Progress In Electromagnetics Research M, Vol. 6, 1–8, 2009

A FORMAL APPROACH FOR CALCULATING THE RADIATION FIELDS OF A LINEAR WIRE ANTENNA

A. J. Zozaya

Laboratorio de Electromagnetismo Aplicado
Universidad de Carabobo
Bárbula, Av. Universidad, Valencia, Edo. Carabobo, Venezuela

Abstract—In classical antenna books, the field radiated by a filamentary antenna is calculated by integrating the electrical current induced over the wires as if it is a primary (impressed) source. This is not technically incorrect, but is not rigorous. In this paper some formal steps are added to the classical procedure to do it more rigorously.

1. INTRODUCTION

In recent antenna books and papers [1, 2] and most of the classical ones [3–5], the radiation field is calculated by integrating the electrical current on the wire surface as if the wire itself does not exist. The electrical current induced over the wire is treated as an impressed source. This is not formally correct, unless the Uniqueness and the Equivalence Theorems are properly invoked. The electrical current on the wire surface is induced by the field itself and, in the electromagnetic problem, must be treated as a boundary condition and not as an impressed source. In some old antenna books, as in [6], this fact is mentioned briefly. Nevertheless, it is well known that the radiation process occurs because of the presence of wires that scatter the radiation field. In this paper, a formal procedure is proposed to give completeness to the classical approach for calculating the radiation fields from a wire antenna.

This paper is organized in the following manner: in Section 2 the classical procedure for calculating the radiation field from a wire antenna is revisited; in Section 3 some formal steps are added to the classical procedure to make it more rigorous.

Corresponding author: A. J. Zozaya (azozaya@uc.edu.ve).

2. RADIATION FIELDS OF A LINEAR WIRE ANTENNA: THE CLASSICAL APPROACH

Given a linear wire antenna, infinitesimal, small or finite length one, positioned symmetrically at the origin of the coordinate system and oriented along the z axis, as shown in Fig. 1. A known filamentary electrical current is supposed to exist in the form of $\vec{J} = I(z')\delta(x)\delta(y)\vec{a_z}$. The form of $I(z')$ has been studied intensively and widely. Several analytical and numerical expressions for the current distribution on an antenna have been obtained from the integral equation formulation [7–10].

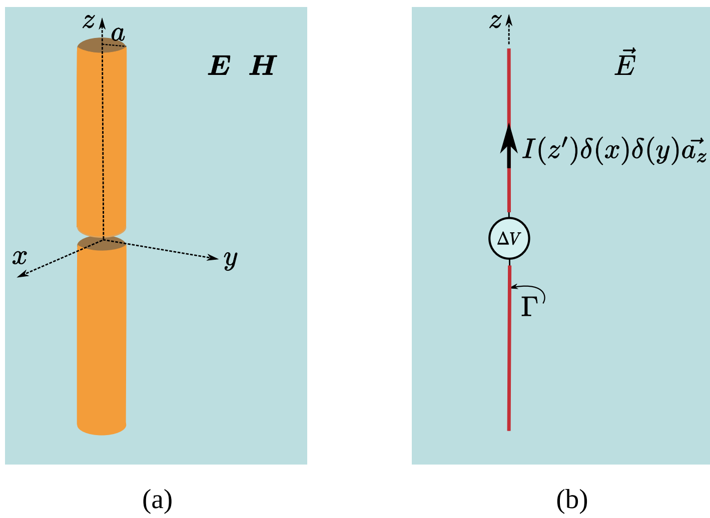

Figure 1. A generic linear wire antenna. (a) Cylindrical antenna of radio $a \ll \lambda$. (b) When $a \ll \lambda$ the cylindrical antenna becomes a linear wire antenna.

To calculate the field radiated by the antena, the conductive wires are suppressed and the electrical current is let alone to generate an electrical field $\vec{E}$ given by:

$$\vec{E} = -\jmath\omega \left[\frac{1}{\kappa^2} \nabla(\nabla \cdot \vec{A}) + \vec{A} \right] \tag{1}$$

where ω is the angular frequency, κ is the wave number and $\vec{A}$ is the Magnetic Vector Potential.

The Magnetic Vector Potential A is related to the current $I(z')$

by a convolutional integral

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_\Gamma I(z')\vec{a_z} \frac{\mathrm{e}^{-\jmath\kappa|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \, \mathrm{d}z' \tag{2}$$

where Γ is the locus of the electrical current.

The Magnetic Vector Potential is the solution of the inhomogeneous Helmholtz equation

$$\nabla^2\vec{A} + \kappa^2\vec{A} = -\mu_0 I(z')\delta(x)\delta(y)\vec{a}_z \tag{3}$$

where $I(z')$ radiates alone in free space as if the wire does not exist.

