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SIMULATION AND EXPERIMENTAL EVALUATION OF THE RADAR SIGNAL PERFORMANCE OF CHAOTIC SIGNALS GENERATED FROM A MICROWAVE COLPITTS OSCILLATOR

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Abstract—The ambiguity function of a kind of chaotic signal radar using Colpitts oscillator is investigated and compared in several aspects. The Colpitts oscillator with specific value of capacitance, inductance and resistance can generate chaotic signal with frequency band from direct current to several gigahertz. The chaotic signal

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is obtained from simulation and experiment. The auto-ambiguity functions of the chaotic signal show that the chaotic signal of such oscillator is ideal for radar application with both high range and range rate resolution. The cross-ambiguity function also indicates the chaotic signal has excellent capabilities in the electronic counter-countermeasures (ECCM). We also present the resolution of range with the spectrum from experiment.

1. INTRODUCTION

Random signal radars have been widely investigated in the recent years [1–3, 21–23]. It is known that random signal has high resolution in both range and range rate, making it ideal to radar applications. Compared with conventional radar signals such as pulse, chirp, continuous wave (CW) and so force, random signal also has excellent capabilities in the electronic counter-countermeasures (ECCM) and low probability intercept (LPI).

Chaotic signal has aperiodic waveform in time domain and noise-like wide frequency band in frequency domain. Superior to the noise signal, the chaotic signal is controllable and can be synchronized between different radar systems with the same circuit parameters [4, 5], which makes it more attractive. Performance of several kinds of radar using chaotic signal have been reported [6–10].

Recently, chaotic circuits with fundamental frequency more than 1 GHz were implemented [11–13], where the Colpitts oscillator is the major candidate due to its simple circuit structure [13, 14]. Using the microwave chaotic signals generated from Colpitts oscillator, the advantage of the direct chaotic radar is, by varying the operating condition, the dynamics in a Colpitts oscillator can be easily switched among different states, and the corresponding diverse waveforms with different characteristics are highly desirable, since it provides the property of "multi-user", which is required when a large number of radars with the same scheme co-exist, in applications such as vehicular collision avoidance system [15]. When compared with laser-based chaotic radar [16, 17], the chaotic radar employing Colpitts oscillator has the advantages of simple circuit structure and low fabrication cost. The chaotic signal used in this paper is generated from the Colpitts oscillator. The basic working frequency of the Colpitts oscillator ranged from 1 GHz to 3.5 GHz, with ultra-wide frequency band from direct current to about 10 GHz.

The radar ambiguity function is generally applied as a means of studying different waveforms, which describes the detection property of a radar system in both range and range rate domain [18]. Recent research also brings forward the concept of acceleration ambiguity [19] to investigate the resolution of acceleration. In this paper, the performance of the chaotic signal obtained from both simulation and experiment will be studied, comparison of the slices of the ambiguity functions along the principal axes in different parameters is also shown. Finally we present the self-correlation for the case of two targets with experimental spectral results, which show that the chaotic RADAR utilizing such signal would have fairly good resolving ability.

2. COLPITTS OSCILLATOR CIRCUIT

The basic configuration of the Colpitts oscillator used as the microwave chaotic oscillator source is shown in Fig. 1(a). It contains a bipolar junction transistor (BJT) as the gain element and a resonant network consisting an inductor and a pair of capacitors [20]. The transistor is modeled with a voltage-controlled nonlinear resistor R_E and a linear current-controlled current source as shown in Fig. 1(b), neglecting the base current. The driving-point characteristics of the nonlinear resistor R_E can be expressed as

$$I_E = f(V_{BE}) = I_s \left(\exp\left(\frac{V_{BE}}{V_T} - 1 \right) \right), \tag{1}$$

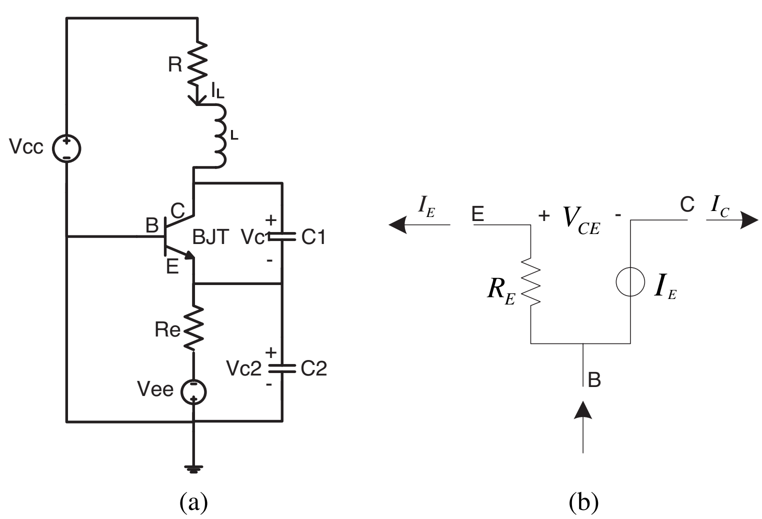


Figure 1. (a) Schematic setup of the Colpitts oscillator circuit. (b) Equivalent circuit model of the BJT in (a).

