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ANALYSIS OF SHADOWING PROCESSING TECHNIQUE BASED ON MODELING USING NURBS

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Abstract—The shadowing relationship between facet elements can be determined rapidly through analytical expressions. On the basis of modeling using curved surfaces, an effective shadowing processing algorithm is proposed which is in combination with that used in the shadowing judgement of facet elements. Firstly several sampling points are taken on the ergodic curved surface element to construct a group of facet elements, which can replace the curved surface element. Then the shadowing processing between the stationary phase point and the ergodic curved surface element is converted to that between the stationary phase point and several facet elements, thus avoiding utilizing optimization method and can increase the computation speed. Similarly, the shadowing processing between the stationary phase segment and the ergodic curved surface element is converted to that between the stationary phase segment and several facet elements. And the trimming algorithm is used to accurately find the visible part of the stationary phase segment, which gets rid of the rough shadowing processing technique that determines the visibility of the whole stationary phase segment through the visibility of the center of the stationary phase segment. Therefore, the computation precision is greatly improved. When there exists a huge number of curved surfaces, maximum-minimum preprocessing is utilized to increase the computation speed. Examples show that this novel algorithm is superior to the traditional one in both computation speed and precision.

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1. INTRODUCTION

With the development of CAGD, at present various targets, for example aircrafts and ships, can be represented by the combination of a group of curved surfaces, such as NURBS surfaces [1], Bi-cubic Spline Surfaces and Bi-cubic B-Spline Surfaces. The NURBS format has found wide applications in the field of CAGD due to its tremendous advantages in geometric representation. Modeling using curved surfaces make it possible to represent targets more precisely with relatively fewer curved surface elements, which not only decreases the facet noise, but also saves the memory [1–5].

The NURBS patches are classified into four types, which are polygonal plane patches (such as the plane), plane patches with curved boundaries (such as the disk), singly curved patches and doubly curved patches, respectively. Different calculation methods are used in electromagnetic computation according to different patch types. Gordon's Method [6] is utilized for plane patches, while stationary phase method is utilized for curved patches [7]. However, shadowing processing is necessary no matter what method is used for computation. There are two types of shadowing processing. One is to judge whether a patch is visible to the incident wave, which means self-occlusion. The other is to judge whether a patch is visible to other patches, which means mutual occlusion. As the judgment of self-occlusion is simple, this paper will focus mainly on the research of mutual occlusion. There are three aspects to deal with in terms of the shadowing processing:

1) Polygonal plane patches: The PO integral can be calculated analytically using Gordon's Method on the polygonal plane patches. It is necessary to use trimming algorithm in the shadowing processing. The trimming algorithm is discussed in literature [8], thus it will not be described in detail in this paper.

2) Critical points: The RCS of the doubly curved surface is calculated using stationary phase method. The search of critical points on the curved surface plays an important role because the contributions to the PO integral come only from the vicinity of the critical points. Therefore, the shadowing processing of doubly curved surfaces can be done by determining the visibility of the critical points.

3) Stationary phase segment: The calculation of the RCS of the singly curved surface is similar to that of the doubly curved surface except that for the singly curved surface it is the stationary phase segment that functions. Thus the shadowing processing of singly curved surfaces can be done by determining the visibility of the stationary phase segment. However, it is possible that only a part of

the stationary phase segment is shadowed, which deserves to be noted.

Based on modeling using curved surfaces, the shadowing processing technique used in RANURS software by M. Domingo [9] is as follows:

1) The visible part of plane patches is obtained using trimming operation in computer graphics;

2) The shadowing processing of doubly curved surfaces is implemented in terms of the visibility of the critical points. Here is the concrete operation. Firstly find a projective plane that is perpendicular to the direction of the incident wave. Then project the sampling points and critical points on the curved surfaces to the projective plane. Finally determine whether the critical points locate in the area of the polygon that is formed by the sampling points. Meanwhile it is necessary to take the depths of the critical points and the curved surface into consideration;

3) The shadowing processing of singly curved surfaces is implemented in terms of the visibility of the center of the stationary phase segment. The concrete operation is similar to that described in 2), while the only difference is to replace the critical points by the center of the stationary phase segment.

Results show that the shadowing processing technique proposed by M. Domingo is very high in computation speed but bad in precision, especially for singly curved surfaces, which will surely bring about a large error in the calculation of RCS.