As seen in Eq. (1) the electrical field has two components: the quasi-static one, given by the expression: $-\frac{\jmath\omega}{\kappa^2}\nabla\nabla\cdot\vec{A}$, and the dynamic component, given by $-\jmath\omega\vec{A}$. The former varies with $\frac{1}{r^2}$, and the latter with $\frac{1}{r}$. In the far zone, the dynamic component predominates over the quasi-static one, and the electrical field can be approximated in the following way [3, 11]:

$$E_r(\vec{r}) \approx 0 \tag{4a}$$

$$E_\theta(\vec{r}) \approx -\jmath\omega A_\theta(\vec{r}) \tag{4b}$$

$$E_\varphi(\vec{r}) \approx -\jmath\omega A_\varphi(\vec{r}) \tag{4c}$$

where $\vec{r}$ must be in the far zone and $A_{\theta,\varphi} = \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{-\jmath\kappa r}}{r}N_{\theta,\varphi}$, where $\vec{N}(\vec{r})$ is the Radiation Vector. The Radiation Vector is defined as:

$$\vec{N} = \int_\Gamma I(z')\vec{a_z}\mathrm{e}^{\jmath\kappa z'\cos\theta} \, \mathrm{d}z' \tag{5}$$

By postulating a known function for the electrical current $I(z')$, or by measuring it, the Radiation Vector is calculated using Eq. (5) and then the Electrical Field $\vec{E}$ is obtained through Eqs. (4). The Magnetic Field is determined as $\vec{H} = \vec{a_r} \times \vec{E}/\eta$.

3. RADIATION FIELDS OF A LINEAR WIRE ANTENNA: A FORMAL APPROACH

Given a linear wire antenna, as shown in Fig. 2, fed across a small gap $\Delta\zeta$ with an impressed voltage ΔV, the excited field is the solution of a boundary value problem: an impressed source radiating in the presence of wires.

The resulting total electrical field $\vec{E}$ can be separated in two components: the impressed field $\vec{E}^i$ and the scattered field $\vec{E}^s$.

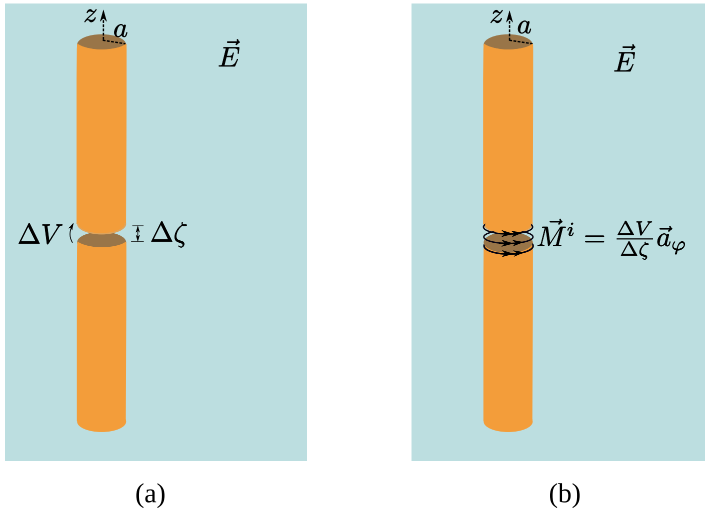

Figure 2. A generic linear wire antenna fed across a small gap $\Delta\zeta$ with an impressed voltage ΔV. (a) Linear wire antenna $-a \ll \lambda-$. (b) Field equivalent source for the delta-gap voltage.

Assuming the wires as perfect electric conductor bodies, the impressed and scattered fields can be calculated by splitting the original problem in two sub-problems as shown in Fig. 3.

The impressed and scattered fields can be viewed like the particular and homogenous solutions, respectively, of the original boundary problem.

In sub-problem A (see Fig. 3(a)) the wires are suppressed and the impressed sources are retained so that they radiate in free space. This problem is governed by the equation:

$$\nabla^2 \vec{F}^i + \kappa^2 \vec{F}^i = -\varepsilon_0 \vec{M}^i \tag{6}$$

where $\vec{F}^i$ is the Electric Vector Potential and $\vec{M}^i = \frac{\Delta V}{\Delta\zeta}\vec{a}_\varphi$, for $-\Delta\zeta/2 \leq z' \leq \Delta\zeta/2 \leq$ and $\rho = a$ is an impressed magnetic current density equivalent to the delta-gap impressed voltage (see Fig. 2(b)) [1]. The solution of Eq. (6) is:

$$\vec{F}^i = \frac{\varepsilon_0}{4\pi} \int_{S_{gap}} \frac{\Delta V}{\Delta\zeta} \vec{a}_\varphi \; \frac{\mathrm{e}^{-\jmath\kappa|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \mathrm{d}s' \tag{7}$$

The impressed electric field is thus obtained as $\vec{E}^i = -\frac{1}{\varepsilon_0}\nabla \times \vec{F}^i$.