where I_L is the inverse saturation current and $V_T \simeq 26\,\mathrm{mV}$ at room temperature. The state equations for colpitts oscillator as shown in Fig. 1(a) are

$$C_1 \frac{dV_{C_1}}{dt} = -f\left(-V_{C_2}\right) + I_L \tag{2}$$

$$C_2 \frac{dV_{C_2}}{dt} = I_L - \frac{V_{C_1} + V_{ee}}{Re} \tag{3}$$

$$L \frac{dI_L}{dt} = -V_{C_1} - V_{C_2} - I_L R + V_{CC}, \tag{4}$$

where I_L is the current through the inductor L, V_{C_1} and V_{C_2} are the voltages across the capacitors C_1 and C_2. It has been verified [20] with proper value of L, C_1 and C_2 list in Fig. 1(a), the chaotic signal with fundamental frequency more than 1 GHz can be generated. We obtain

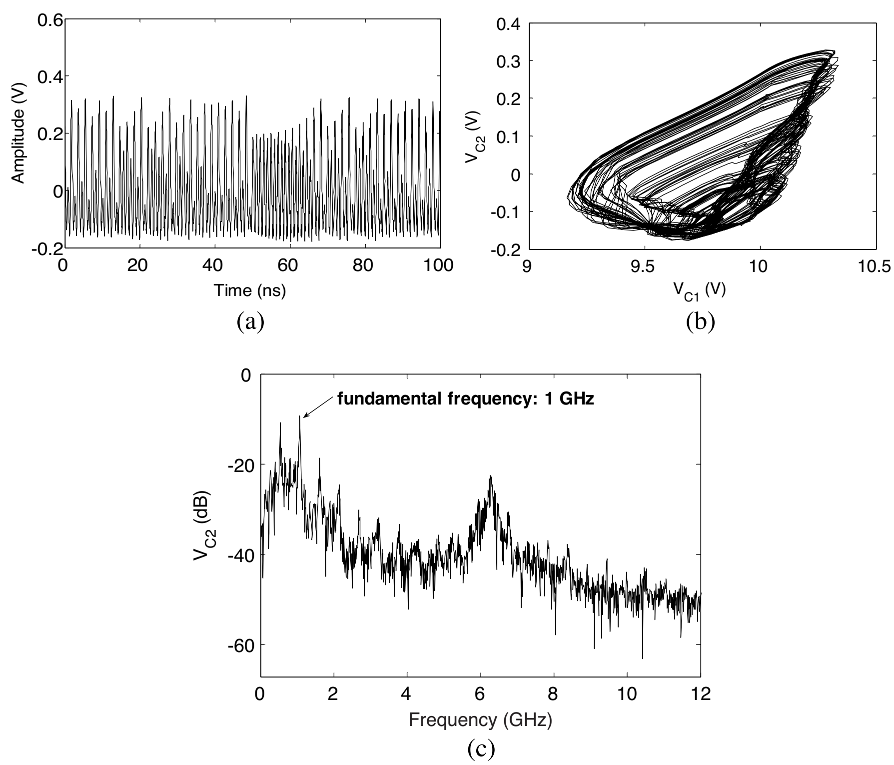


Figure 2. Simulation results (a) time domain waveform of V_{C_2}. (b) Attractor in the $V_{C_2} - V_{C_1}$ plane. (c) Spectrum of V_{C_2}.

the chaotic signal with fundamental frequency about 1 GHz both in simulation and experiment as shown in Fig. 2, here in the Colpitts circuit the $V_{C_1} = V_{C_2} = 3.9\,\text{pF}$, $L = 4.6\,\text{nH}$.

