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ZERO-AXIAL-PARAMETER (ZAP) MEDIUM SHEET

I. V. Lindell and A. H. Sihvola

Electromagnetics Group
Department of Radio Science and Engineering
Helsinki University of Technology
Box 3000, Espoo 02015 TKK, Finland

Abstract—Plane-wave reflection from and transmission through a slab of uniaxial anisotropic medium is studied and the concept of ZAP (Zero-Axial-Parameter) medium sheet is defined as the limiting case when the axial parameters and the thickness of the slab vanish simultaneously. It is shown that the ZAP sheet may act as a spatial filter for the incident waves with transmission in a narrow cone around normal direction. Such a sheet may find application in narrowing the radiation beam and reducing sidelobes of an antenna or as a computer privacy filter in optical frequencies.

1. INTRODUCTION

It has been recently found [1, 2] that boundary conditions of the simple form

$$\mathbf{n}\cdot\mathbf{D} = 0, \qquad \mathbf{n}\cdot\mathbf{B} = 0, \tag{1}$$

labeled for brevity as conditions of the DB boundary, show quite interesting properties to electromagnetic fields. In fact, it was shown that the planar DB boundary $z = 0$ with normal unit vector $\mathbf{n} = \mathbf{u}_z$, in terms of which any given electromagnetic field can be split in transverse electric (TE) and transverse magnetic (TM) components, reacts differently to the two components so that the TE component sees the boundary as perfect electric conductor (PEC) and the TM component sees the boundary as perfect magnetic conductor (PMC). For fields in an isotropic medium the DB-boundary conditions (1) can be alternatively expressed as

$$\mathbf{n}\cdot\mathbf{E} = 0, \qquad \mathbf{n}\cdot\mathbf{H} = 0. \tag{2}$$

Corresponding author: I. V. Lindell (ismo.lindell@tkk.fi).

Further, it was shown that the planar DB boundary can be realized by the interface of a uniaxially anisotropic medium defined by the permittivity and permeability dyadics of the form

$$\overline{\overline{\epsilon}} = \epsilon_t \overline{\overline{\mathsf{I}}}_t + \epsilon_z \mathbf{u}_z \mathbf{u}_z = \epsilon_t (\overline{\overline{\mathsf{I}}}_t + e \mathbf{u}_z \mathbf{u}_z), \tag{3}$$

$$\overline{\overline{\mu}} = \mu_t \overline{\overline{\mathsf{I}}}_t + \mu_z \mathbf{u}_z \mathbf{u}_z = \mu_t (\overline{\overline{\mathsf{I}}}_t + m \mathbf{u}_z \mathbf{u}_z), \tag{4}$$

with the transverse unit dyadic defined by

$$\overline{\overline{\mathsf{I}}}_t = \mathbf{u}_x \mathbf{u}_x + \mathbf{u}_y \mathbf{u}_y. \tag{5}$$

Here the relative axial parameters are defined by

$$m = \mu_z / \mu_t, \qquad e = \epsilon_z / \epsilon_t. \tag{6}$$

The four medium parameters are here assumed to have real non-negative values and the parameters

$$k_t = \omega \sqrt{\mu_t \epsilon_t}, \qquad \eta_t = \sqrt{\mu_t / \epsilon_t} \tag{7}$$

are assumed real and non-negative as well. Further, the medium is assumed lossless and its frequency dispersion is not taken into account as the first approximation.

In the limiting case of $\epsilon_z \to 0$ and $\mu_z \to 0$, or $e \to 0, m \to 0$, the medium can be called zero axial parameter medium or ZAP medium for short. In such a medium the axial components of the $\mathbf{D}$ and $\mathbf{B}$ fields obviously vanish, $D_z \to 0$ and $B_z \to 0$, whence from continuity the conditions (1) will be satisfied at the interface. Media with zero-valued medium parameters have been studied recently [3–6] and their realization by mixing metamaterials of positive and negative parameter values have been suggested.