Sandy Sefi [10] in Sweden proposed a shadowing processing algorithm based on hybrid modeling, but he did not give the concrete realization procedure. This proposal is implemented in literature [11], in which the shadowing processing is utilized to determine the visibility of the integral element in Gauss integral and Ludwig integral. Its main idea is to find whether the radial from the center of the integral element intersects with the ergodic curved surface by using optimization algorithm. If there exists an intersection, the integral element is shadowed, otherwise it is visible. This idea can be readily extended to the shadowing processing of critical points and stationary phase segments in the stationary phase method [12]. However, it takes a long time to get the results as the optimization algorithm is used.

The main idea of the shadowing processing technique proposed in this paper is to take a few sampling points on the ergodic curved surface element and construct several sampling planes with those sampling points. Then the curved surface element is replaced by the combination of these sampling planes. Different shadowing processing algorithms are introduced according to different types of curved surfaces and calculation methods of RCS. For example, the shadowing processing of critical points on doubly curved surfaces are carried out in combination with the shadowing processing of facet elements; while the shadowing processing of stationary phase segments on singly curved surfaces are implemented with the help of trimming operation between the segment and the facet element. Besides, the maximum-minimum preprocessing method is utilized to increase the efficiency when there are a great many surface elements.

The framework of this paper is as follows. The shadowing processing algorithm of facet elements and doubly curved surfaces are introduced in Section 2 and Section 3, respectively. The possible error brought about in the conventional shadowing processing is analyzed in Section 4, and a novel shadowing processing algorithm of singly curved surfaces is also discussed. The maximum-minimum judging method is illustrated in Section 5. Several examples are given in Section 6 to validate the effectiveness of this novel algorithm.

2. SHADOWING PROCESSING ALGORITHM OF FACET ELEMENTS

The shadowing processing algorithm based on facet elements is simple. And the main procedure is as follows:

Step 1: Find whether an illuminated surface element (denoted as element 1) is shadowed by another surface element (denoted as element 2).

As shown in Fig. 1, let o be the origin of the coordinates. M is the geometrical center of element 1; $\bar{r}_M$ is the position vector of M; the incident wave vector $\hat{k}_i$ that passes M intersects with the plane containing element 2 at point P, $\bar{r}_p$ is the position vector of P; $\bar{r}_V$ is the position vector of any vertex of element 2; $\hat{n}_2$ is the outward unit normal vector. A parameter α is brought in, let

$$\bar{r}_P - \bar{r}_M = \alpha\hat{k}_i \tag{1}$$

where α is obtained as follows:

According to Fig. 1, we have

$$(\bar{r}_V - \bar{r}_P) \cdot \hat{n}_2 = 0 \tag{2}$$

Substituting (1) into (2) and simplifying yields

$$\alpha = \frac{\bar{r}_V \cdot \hat{n}_2 - \bar{r}_M \cdot \hat{n}_2}{\hat{k}_i \cdot \hat{n}_2} \tag{3}$$

If $\alpha > 0$, element 1 is not shadowed by element 2; if $\alpha < 0$, continue to do Step 2.

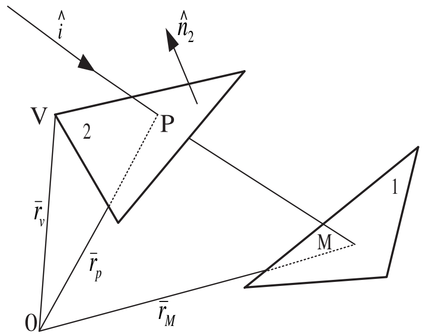


Figure 1. Shadowing relationship.

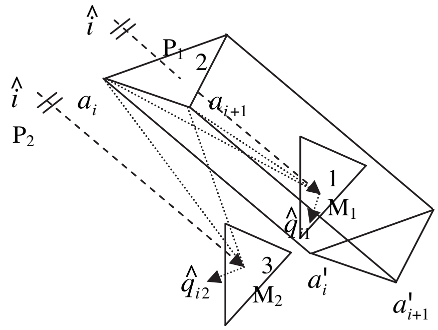


Figure 2. Step 2 of shadowing processing.

Step 2: Determine whether P is in the area of element 2.