Figures 2(a)–(c) are the simulation results, where Fig. 2(a) shows the time series which is noise like signal, Fig. 2(b) plots the projection of the attractor of the phase space into the $V_{C_1} - V_{C_2}$ plane, which is a typical chaotic attractor of Colpitts oscillator, Fig. 2(c) shows power spectra of the signal V_{C_2}, which is like white noise with frequency band of about 1 GHz. Fig. 3 shows the experimental results with similar circuit parameters as in the simulation. Since commercial capacitance and inductance have discrete values, it is difficult to choose identical circuit parameters the same as in the simulation. We see that the frequency spectrum is similar to the simulation result, both the simulation and experimental results have a working band from direct current to 1 GHz and a working band at higher frequency, the fundamental working frequency is a little lower than the simulation result, that is due to the parasitic capacitance and the limited cut-off frequency of the bipolar junction transistor (BJT) used in the experiments. Fig. 4 reveals the chaotic signal from simulation and experiment with smaller circuit parameters than in Fig. 2. It is found that as our expectation, the fundamental frequencies in simulation and experimental results are moved up to around 3.5 to 4 GHz. Because of the cutting off frequency of the bipolar junction transistor (BJT) used in the experiments, the spectrum in experiment has no signal at higher frequency. It is found the fundamental frequency of the Colpitts oscillator can be tuned to be several gigahertz with carefully designed values of C_1, C_2 and L. In the following sections, we present the ambiguity function of the chaotic signal mainly from simulation, as it

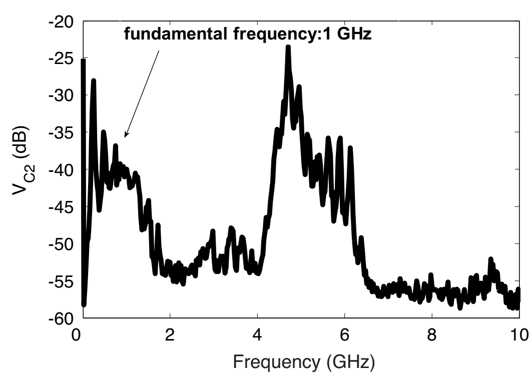
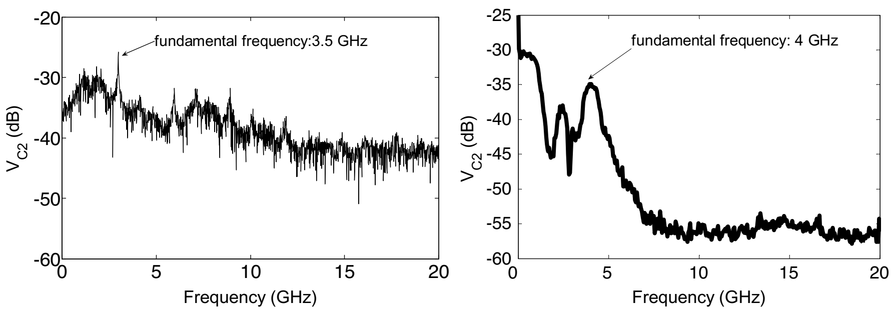


Figure 3. Spectrum of V_{C_2} of the experimental result.

is difficult to obtain the time domain signal of such broad frequency band, and it is found the spectrum of the measured signals are actually similar to the simulations in the range of the working band of the microwave transistor.



(a) (b)

Figure 4. (a) Simulation result for the spectrum of V_{C_2}. (b) Experimental result for the spectrum of V_{C_2}.

3. AMBIGUITY FUNCTION ANALYSIS OF THE CHAOTIC SIGNAL

3.1. Wideband Chaotic Signal Model

Recently, besides the conventional conception of ambiguity function of range and range rate, there has been put forward the idea of ambiguity function of acceleration [19], in this paper the detection property of acceleration will also be discussed combined with the range and range rate domain. For a point target, suppose $s(t)$ is the reference signal and

$$s_r(t) = s(t - \tau(t)) \tag{5}$$

is the reflected signal from the moving target, where $\tau(t)$ is the delay time between the reference and the received signal, and it can be expressed as

$$\tau(t) = \frac{2R(t)}{c+v} \approx \frac{2R(t)}{c} \tag{6}$$

the velocity of the target is much smaller than the microwave signal, R denotes the distance between the source and the target. Let $\tau(0) = \tau_0 = \frac{2R_0}{c}$, we can make Taylor expansion of $\tau(t)$ at $t = 0$

as equation

$$\tau(t) = \tau_0 + \tau_0^{(1)} t + \tau_0^{(2)} \frac{t^2}{2} + \ldots + \tau_0^{(n)} \frac{t^n}{n!} \tag{7}$$