Because a plane wave incident normally to the DB boundary with TEM polarization satisfies the DB conditions identically, it forms a strange anomaly which can only understood by starting from an approximative representation of the boundary, for example in terms of an interface of the anisotropic medium with small but nonzero relative axial parameter values m, e. Instead of considering the half space of infinite extent, the slab of such a medium forms a more realistic problem to be studied. Letting the thickness of the slab become zero simulatneously with the axial parameters of the uniaxial slab the concept of ZAP sheet is obtained. To study the effect of the ZAP sheet to an incident plane wave forms the topic of the present paper.

2. PLANE WAVES IN UNIAXIAL MEDIUM

Let us briefly consider plane-wave propagation in the uniaxial anisotropic medium defined by the permittivity and permeability dyadics (3) and (4). It is well known [7, 8] that fields of a plane wave in any medium satisfy $\mathbf{E}\cdot\mathbf{B} = 0$ and $\mathbf{H}\cdot\mathbf{D} = 0$, whence in the uniaxial medium they satisfy

$$\epsilon_t \mathbf{E}\cdot\mathbf{B} - \mu_t \mathbf{H}\cdot\mathbf{D} = (\epsilon_t\mu_z - \mu_t\epsilon_z)E_z H_z = 0. \tag{8}$$

Thus, unless the parameters satisfy the special condition

$$\mu_t\epsilon_z - \epsilon_t\mu_z = 0, \quad \text{or} \quad m = e, \tag{9}$$

a plane wave in the uniaxially anisotropic medium must be either TE or TM polarized with respect to the axial direction $\mathbf{u}_z$. The TE and TM waves do not interact at an interface perpendicular to the axial direction and can be handled separately. If the special condition (9) is satisfied, the two wave-vector surfaces coincide and the medium can be reduced to an isotropic medium through an affine transformation (linear transformation of spatial coordinates) which transforms the single $\mathbf{k}$-vector surface to a sphere. In this case the polarization of a plane wave can be a combination of TE and TM components. Such a medium was called affine isotropic in [8].

Let us assume that the wave vector of a plane wave in the uniaxially anisotropic medium is of the form

$$\mathbf{k} = \mathbf{u}_z\beta + \mathbf{K}, \quad \mathbf{u}_z\cdot\mathbf{K} = 0, \tag{10}$$

where $\mathbf{K}$ is a real vector transverse to the axis, the dispersion equations for the TE and TM waves are, respectively [8],

$$\beta_{TE}^2 + \frac{1}{m}\mathbf{K}\cdot\mathbf{K} = k_t^2, \tag{11}$$

$$\beta_{TM}^2 + \frac{1}{e}\mathbf{K}\cdot\mathbf{K} = k_t^2. \tag{12}$$

The dispersion diagram for the $\mathbf{k}$ vector is a spheroid in both cases when m and e have finite positive values, as is seen from the form of (11) and (12). For the special case $m = e$ the two spheroids coincide. For $m \to 0$ and $e \to 0$ the medium approaches a ZAP medium in which case the two spheroids become thin needles. Real β is then possible only for small magnitude of $\mathbf{K}$ which corresponds to wave incident in almost axial direction. For $|\mathbf{K}| \gg k_t\sqrt{m}$ and $|\mathbf{K}| \gg k_t\sqrt{e}$ the axial propagation factors are imaginary and their magnitudes approach the values

$$|\beta_{TE}| \approx |\mathbf{K}|/\sqrt{m}, \qquad |\beta_{TM}| \approx |\mathbf{K}|/\sqrt{e}. \tag{13}$$

This means that the waves decay exponentially along the z coordinate for oblique incidence for small values of the parameters e and m.

