conditions, including different temperatures, different reference materials, different intervals of sampling frequencies etc.

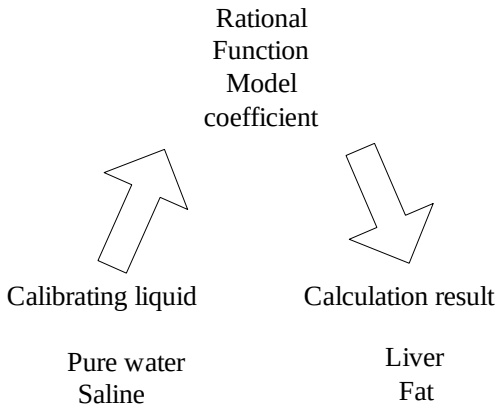

Figure 2. Calibrating and calculating orders.

2.3. Uncertainty Items

2.3.1. Temperature and Material Sensitivity of the RFM

The comparisons have been made to observe the change of the complex permittivity with temperature and the calibrating steps. In our work, the PMCT probe [12] is used for calibration and measurement. According to the above-mentioned theoretical principle, the parameters of RFM model for our PMCT probe should be obtained firstly. Two reference liquids are measured and the reflection coefficients are recorded and calculated, the RFM coefficients are calculated and compared in two cases of different reference liquids, which are illustrated as Fig. 3. We may note that, we use a group of data of the model parameter of the distilled water at 25°C while the frequency range is from 0.45–4.07 GHz as the reference data. All the relative errors of the RFM coefficients from the measured results at 104 frequency points are very little in cases of two different reference liquids and different temperatures, implying the RFM model for our PMCT probe is stable. It should be noted that, the curve with "·" is for the case of distilled water while the ambient temperature changes from 25°C to 37°C, while the curve with "Δ" indicates the relative errors of RFM coefficients at 104 frequency points in case of distilled water and the Saline at 25°C. In this way, the PMCT probe can be used for the further measurement of some biological tissues with certain reliability and the RFM model can be used to extract the permittivity of the biological tissues of our interests.

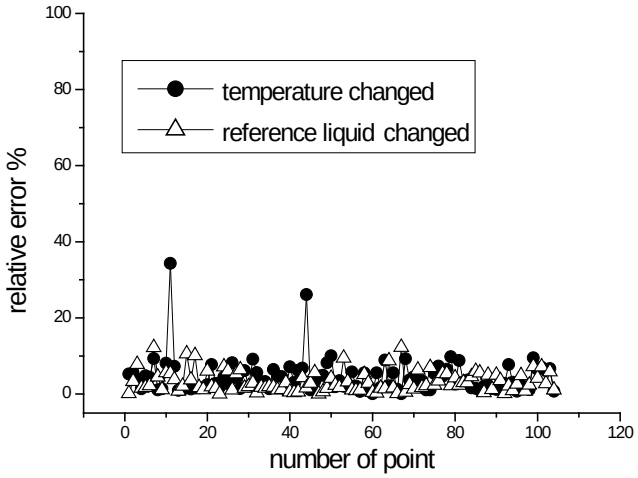

Figure 3. The relative error of the rational function model coefficient.

2.3.2. Frequency Sampling Sensitivity of the RFM

From (1) we can see that the rational function model parameters $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ are quite related with the admittance Y, the complex permittivity ε_r, the inner radius of the coaxial probe and the complex frequency. As the expression was admittance model while $N = M$ and $P = Q$, the relationship between the model parameters $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ uniformed as M and the complex frequency can be expressed as:

$$M = \frac{1}{\frac{1-Y}{Y}\sum_{n=1}^{N}\sum_{p=1}^{P}(\sqrt{\varepsilon_r})^p(sa)^n - Y\sum_{m=1}^{M}(sa)^m} \tag{5}$$

From the theoretical principle in [9] the relative change in the model parameters, namely, the sensitivity of the model, can be expressed as:

$$\frac{\Delta M}{M} = S_S^M\frac{\Delta S}{|S|} + S_Y^M\frac{\Delta Y}{|Y|} + S_{\varepsilon_r}^M\frac{\Delta\varepsilon_r}{|\varepsilon_r|} \tag{6}$$

As the aperture admittance and complex permittivity of biological tissues are both the functions of the frequency, it is thus very interesting to investigate the effect of sampling frequency range and intervals on the measurement errors. We have chosen two different frequency ranges, 0.55 GHz to 5.07 GHz and 0.45 GHz to 14.5 GHz as they can be compared with the results of the reference liquid mentioned previously,

saying, the model parameters calculated through the distilled water at 25°C while sampling frequencies range from 0.45 GHz to 4.07 GHz, both with 104 sampling points. Similarly, the RFM coefficients are calculated from the measured reflection coefficients of the reference liquids, the relative errors of the model parameters at different number of points are illustrated in Fig. 4. Noting that, the curve with "○" corresponds to the case whose sampling frequency range is 0.55–5.07 GHz which is the same frequency range as the reference group to test the sensitivity of the frequency sampling intervals, while the curve with "■" represents the case in which both the sampling frequency range and sampling interval change. In addition, we may note that frequency range of this group is from 0.45–14.5 GHz.