Suppose that element 2 is a polygon surface, which has N edges. $\vec{r}_i (i = 1, 2, \ldots, N)$ is the position vector of each vertex of element 2. According to Fig. 2, let

$$\beta_i = \left[\left(\vec{r}_M - \vec{r}_{i+1}\right) \times \left(\vec{r}_M - \vec{r}_i\right)\right] \cdot \hat{i}, \quad i = 1, 2, \ldots, N \tag{4}$$

where $\vec{r}_{N+1} = \vec{r}_1$.

If $\beta_i > 0$ for each i from 1 to N, P is in the area of element 2, thus element 1 is shadowed by element 2; otherwise, P is not in the area of element 2 and element 1 is visible.

3. SHADOWING PROCESSING ALGORITHM OF DOUBLY CURVED SURFACES

The shadowing processing algorithm of doubly curved surfaces is simple. It determines the visibility of the doubly curved surfaces in terms of the visibility of the critical points (stationary phase points) on the surfaces.

There are three types of surfaces that can be used to represent targets. They are planes, singly curved surfaces and doubly curved surfaces. Thus each of these surfaces may form a shadowing relationship with the doubly curved surface to be dealt with. For planes the shadowing processing of critical points can be implemented using analytical expressions that are discussed in Section 2, which is simple and also fast in speed. However, if the shadowing surface is a curved surface, the optimization algorithm is utilized to find whether the radial passing the critical point goes through the shadowing surface. The optimization algorithm is more complicated and consumes much more time compared with the analytical algorithm introduced in Section 2.

Therefore, the main idea of our shadowing processing algorithm is to take several sampling points on the curved surface element and construct a group of facet elements with those sampling points to replace the original curved surface element. Then determine the shadowing relationship between the critical points and the group of facet elements. The shadowing surfaces are classified into two types in the shadowing processing. These two types are planes and curved surfaces. The concrete shadowing processing procedure is as follows:

1) Take sampling points on the surface according to the type of the surface. If the shadowing surface is a plane, the four vertexes of the plane are chosen as four sampling points. Otherwise, sample nine points on the curved surface, the locations of which are shown in Fig. 4. Four quadrilateral facets connected with each other are formed by these sampling points.

2) Regard the opposition to the direction of the incident wave as z-direction. The critical points and sampling points on each curved surface element are given coordinate transformation and then projected into XOY plane.

3) Select curved surface elements which may shadow the critical points according to the maximum-minimum judging method. The details are described in Section 4.

4) Determine whether the critical points are shadowed using the shadowing processing algorithm introduced in Section 2. The critical point is thought to be invisible when it is shadowed by at least one of the four quadrilateral facets.

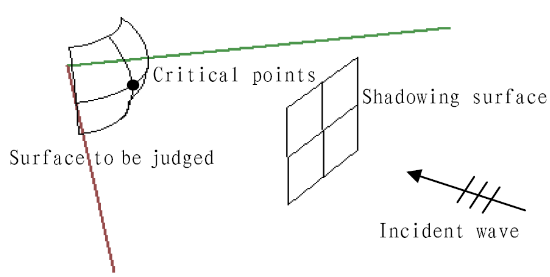


Figure 3. Shadowing of doubly curved surfaces.

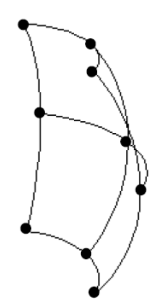


Figure 4. The locations of sampling points on the surface.

4. SHADOWING PROCESSING ALGORITHM OF SINGLY CURVED SURFACES

The shadowing processing of singly curved surfaces is more complicated compared to that of doubly curved surfaces, because stationary phase segments rather than critical points are dealt with for singly curved surfaces. The problem is described in the following figure.

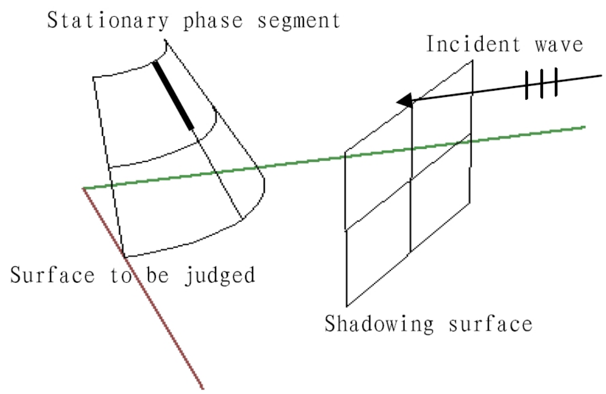


Figure 5. Shadowing relationship of singly curved surfaces.