3. SLAB OF UNIAXIAL MEDIUM

Let us now consider the problem of a slab $(0 > z > -d)$ of uniaxial medium between two isotropic half spaces 1 $(z > 0)$ and 2 $(z < -d)$ defined by the respective medium parameters μ_1, ϵ_1 and ϵ_2, μ_2, Figure 1. The medium dyadics of the slab are given by (3) and (4). Because the TE and TM components of the waves do not interact, they can be handled separately.

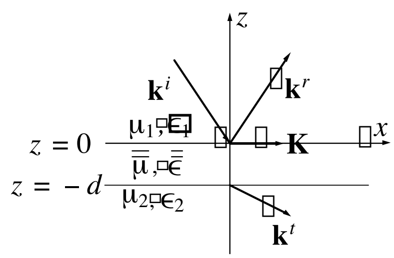

Figure 1. Incident, reflected and transmitted plane waves at a slab of uniaxial anisotropic medium.

3.1. TE Wave

Assuming a TE wave incident and reflected in medium 1,

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \qquad \mathbf{H}^i(\mathbf{r}) = \mathbf{H}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \tag{14}$$

$$\mathbf{E}^r(\mathbf{r}) = \mathbf{E}^r e^{-j\mathbf{k}^r\cdot\mathbf{r}}, \qquad \mathbf{H}^r(\mathbf{r}) = \mathbf{H}^r e^{-j\mathbf{k}^r\cdot\mathbf{r}}, \tag{15}$$

with the wave vectors

$$\mathbf{k}^i = -k_{1z}\mathbf{u}_z + \mathbf{K}, \qquad \mathbf{k}^r = k_{1z}\mathbf{u}_z + \mathbf{K}, \tag{16}$$

satisfying the condition

$$\mathbf{k}^i\cdot\mathbf{k}^i = \mathbf{k}^r\cdot\mathbf{k}^r = k_{1z}^2 + \mathbf{K}\cdot\mathbf{K} = k_1^2, \qquad k_1 = \omega\sqrt{\mu_1\epsilon_1}, \tag{17}$$

the waves in the slab and medium 2 also have TE polarization. The wave transmitted through the slab into medium 2 is of the form

$$\mathbf{E}^t(\mathbf{r}) = \mathbf{E}^t e^{-j\mathbf{k}^t\cdot\mathbf{r}}, \qquad \mathbf{H}^t(\mathbf{r}) = \mathbf{H}^t e^{-j\mathbf{k}^t\cdot\mathbf{r}}, \tag{18}$$

$$\mathbf{k}^t = -k_{2z}\mathbf{u}_z + \mathbf{K}, \tag{19}$$

$$\mathbf{k}^t \cdot \mathbf{k}^t = k_{2z}^2 + \mathbf{K}\cdot\mathbf{K} = k_2^2, \qquad k_2 = \omega\sqrt{\mu_2\epsilon_2}. \tag{20}$$

The wave components in the slab are

$$\mathbf{E}^+(\mathbf{r}) = \mathbf{E}^+ e^{-j\mathbf{k}^+\cdot\mathbf{r}}, \qquad \mathbf{H}^+(\mathbf{r}) = \mathbf{H}^+ e^{-j\mathbf{k}^+\cdot\mathbf{r}}, \tag{21}$$

$$\mathbf{E}^-(\mathbf{r}) = \mathbf{E}^- e^{-j\mathbf{k}^-\cdot\mathbf{r}}, \qquad \mathbf{H}^-(\mathbf{r}) = \mathbf{H}^- e^{-j\mathbf{k}^-\cdot\mathbf{r}}, \tag{22}$$

$$\mathbf{k}^+ = -\beta_{TE}\mathbf{u}_z + \mathbf{K}, \qquad \mathbf{k}^- = \beta_{TE}\mathbf{u}_z + \mathbf{K}. \tag{23}$$

The propagation factor β_{TE} can be obtained from the dispersion Equation (11) for any given $\mathbf{K}$ vector of the incident wave.