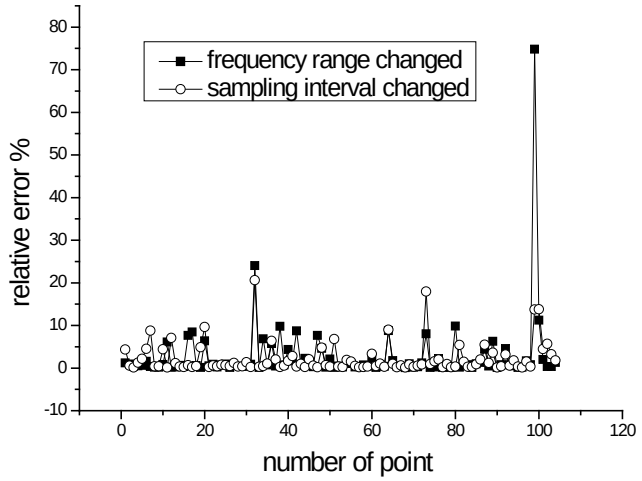

Figure 4. The relative error of the rational function model coefficient.

From Fig. 4, we can easily find the model is quite sensitive to the frequency range as this change is combined with the change of sampling interval, for the number of calculation points was restricted.

It has been observed that the model coefficients are more sensitive to the frequency variation. This is not difficult to understand, for the aperture admittance and the complex permittivity in previous are functions of the complex frequency also. To get a stable RFM model for further permittivity measurement of the biological tissues of our interest, the ultimate RFM coefficients have been chosen on the basis of considering the reference materials, the ambient temperature, the frequency range and the sampling intervals etc.

3. RESULTS AND DISCUSSIONS

3.1. Experimental Setup

The measurements are conducted using an Agilent 8722ES Vector network analyzer (VNA) with a HP notebook connected through the Agilent 82357A USB/GPIB interface. As stated previously, the PMCT probe shown in Fig. 5 has been used in our experiments, which is connected with the VNA through a TNC-SMA connector. The frequency range of the experiment is 0.45–14.5 GHz with 401 sampling points. Corresponding temperatures are recorded within a certain time range and averaged to eliminate the errors caused by the dynamic temperature variation.

The PMCT probe was a thin coaxial-fed short monopole antenna, the outer radius of the coaxial line is 0.55 mm while the inner radius is 0.125 mm. The PMCT probe was used in invasive measurement.

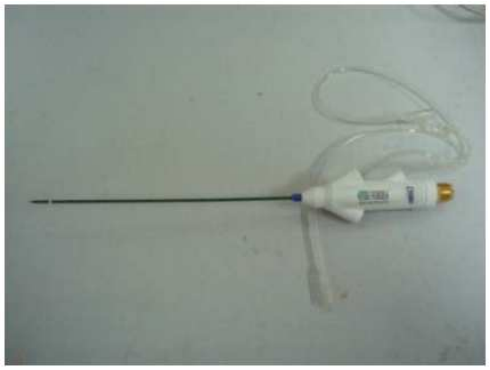

Figure 5. The picture of the PMCT probe.

3.2. RFM Coefficients of PMCT Probe

For further permittivity measurement of the biological specimens under test, the parameters $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ of the RFM for PMCT are required to obtain firstly. As we know, the complex permittivity of 2 reference liquids (distilled water and Saline) can be found from the Cole-Cole equation [11] and the measured S-parameters, $\hat{\alpha}_{np}$ and $\hat{\beta}_{mq}$ of the PMCT probe can thus be obtained by RFM method, which are averaged and listed in Tables 1 and 2.