One solution to this problem is to sample one point at the center of the stationary phase segment and utilize the shadowing processing algorithm of the doubly curved surfaces. This method is used in the software RANURBS by M. Domingo [9]. However, it is found that this method is rough in precision in that the computation results are not accurate in some angle range. Two examples are given below to illustrate this problem. The algorithm proposed in this paper is denoted as algorithm 1, while the algorithm in which the shadowing relationship of the whole stationary phase segment is determined in terms of the visibility of its center point is denoted as algorithm 2.

As shown in Fig. 12 and Fig. 14, the precision of algorithm 2 is low, especially in some angle range, while the novel algorithm, namely algorithm 1, can accurately determine the visible part of the stationary phase segment. It is also seen that the algorithm l is much faster than shadowing processing technique using optimization algorithm.

The concrete procedure of the novel algorithm is as follows:

1) Take sampling points on the shadowing surface, and this is the same as that described in procedure 1) in Section 2.

2) Regard the opposition to the incident wave direction as z-direction. The two ends of the stationary phase segment and the

sampling points on each shadowing surface are given coordinate transformation and then projected into XOY plane.

3) Select surface elements which may shadow the stationary phase segment according to maximum-minimum judging method. The details are introduced in Section 4.

4) Carry out trimming operation between the stationary phase segment and quadrilaterals projected into XOY plane. Readers can refer to literature [13] about the details.

5. MAXIMUM-MINIMUM JUDGING METHOD

The number of surfaces of a complex target is large, even if the target is based on modeling using curved surfaces. So if the trimming operation or shadowing processing is implemented for each surface, the computation time will be unacceptable. Thus the following preprocessing, namely maximum-minimum judging method, is carried out before other operations are done. In this way most of the surface elements are excluded, which means operations such as trimming are implemented only on the surfaces that may cause shadowing effect.

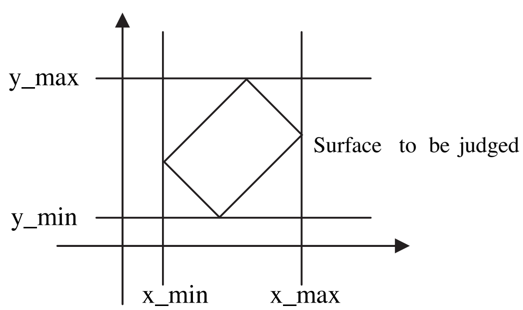


Figure 6. Maximum-minimum judging method.

The concrete procedure is simple. If an ergodic surface element satisfies the following expression, it is excluded, and trimming operation will not be carried out for this surface. Suppose

$a_{ix}, i = 1, \ldots, 9$, is the x coordinate of sampling point i on the ergodic surface element

$a_{iy}, i = 1, \ldots, 9$, is the y coordinate of sampling point i on the ergodic surface element

$y\_\max$ is the maximum value in y direction on the surface to be dealt with

$y\_\min$ is the minimum value in y direction on the surface to be dealt with

$x\_\max$ is the maximum value in x direction on the surface to be dealt with

$x\_\min$ is the minimum value in x direction on the surface to be dealt with

Thus the judging procedure is as follows:

if ($a_{ix} \leq x\_\min$ or $a_{ix} \geq x\_\max$ or $a_{iy} \leq y\_\min$ or $a_{iy} \geq y\_\max$),
exclude this surface element, and go on dealing with the next one,
else,
carry out trimming operation or shadowing processing,
end.

For shadowing processing of critical points, it is necessary to substitute the corresponding coordinates of critical points into $y\_\max$, $y\_\min$, $x\_\max$ and $x\_\min$; while for shadowing processing of stationary phase segment, the maximum and minimum values in both x and y direction of the stationary phase segment are substituted correspondingly into $y\_\max$, $y\_\min$, $x\_\max$ and $x\_\min$.

The visible surfaces determined by the shadowing processing algorithm described above are used in the calculation of the scattering field. Several examples are given to validate the effectiveness of this novel shadowing processing algorithm.