Let us assume that $\mathbf{K}$ is the real vector

$$\mathbf{K} = \mathbf{u}_x K, \tag{24}$$

whence the electric fields are polarized along the y coordinate:

$$\mathbf{E}^i = \mathbf{u}_y E_y^i, \qquad \mathbf{E}^r = \mathbf{u}_y E_y^r, \tag{25}$$

$$\mathbf{E}^\pm = \mathbf{u}_y E_y^\pm, \qquad \mathbf{E}^t = \mathbf{u}_y E_y^t. \tag{26}$$

From the Maxwell equations we can now write the x components of the magnetic fields as

$$H_x^i = \frac{k_{1z}}{k_1\eta_1}E_y^i, \qquad H_x^r = -\frac{k_{1z}}{k_1\eta_1}E_y^r, \tag{27}$$

$$H_x^+ = \frac{\beta_{TE}}{k_t\eta_t}E_y^+, \qquad H_x^- = -\frac{\beta_{TE}}{k_t\eta_t}E^-, \tag{28}$$

$$H_x^t = \frac{k_{2z}}{k_2\eta_2}E_y^t. \tag{29}$$

Since the total fields tangential to the interfaces are continuous, we obtain conditions for the wave amplitudes at $z = 0$,

$$E_y^i + E_y^r = E_y^+ + E_y^-, \tag{30}$$

$$\frac{k_{1z}}{k_1\eta_1}\left(E_y^i - E_y^r\right) = \frac{\beta_{TE}}{k_t\eta_t}\left(E_y^+ - E_y^-\right), \tag{31}$$

and at $z = -d$,

$$E_y^+ e^{-j\beta_{TE}d} + E_y^- e^{j\beta_{TE}d} = E_y^t e^{-jk_{2z}d}, \tag{32}$$

$$\frac{\beta_{TE}}{k_t\eta_t}\left(E_y^+ e^{-j\beta_{TE}d} - E_y^- e^{j\beta_{TE}d}\right) = \frac{k_{2z}}{k_2\eta_2} E_y^t e^{-jk_{2z}d}. \tag{33}$$

Expressing

$$E_y^- = R_2 E_y^+, \qquad E_y^r = R_{TE} E_y^i, \tag{34}$$

from (32), (33) we obtain

$$R_2 = \frac{\beta_{TE}k_2\eta_2 - k_{2z}k_t\eta_t}{\beta_{TE}k_2\eta_2 + k_{2z}k_t\eta_t} e^{-2j\beta_{TE}d}. \tag{35}$$

while (30) and (31) yield the relation

$$\frac{\beta_{TE}k_1\eta_1(1+R_{TE})}{k_{1z}k_t\eta_t(1-R_{TE})} = \frac{1+R_2}{1-R_2}. \tag{36}$$

Substituting (35), the reflection coefficient at the interface $z = 0$ can be solved as

$$R_{TE} = \frac{\begin{array}{c}\beta_{TE}k_t\eta_t\left(k_{1z}k_2\eta_2 - k_{2z}k_1\eta_1\right)\cos\beta_{TE}d \\ -\,j\left(\beta_{TE}^2 k_1k_2\eta_1\eta_2 - k_{1z}k_{2z}k_t^2\eta_t^2\right)\sin\beta_{TE}d\end{array}}{\begin{array}{c}\beta_{TE}k_t\eta_t\left(k_{1z}k_2\eta_2 + k_{2z}k_1\eta_1\right)\cos\beta_{TE}d \\ +\,j\left(\beta_{TE}^2 k_1k_2\eta_1\eta_2 + k_{1z}k_{2z}k_t^2\eta_t^2\right)\sin\beta_{TE}d\end{array}}. \tag{37}$$

As a simple check, for $d = 0$ this reduces to the familiar expression corresponding to the reflection from the interface of two isotropic half spaces.