Table 1. The coefficient $\hat{\alpha}_{np}$ of the RFM of the PMCT probe.

p \ n	$\hat{\alpha}_{np}$							
	1	2	3	4	5	6	7	8
1	0.242361145	0.244204309	0.643852136	7.10E-02	-1.89E-01	-0.104375755	-0.699900245	0.259451845
2	-0.365999527	-3.15E-01	-3.73E-01	4.61E-02	-1.16E-01	-0.724306718	-0.640028318	0.614762236
3	0.245217891	-0.704752482	-8.35E-02	-6.15E-01	3.16E-01	-0.212755327	0.636038018	-0.213211355
4	0.003761278	3.69E-01	5.72E-01	1.20E-01	-1.63E-01	0.185477918	-0.446687291	0.595529182
5	-0.405530982	0.616206009	2.66E-01	-4.63E-01	7.10E-01	0.169863618	-0.104931753	-4.0332508
6	3.92E-02	3.67E-01	-4.63E-01	2.07E-01	8.53E-02	0.395957845	9.95E-02	-0.154162845

Table 2. The coefficient $\hat{\beta}_{mq}$ of the RFM of the PMCT probe.

q \ m	$\hat{\beta}_{mq}$							
	1	2	3	4	5	6	7	8
0	-0.138491945	0.910324591	0.121791927	-0.767747345	0.108991032	-3.80E-02	-1.05E-01	1.88E-01
1	-0.539973564	-0.575376591	0.420381109	0.142684436	0.190915882	-2.31E-02	-4.49E-01	2.81E-01
2	-0.465738918	-9.20E-02	-0.635267882	0.117439691	-0.640792864	-1.21E-01	-6.88E-01	-1.04E-02
3	-0.794306155	0.519282236	0.360557091	0.511512773	-0.194566882	5.50E-01	2.07E-01	-2.07E-02
4	0.127721482	-0.170070191	0.129421382	0.415039255	-0.440654255	2.66E-01	-5.41E-01	-2.57E-01
5	-0.3046541	-6.37E-02	0.366332027	3.99E-02	-5.33E-03	1.59E-01	1.47E-01	-2.06E-01
6	-0.458220118	0.280471191	4.05E-02	0.494543627	4.20E-02	-4.01E-01	-8.10E-01	-7.16E-01

3.3. Resultant Complex Permittivity of the Biological Tissues and Discussions

As stated in Fig. 2, the admittance related with the S-parameters of the specimen is measured through the VNA. The RFM technique is used to convert the reflection coefficient at aperture plane to the frequency-dependent complex permittivity of the specimen. In our work, two kinds of animal tissues have been measured, saying, the porcine fat and liver. In addition, the measurements have been conducted in cases of different temperatures, 25°C and 37°C. The resultant real and imaginary parts for these two tissues are shown in Figs. 6–9. For purpose of comparison, the complex permittivity of these two tissues appeared in Reference [15] by Cole-Cole method is also shown inside the figures. Noting that, Figs. 6 and 8 indicate the real part of the complex permittivity, saying, the dielectric constant, while Figs. 7 and 9 show the imaginary part of the complex permittivity, i.e., the loss factor, which is a function of loss tangent.

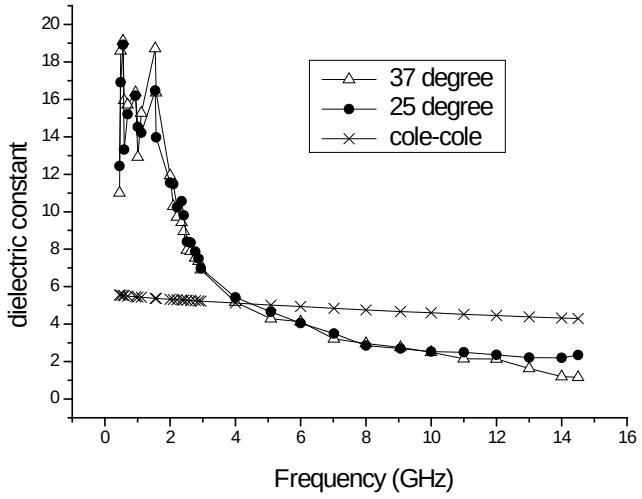

Figure 6. The real part of the permittivity of fat versus frequency in case of different temperatures.

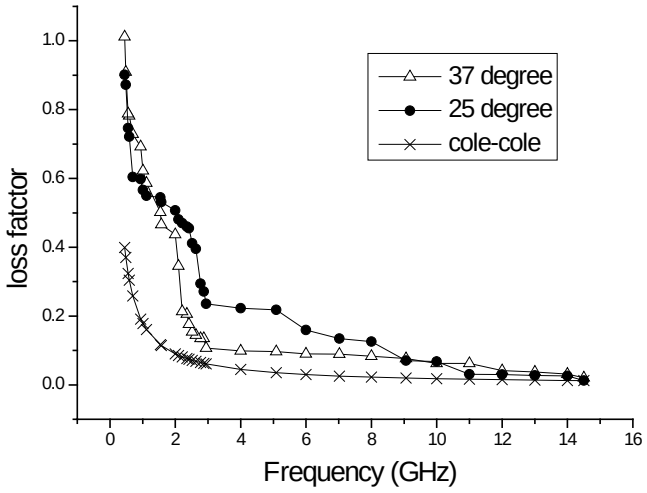

Figure 7. The imaginary part of the complex permittivity of fat versus frequency in case of different temperatures.