6. NUMERICAL RESULT

6.1. Example 1: The Combination of a Sphere and a Plane

The model is shown in Fig. 7. The center of the sphere locates at the origin. The radius of the sphere is 1 m, and its flare angles in the azimuth plane and the elevation plane are both 90 degrees. The size of the plane is $1.5\,\text{m} \times 1.5\,\text{m}$. The distance between the sphere and the plane is 1 m. The frequency of the incident wave is 3 GHz, and the scanning range is $\phi = 45°$, $0° \leq \theta \leq 180°$. The calculation results of monostatic RCS is presented in Fig. 8, where the line is the result obtained from the stationary phase method while the dotted line is the result obtained from software FEKO (PO method). It can be observed that the two results are compatible.

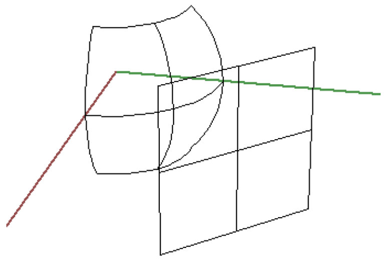


Figure 7. The model of Example 1.

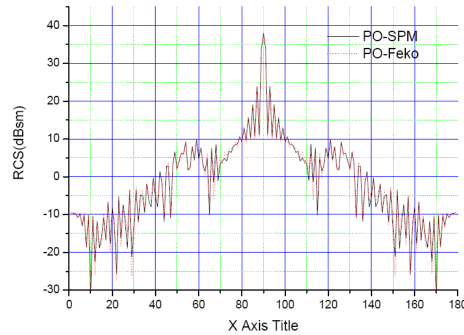


Figure 8. The RCS of Example 1.

6.2. Example 2: The Combination of Two Spheres

The model is achieved by duplicating and moving the part of sphere presented in Example 1. The distance between the two spheres is 1 m. The frequency of the incident wave is 3 GHz, and the scanning range is $\phi = 45°$, $0° \leq \theta \leq 180°$. The calculation results of monostatic RCS is shown in Fig. 10, where the line is the result obtained from the stationary phase method while the dotted line is the result obtained from software FEKO (PO method). It can be seen that the two results are almost identical.

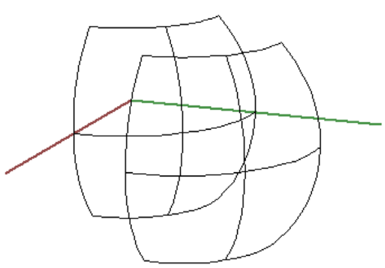


Figure 9. The model of Example 2.

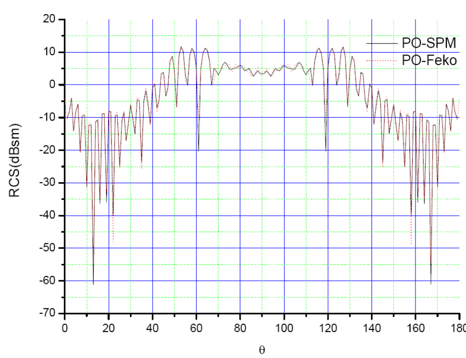
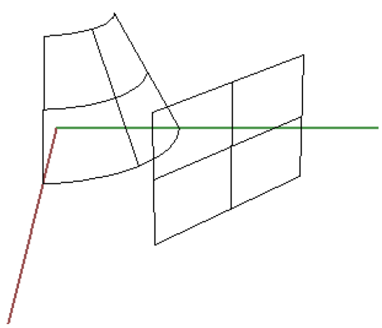


Figure 10. The RCS of Example 2.

6.3. Example 3: The Combination of a Cone and a Plane

The model is presented in Fig. 11. The center of the bottom cross section of the cone locates at the origin, and the central angle is $\varphi = 90°$. The height of the cone is 1 m. The radius of the top cross section and the bottom cross section of the cone are 0.5 m and 1 m, respectively. The size of the plane is $1.5\,\text{m} \times 1\,\text{m}$. The distance between the plane and the cone is 1 m. The frequency of the incident wave is 3.0 GHz, and the scanning range is $\phi = 45°$, $0° \leq \theta \leq 150°$. The calculation results of monostatic RCS are shown in Fig. 12, where the line is the result obtained from the stationary phase method utilizing shadowing processing described in algorithm 1, the dotted line is the result obtained from software FEKO (PO method), and the dashed line is the result obtained from the stationary phase method utilizing shadowing processing described in algorithm 2. It can be observed that the stationary phase method which uses algorithm 2 to accomplish shadowing processing is low in precision. However, the precision of the stationary phase method using algorithm 1 in shadowing processing is improved obviously.