The field transmitted through the slab can be expressed as

$$E_y^t = T_{TE} E_y^i, \tag{38}$$

in terms of the transmission coeffient T_{TE}. From (30)–(33) we obtain

$$T_{TE} = \frac{2\beta_{TE}k_t\eta_t k_{1z}k_2\eta_2 e^{jk_{2z}d}}{\begin{array}{c}\beta_{TE}k_t\eta_t\left(k_{1z}k_2\eta_2 + k_{2z}k_1\eta_1\right)\cos\beta_{TE}d \\ +\,j\left(\beta_{TE}^2 k_1k_2\eta_1\eta_2 + k_{1z}k_{2z}k_t^2\eta_t^2\right)\sin\beta_{TE}d\end{array}}. \tag{39}$$

From (37) and (39) one can show that, for $\eta_1 = \eta_2 = \eta_t$ and $k_1 = k_2$ the slab acts as a spatial filter for small values of the relative axial permeability m allowing transmission of TE waves in a narrow cone around normal direction and with almost total reflection ($R_{TE} \approx -1$) outside this cone. This can be seen from Figures 2 and 3. One can easily show that the property

$$|R_{TE}|^2 + |T_{TE}|^2 = 1 \tag{40}$$

is valid in this case.

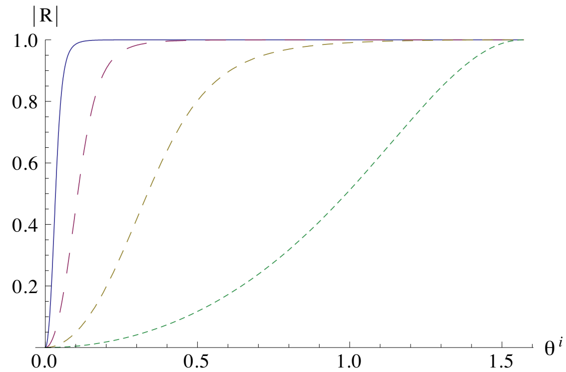

Figure 2. Magnitude of the reflection coefficient for a plane wave incident with TE polarization to a slab of uniaxial medium for angles of incidence $\theta^i = 0, \ldots, \pi/2$. The parameters satisfy $\mu_t = \mu_1 = \mu_2$, $\epsilon_t = \epsilon_1 = \epsilon_2$ and $k_1 d = 0.1$. The relative axial parameter $m = \mu_z/\mu_t$ is varied as $0.1,\ 0.01,\ 0.001$ and 0.0001. For $m \to 0$ the slab acts as a PEC plane with $R_{TE} = -1$ except for almost normally incident waves.

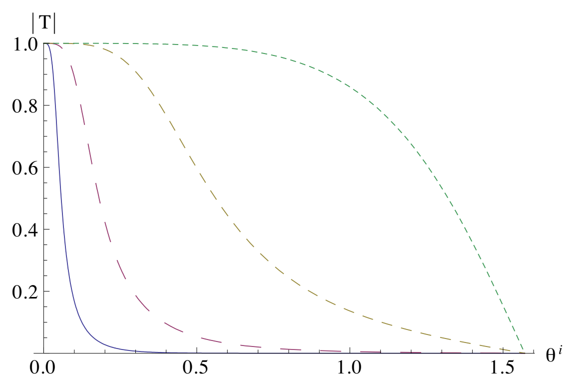

Figure 3. Magnitude of the transmission coefficient for a plane wave incident with TE polarization to a slab of uniaxial medium for angles of incidence $\theta^i = 0, \ldots, \pi/2$. The parameters satisfy $\mu_t = \mu_1 = \mu_2$, $\epsilon_t = \epsilon_1 = \epsilon_2$ and $k_1 d = 0.1$. The relative axial parameter $m = \mu_z/\mu_t$ is varied as $0.1,\ 0.01,\ 0.001$ and 0.0001. The slab acts as a narrow-beam spatial filter for small m.