From Figs. 6–9, we can see that at the vicinity of lower band of the frequency range, for example from 450 MHz to 1.5 GHz, the calculation results are not stable enough; the agreements with the Cole-Cole results are not very good. In addition, the agreement in

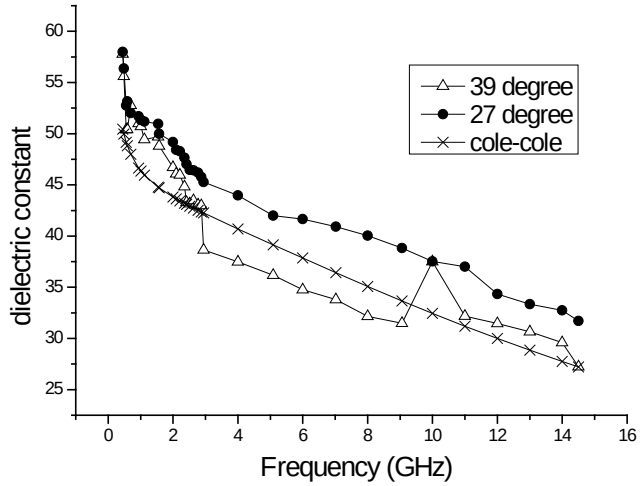

Figure 8. The real part of the complex permittivity of liver versus frequency in case of different temperatures.

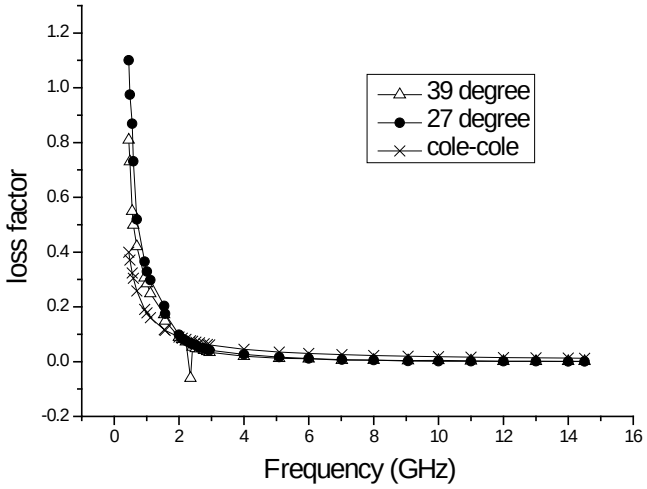

Figure 9. The imaginary part of the complex permittivity of liver versus frequency in case of different temperatures.

case of liver is better than the case of fat. The reasons may lie in that, the PMCT probe was actually used for microwave coagulation therapy of liver cancer, the probe obviously has better matching in liver than in fat. In addition, the liver has higher complex permittivity than fat;

the working wavelengths in these two tissues are quite different. These factors would certainly cause obvious measurement deviations. In this way, the RFM model should be modified a little for cases of different tissues for more accurate measurement.

The limitation of the PMCT probe used in the measurement of the biological tissue can be observed also in above figures, especially its stability. As the matching of the PMCT probe is not so good at higher frequencies, the stability of measurement with PMCT in a wide frequency range is still an important work to do in the near further. If the PMCT probe is expected to be used for real-time permittivity measurement of the biological tissue while the microwave coagulation therapy of the liver tissue is conducting, the stabilities to frequency and temperature of the PMCT must be resolved.

4. CONCLUSIONS

The theoretical principle of RFM method is introduced firstly in this paper. The highlight in this work lies in the fact that the PMCT probes are employed for permittivity measurement of biological tissues. The RFM model for PMCT probe is developed based on 2 different reference liquids. The permittivity of two biological tissues (porcine fat and liver) are obtained from the measured reflection coefficients and thus calculated by the known RFM model. The uncertainties in the measurement such as the temperature and different reference liquids have also been discussed. The agreements between the resultant complex permittivity of the tissues under test and the data in references are not good enough; however, these results have still demonstrated the validity of the RFM of the PMCT probe. Some further works are required to improve the accuracy and stability while using the PMCT probe in the measurement and calculation. It is believed that this work will provide potential guidance for the simultaneous permittivity measurement with higher accuracy in case of medical treatment of human cancer.

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