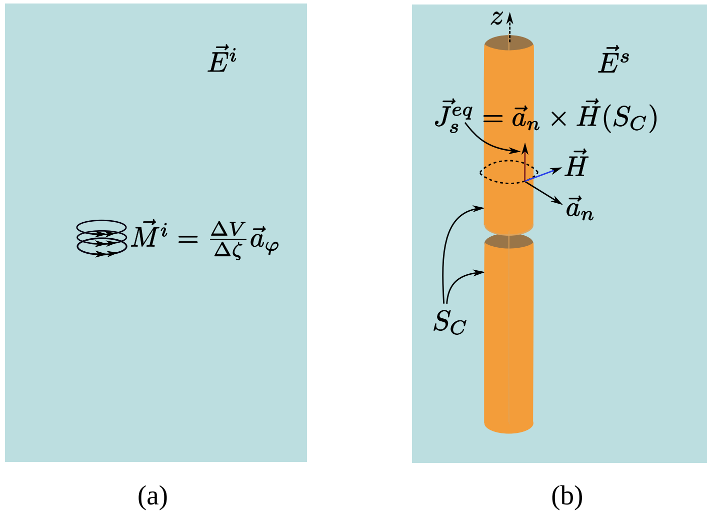

Figure 3. Subdivision of the original problem in two sub-problems because of linearity. (a) Sub-problem A: impressed source $\vec{M}^i$ radiates in an unbounded free space generating the electrical field $\vec{E}^i$. (b) Sub-problem B: the current on wire surface gives up a boundary problem, the solution of which is the scattered electrical field $\vec{E}^s$.

In sub-problem B (see Fig. 3(b)) the wires and their induced surface current are retained and the impressed sources are suppressed. A homogeneous boundary problem is obtained. While the scattered magnetic field $\vec{H}^s$ is the solution of the homogeneous equation

$$(\nabla^2 + \kappa^2)\vec{H}^s = 0 \tag{8}$$

with a boundary condition given by $\vec{J}_s = \vec{a}_n \times \vec{H}(S_C)$, where $\vec{H}(S_C)$ is the total Magnetic Field over the wire surface; the scattered electric field is the solution of the homogenous equation

$$(\nabla^2 + \kappa^2)\vec{E}^s = 0 \tag{9}$$

with a boundary condition given by $\vec{a}_n \times \vec{E}(S_C) = 0$, where $\vec{E}(S_C)$ is the total Electric Field over the wire surface.

It is known that only one of these two equations (Eqs. (8) and (9)) need to be solved. A simple analytical solution to the problem given by Eq. (8) (or Eq. (9)) doesn't exist [12], but assuming that $I(z')$ is known and wires are PEC bodies, the classical approach for finding the radiation fields can be still used. To do that, some considerations

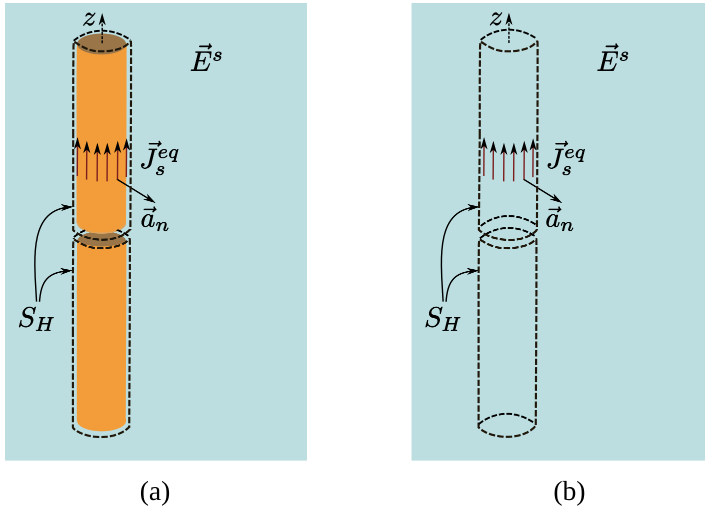

Figure 4. Equivalence Principle formulation. (a) Definition of a Huygens surface around the wires. (b) Suppression of wires.

which are described next must be made. First of all, the Uniqueness theorem must be invoked in the sense that if $\vec{J}_s$ is known, it implies that the tangential component of the magnetic field is also known, and assuming that fields vanish at infinity, the solution to the problem is unique.