The velocity and acceleration of the target can be decided as $v(t) = -R^{(1)}(t)$ and $a(t) = -R^{(2)}(t)$ respectively, consider the positive direction is towards the microwave source. From equation, $\tau^{(1)} = -\frac{2v}{c}$, $\tau^{(2)} = \frac{2a^2}{c}$. Neglecting the higher order term when the acceleration is constant, the equation is rewritten as

$$\tau(t) = \tau + \frac{2v}{c} t + \frac{2a}{c} \frac{t^2}{2} \tag{8}$$

Substituting it into equation, it is obtained

$$s_r(t) = s\left(\alpha_a t^2 + (1+\alpha)\, t - \tau\right) \tag{9}$$

where $\alpha_a = -\frac{2a}{c}$ and $\alpha = \frac{2v}{c}$. When the velocity is constant, that is the acceleration equals zero, the equation becomes the conventional expression

$$s_r(t) = s((1+\alpha)t - \tau_0) \tag{10}$$

3.2. Ambiguity Function

3.2.1. Auto-ambiguity Function

In the Colpitts oscillator chaotic signal radar system, the broadband microwave chaotic waveforms are transmitted and received as baseband signals. The ambiguity function is defined as

$$\langle \chi(\tau, \alpha, \alpha_a) \rangle = \int_t^{t+T} s(t) s_r(t) dt \tag{11}$$

where $s_r(t)$ is defined from equation, and T is the correlation interval. When the velocity is constant, that is $\alpha_a = 0$, equation is rewritten as

$$\langle \chi(\tau, \alpha, 0) \rangle = \int_t^{t+T} s(t) s((1+\alpha)t - \tau) dt \tag{12}$$

Figure 5 shows the auto-ambiguity function of range and range rate, with correlation internal $T = 10\,\mu\text{s}$, for signal with a fundamental frequency of 1 GHz as shown in Fig. 3. As can be seen, the ambiguity function of $\langle\chi(\tau,0,0)\rangle$ has many side lobes. This is because the chaotic signal generated by Colpitts oscillator possesses somewhat periodicity essentially, which makes the chaotic signal not ideal random waveform. From the time-domain view, the chaotic signals with time distance of τ_0, $2\tau_0$, $\ldots$, $n\tau_0$, have similarities which result in the side lobes. The auto-ambiguity function of signal with a fundamental frequency of 3.5 GHz in Fig. 4 is shown in Fig. 6. It should be emphasized that $\langle\chi(\tau,0,0)\rangle$ is negligible when the frequency band is broader, as can be seen in Fig. 7(a).

We compare the slices of $\langle\chi(0,\alpha,0)\rangle$ and $\langle\chi(0,0,\alpha_a)\rangle$ of chaotic signal with different fundamental frequency in Figs. 7(b) and (c). The width of the slices is narrower with the higher fundamental frequency. The chaotic signal with higher fundamental frequency has broader

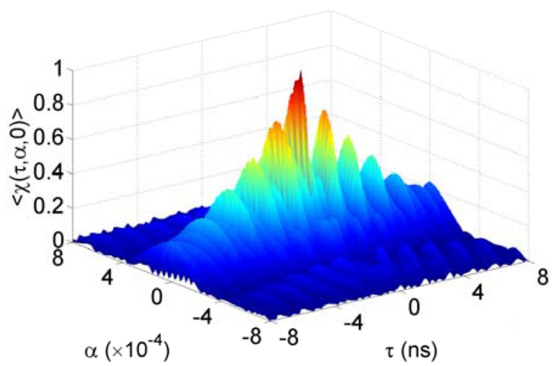


Figure 5. Auto-ambiguity function of the Colpitts oscillator with 1 GHz fundamental frequency.

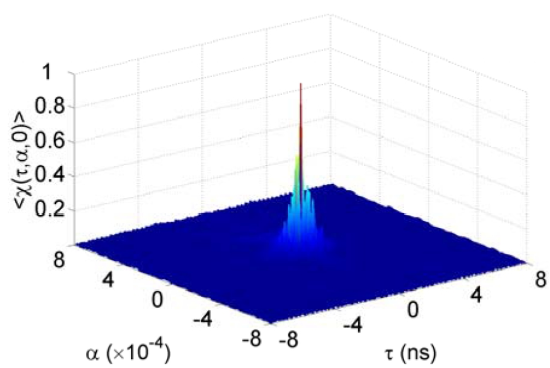


Figure 6. Auto-ambiguity function of the Colpitts oscillator with 3.5 GHz.

frequency band which is much more like random signal. For the chaotic signal with a fundamental frequency of 3.5 GHz, from the full-width at half maximum (FWHM) of the peak in the delay and delay rate axes, range and range rate resolution are 0.1 m and 7.5 km/s, respectively, the acceleration resolution is $1.11 \times 10^9\,\mathrm{m/s^2}$, it should be noticed that the acceleration resolution is not feasible in reality.