3.2. TM Wave

Applying the duality substitution for the fields

$$\mathbf{E} \to \mathbf{H}, \qquad \mathbf{H} \to \mathbf{E}, \tag{41}$$

and for the media

$$\overline{\overline{\epsilon}} \to -\overline{\overline{\mu}}, \quad \overline{\overline{\mu}} \to -\overline{\overline{\epsilon}}, \tag{42}$$

the pair of Maxwell equations remains invariant. The substitutions imply

$$m \to e, \quad k \to k, \quad \eta \to -1/\eta, \quad \beta_{TE} \to \beta_{TM}, \tag{43}$$

which inserted in the reflection and transmission coefficient expressions (37), (39) yields the reflection and transmission coefficients for the magnetic field of the TM wave. Because the transmission coefficient for the electric field is the same as for the magnetic field while the reflection coefficient for the electric field is the negative of that of the magnetic field. Thus the coefficients for the electric field of the TM wave can be expressed as

$$R_{TM} = -\frac{\begin{array}{c}\beta_{TM} k_t \eta_t^{-1} \left(k_{1z} k_2 \eta_2^{-1} - k_{2z} k_1 \eta_1^{-1}\right) \cos \beta_{TM} d \\ - j \left(\beta_{TM}^2 k_1 k_2 \eta_1^{-1} \eta_2^{-1} - k_{1z} k_{2z} k_t^2 \eta_t^{-2}\right) \sin \beta_{TM} d\end{array}}{\begin{array}{c}\beta_{TM} k_t \eta_t^{-1} \left(k_{1z} k_2 \eta_2^{-1} + k_{2z} k_1 \eta_1^{-1}\right) \cos \beta_{TM} d \\ + j \left(\beta_{TM}^2 k_1 k_2 \eta_1^{-1} \eta_2^{-1} + k_{1z} k_{2z} k_t^2 \eta_t^{-2}\right) \sin \beta_{TM} d\end{array}}, \tag{44}$$

$$T_{TM} = \frac{2 \beta_{TM} k_t \eta_t^{-1} k_{1z} k_2 \eta_2^{-1} e^{j k_{2z} d}}{\begin{array}{c}\beta_{TM} k_t \eta_t^{-1} \left(k_{1z} k_2 \eta_2^{-1} + k_{2z} k_1 \eta_1^{-1}\right) \cos \beta_{TM} d \\ + j \left(\beta_{TM}^2 k_1 k_2 \eta_1^{-1} \eta_2^{-1} + k_{1z} k_{2z} k_t^2 \eta_t^{-2}\right) \sin \beta_{TM} d\end{array}}. \tag{45}$$

Because of the simple analogy between the TM and TE cases, we will mainly concentrate on the TE case in the sequel.

4. SHEET OF ZAP MEDIUM

By letting the thickness of the uniaxial slab vanish, $d \to 0$, simultaneously with vanishing axial parameters, $m \to 0$ and $e \to 0$, the uniaxial slab becomes a ZAP sheet. Let us assume that the limiting processes are related by

$$k_t d = c_e \sqrt{e} = c_m \sqrt{m}, \tag{46}$$

with some finite dimensionless thickness parameters c_e and c_m. For $\mathbf{K} \cdot \mathbf{K} = k_1^2 \sin^2 \theta^i \neq 0$, we then have the finite limits

$$\beta_{TE} d \to c_m \sqrt{m - \left(k_1^2/k_t^2\right) \sin^2 \theta^i} \to -j c_m \left(k_1/k_t\right) \sin \theta^i, \tag{47}$$

$$\beta_{TM} d \to c_e \sqrt{e - \left(k_1^2/k_t^2\right) \sin^2 \theta^i} \to -j c_e \left(k_1/k_t\right) \sin \theta^i, \tag{48}$$

$$k_{2z} d \to 0. \tag{49}$$

As an example, let us consider the special case when medium 2 equals medium 1, whence

$$\eta_2 = \eta_1, \quad k_2 = k_1, \quad k_{2z} = k_{1z} = k_1 \cos\theta^i. \tag{50}$$