Then, we can use the Equivalence Principle (see Fig. 4) to formulate an equivalent problem [13]. To do that, a Huygens surface S_H around the cylindrical wires, infinitesimally separated from them, is constructed as shown in Fig. 4(a). The surface current $\vec{J}_s = \vec{a}_n \times \vec{H}(S_C)$ on S_H is retained acting like an equivalent source. Wires are then suppressed (Fig. 4(b)) and, finally, a zero field inside S_H is set up. Such an equivalent current $\vec{J}_s^{eq} = \vec{a}_n \times \vec{H}(S_C)$ radiates in free space in the absence of wires, supporting the scattered electrical field $\vec{E}^s$.

The scattered electric field is given by

$$\vec{E}^s = -\jmath\omega \left[\frac{1}{\kappa^2} \nabla(\nabla \cdot \vec{A}^s) + \vec{A}^s \right] \tag{10}$$

where $\vec{A}^s$ is the (scattered) Magnetic Vector Potential

$$\vec{A}^s(\vec{r}) = \frac{\mu_0}{4\pi} \int_\Gamma [J_s^{eq}(z') 2\pi a] \vec{a}_z \frac{\mathrm{e}^{-\jmath\kappa|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \, \mathrm{d}z' \tag{11}$$

where $\vec{r}$ must be taken outside of wires and $I(z') = J_s^{eq}(z')2\pi a$ is the equivalent current acting like a filamentary one.

At this point, the procedure described in Section 2 can be used in determining the radiation component of the scattered electrical field.

It is well known that the impressed voltage ΔV across the delta-gap generates an impressed field which is mostly an induction field, with an intangible (practically) radiation component. The converse occurs to the scattered field, which is mostly dynamic. Due to this fact, in the far zone of the wire antenna, the radiation field is approximatively equal to the scattered field: $\vec{E}(\vec{r}) \approx \vec{E}^s(\vec{r})$, with $\vec{r}$ in the far zone.

4. CONCLUSION

In almost all antenna text books the current on wires is treated as impressed source. Even though this leads to a correct result, it is not rigorous. In this paper some considerations for the calculation of radiation fields from a wire linear antenna were suggested. By invoking the Uniqueness and Equivalence theorems the completeness of the classical approach was achieved.

REFERENCES

1. Balanis, C. A., *Antenna Theory Analysis and Design*, John Wiley & Sons, 2005.
2. Jackson, J. D., "How an antenna launches its input power into radiation: The pattern of the poynting vector at and near an antenna," *Am. J. Phys.*, Vol. 74, No. 4, 280–288, April 2006.
3. Schelkunoff, S. A., "A general radiation formula," *Proceedings of the I.R.E.*, 660–666, October 1939.
4. King, R. W., "The linear antenna — Eighty years of progress," *Proceedings of the IEEE*, Vol. 55, No. 1, 2–16, January 1967.
5. Collin, R. E., *Antennas and Radiowave Propagation*, McGraw-Hill Book Company, 1985.
6. Weeks, W. L., *Antenna Engineering*, McGraw-Hill Book Company, 1968.
7. Hallén, E., "Theoretical investigations into the transmitting and receiving qualities of antennae," *N. A. R. Soc. Sci. Upsal., Series IV*, Vol. 11, 1–44, 1938.
8. Mei, K. K., "On the integral equations of thin wire antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 13, No. 3, 374–378, May 1965.

9. Thiele, G. A., "Calculation of the current distribution on a thin linear antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 14, No. 5, 798–799, September 1966.
10. Harrington, R. F., *Field Computation by Moment Methods*, IEEE Press Series on Electromagnetic Wave Theory, Wiley-IEEE Press, 1991.
11. Balanis, C. A., *Advanced Engineering Electromagnetics*, John Wiley & Sons, 1989.
12. Stratton, J. A., *Electromagnetic Theory*, McGraw-Hill Book Company, 1941.
13. Harrington, R. F., *Time-Harmonic Electromagnetic Fields*, John Wiley & Sons, 2001.

Erratum to A FORMAL APPROACH FOR CALCULATING THE RADIATION FIELDS OF A LINEAR WIRE ANTENNA

by A. J. Zozaya, in *Progress In Electromagnetics Research M*, Vol. 6, pp. 1–8, 2009

On page 6 the Equivalence Principle was incorrectly formulated.

"Wires are then suppressed (Fig. 4(b)) and, finally, a zero field inside S_H is set up. Such an equivalent current $\vec{J}_s^{eq} = \vec{a}_n \times \vec{H}(S_C)$ radiates in free space in the absence of wires, supporting the scattered electrical field $\vec{E}^s$." should be changed to:

Wires are then suppressed (Fig. 4(b)) and, finally, a field $-\boldsymbol{E}^{\boldsymbol{i}}$ (and $-\boldsymbol{H}^{\boldsymbol{i}}$) inside S_H is set up. Such an equivalent current $\vec{J}_s^{eq} = \vec{a}_n \times \vec{H}(S_C)$ radiates in free space, supporting the scattered electrical field $\vec{E}^s$ outside the wires.