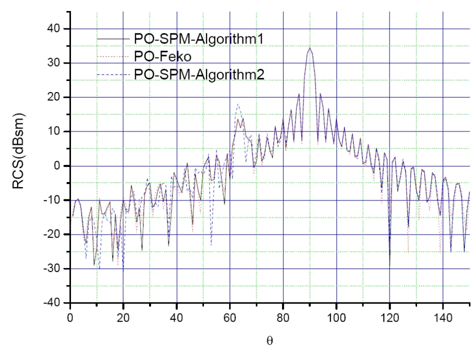


Figure 11. The model of Example 3.

Figure 12. The RCS of Example 3.

6.4. Example 4: The Combination of a Sphere and a Cone

The model is shown in Fig. 13, in which the cone is the same as that in Example 3. The radius of the sphere is 1 m, and its flare angles in the azimuth plane and the elevation plane are both 90 degrees. The distance between the cone and the sphere is 1 m. The frequency of the incident wave is 3.0 GHz, and the scanning range is $\phi = 45°$, $0° \leq \theta \leq 150°$. The calculation results of monostatic RCS are presented in Fig. 14, where the line is the result obtained from the stationary phase method utilizing shadowing processing described in algorithm 1, the dotted line is the result obtained from software FEKO (PO method), and the dashed line is the result obtained from the stationary phase method utilizing shadowing processing described in algorithm 2. It can be seen that the stationary phase method which uses algorithm 2 to accomplish shadowing processing is low in precision. However, the precision of the stationary phase method using algorithm 1 in shadowing processing is greatly improved.

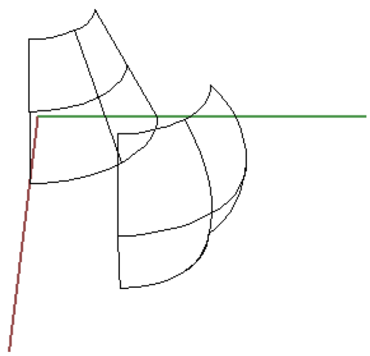


Figure 13. The model of Example 4.

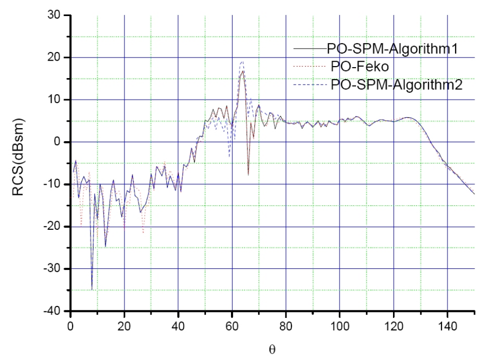


Figure 14. The RCS of Example 4.

6.5. Example 5: The Comparison of Computation Time

The computation time of two methods are given. The shadowing processing technique used in each method is described as follows:

Method 1: The shadowing relationship of doubly curved surfaces is determined in terms of the visibility of the critical points. And the optimization algorithm is used to judge whether the radial from the critical point intersects with the ergodic surface. The visible part of the stationary phase segment on the singly curved surface is found out. For singly curved surfaces, the optimization algorithm is called by the recursive algorithm for many times to search for the visible part of the stationary phase segment on the singly curved surface.

Method 2: The method which is proposed in this paper.

It should be noted that the computation time includes the time consumed both in the search of stationary phase points and in the shadowing processing.

Take the above Examples 1 to 4 as examples, the computation time is given as follows:

Table 1. Computation time of different models using Method 1 and Method 2.

	Example 1	Example 2	Example 3	Example 4
Method 1	3.2 s	7.6 s	5.4 s	23.7 s
Method 2	0.5 s	0.8 s	0.6 s	0.7 s

7. CONCLUSION

A novel shadowing processing technique is proposed in this paper, in which the curved surface is replaced by a group of planes and the trimming operation is utilized. This novel technique not only improves the computation precision compared to the conventional one put forward by M. Domingo, but also increases the computation speed compared with the shadowing processing technique using optimization algorithms. Therefore, this new technique is of great value in the fast calculation of RCS of electrically large targets based on modeling using NURBS.

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