Also, let us also assume that the uniaxial anisotropic medium is restricted by the condition (9) implying

$$c_e = c_m = c, \quad m = e = (k_t d/c)^2, \quad \eta_t = \eta_1, \tag{51}$$

whence the anisotropic medium is actually affine isotropic. In such a symmetric case we have $\beta_{TE} = \beta_{TM}$ and, for $\theta^i \neq 0$,

$$R_{TE} = -R_{TM} = -1, \tag{52}$$

$$T_{TE} = T_{TM} = 0. \tag{53}$$

Thus, the ZAP sheet appears as a PEC sheet for the TE wave and PMC sheet for the TM wave. In the case $\theta^i = 0$ the wave is TEM and the coefficients are

$$R_{TE} = R_{TM} = 0, \tag{54}$$

$$T_{TE} = T_{TM} = 1, \tag{55}$$

or the wave does not see the ZAP sheet at all. As a mental picture we can think that there is a hole in the ZAP sheet for normal incidence. Because only waves with exactly normal incidence will leak through the hole, for a continuous spectrum of waves the leakage corresponds to zero energy and can be neglected. Thus, the ideal ZAP sheet acts as a strange combination of PEC and PMC boundaries.

In practice, $m = 0$ and $e = 0$ can only achieved approximately. In the approximate case, the ZAP sheet is penetrable in a cone around the normal incidence. Considering the TE case, the penetration depends on two parameters, the (small) relative axial parameter m and the parameter $c = c_m$ defined by (46). In particular, the beamwidth of the transmission filter depends not only on the relative axial parameter m but also on the thickness parameter c_m. Obviously, for a given value of m the beamwidth becomes smaller the larger the parameter c_m. For d infinite we have the half space and the beamwidth is defined by the angle $\theta^i = \theta_{TE}$ satisfying

$$\sin\theta_{TE} = \sqrt{m}, \tag{56}$$

whence $\theta_{TE} \approx \sqrt{m}$ for small m. An example of the effect of the thickness parameter is given in Figure 4. For $c_m = 30$ the thickness equals $d \approx 0.15\lambda_1$.

If a source in the half space $z > 0$ radiates towards the nonideal (approximate) ZAP sheet, the radiation is filtered by the transmission factor of the sheet. Thus, the ZAP sheet can be used to remove sidelobes and reduce the beamwidth of the main lobe. In optical frequencies such a sheet could be used as a privacy filter for a laptop computer, for example. Because the ideal ZAP sheet satisfies the DB boundary conditions (1), the Poynting vector does not have a tangential component at the surface. Because electromagnetic power cannot propagate along such a surface, it can also be called an isotropic soft surface ("stop surface") [9] which has been applied as a mathematical model for certain band-gap surfaces [10].

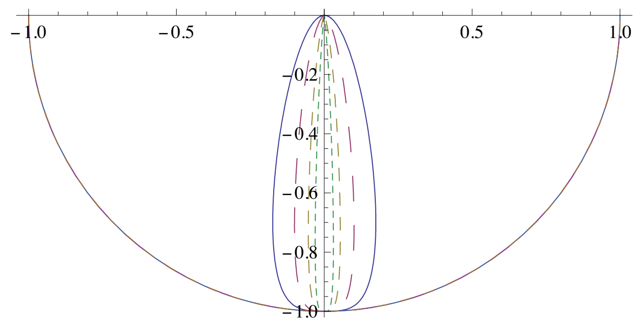

Figure 4. Polar plot depicting the magnitude of the transmission coefficient for a plane wave incident onto a ZAP sheet for angles of incidence $\theta^i = -90, \ldots, 90°$. The z axis is vertical and the parameters are $\mu_t = \mu_1 = \mu_2$, $\epsilon_t = \epsilon_1 = \epsilon_2$. The relative axial parameter is $m = \mu_z/\mu_t = 0.001$ and the thickness parameter $c_m = 1, 3, 10$ and 30. For large c_m the ZAP sheet acts as a narrow-beam spatial filter.

In the Introduction it was assumed that the relative axial permeability parameter m approaces zero from the positive side. Let us briefly consider the case when m is real and negative. It turns out that, in such a case, in addition to the normal incidence, the ZAP sheet appears penetrable for a certain set of discrete angles of incidence as well. For example, in Figure 5 for $m = -0.001$ there are five distinct conic penetration zones around the main lobe normal to the boundary.

This phenomenon can be verified from the reflection coefficent expression (37). In fact, requiring $R_{TE} = 0$ and assuming $k_1 = k_2$, $\eta_1 = \eta_2 = \eta_t$ this reduces to

$$(k_t^2 - k_1^2/m)\sin^2\theta^i \sin\beta_{TE}d = 0. \tag{57}$$

Omitting the case $k_t^2 = k_1^2/m$ we have in addition to $\theta^i = 0$ the solutions θ^i satisfying

$$\sin\beta_{TE}d = 0, \quad \Rightarrow \quad \sin^2\theta^i = m(1 - (n\pi/k_1 d)^2), \tag{58}$$

where $n > 0$ is an integer. For a thin ZAP sheet we have $k_1 d < \pi$, whence there are no real solutions for positive values of m while for negative values of m there exist real solutions

$$\theta_n^i = \sin^{-1}\left(|m|\left((n\pi/k_1 d)^2 - 1\right)\right) \tag{59}$$

for all integers n satisfying

$$n < \frac{k_1 d}{\pi}\sqrt{(1+|m|)/|m|}. \tag{60}$$

For example, the parameter values of Figure 5 yield $n < 5.03$, whence there are five filtering lobes in addition to the axial lobe.

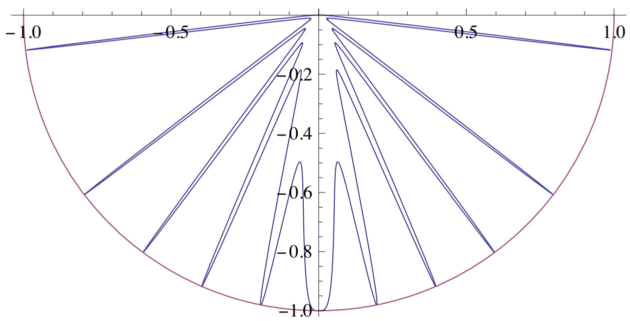

Figure 5. Same as Figure 4 except that m has the negative value $m = -0.001$ and the thickness parameter is $k_1 d = 0.5$. The ZAP sheet appears totally penetrable for a set of certain angles of incidence.

One may note that the angles of the transmission lobes are so distributed that the sines of the adjacent lobe angles are equidistant from one another. This kind of comb-filtering property of the ZAP sheet with negative relative axial parameters may have application in separating discrete wave components from a mixture of plane waves.

5. CONCLUSION

The concept of ZAP (Zero-Axial-Parameter) medium sheet was defined as the limiting case when the axial parameters and the thickness of a slab of uniaxial anisotropic medium vanish simultaneously. The ideal ZAP sheet has the property of reflecting waves incident with TE or TM polarization as from PEC or PMC boundaries, respectively. A nonideal ZAP sheet with small but finite axial parameters and thickness may act as a spatial filter for the incident waves with transmission in a narrow cone around normal direction. Such a sheet may have application in narrowing radiation beams and reducing sidelobes of antennas. Also, because there is no energy propagation along the surface of the ideal ZAP sheet, coupling between radiating elements at the sheet is reduced. There may also be potential application offered by a ZAP sheet with negative axial parameters due to its multi-lobed filtering property

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