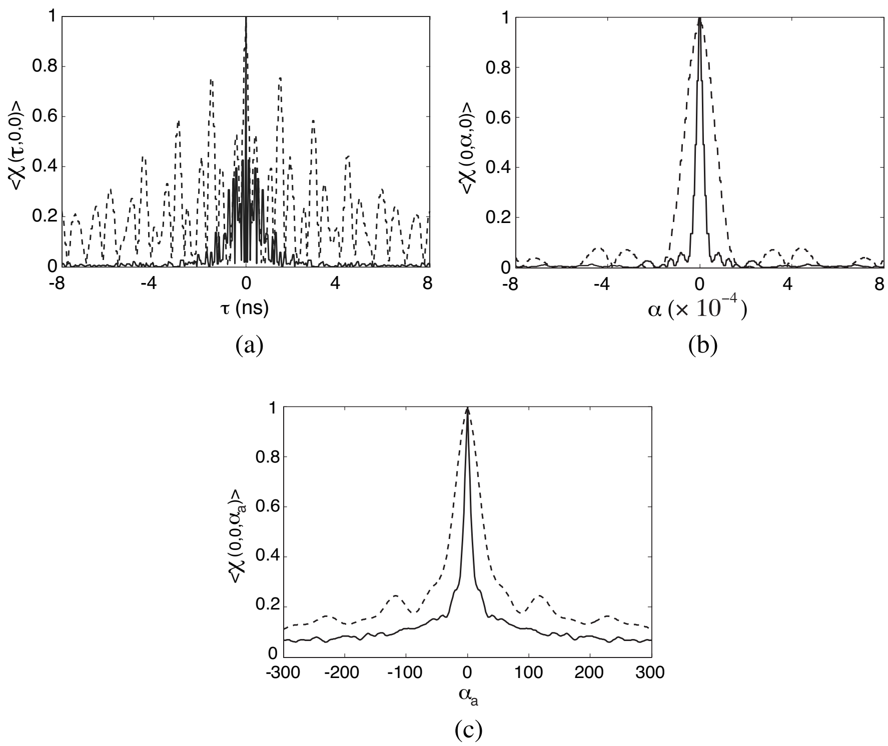


Figure 7. Slices of the auto-ambiguity function in Fig. 5 and Fig. 6. The solid line is the 3.5 GHz fundamental frequency signal, the dash line is the 1 GHz fundamental frequency signal. (a) Slices of $\langle\chi(\tau,0,0)\rangle$. (b) Slices of $\langle\chi(0,\alpha,0)\rangle$. (c) Slices of $\langle\chi(0,0,\alpha_a)\rangle$.

3.2.2. Cross-ambiguity Function

Next we investigate the characteristics of the chaotic signal that affects the ambiguity function. Fig. 8 shows the slices of $\langle\chi(0,\alpha,0)\rangle_T$ for the chaotic signal with a 3.5 GHz fundamental frequency, with $T = 10$, 2, and 1 µs, the widths of the slices narrow when the correlation interval increases. As can be seen from Fig. 8, the FWHM of the slices are

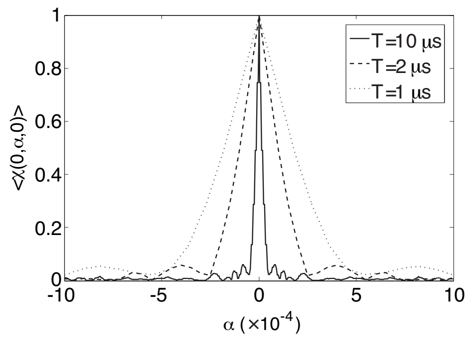


Figure 8. Slices of $\langle\chi(0, \alpha, 0)\rangle$ for different correlation interval T.

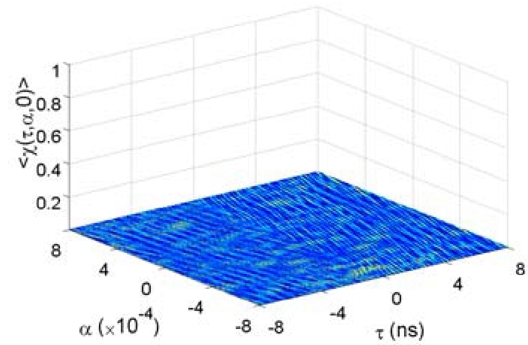
inversely proportional to the correlation interval, which means the range rate resolution can be improved linearly with the correlation interval.

Another merit of the chaotic signal of Colpitts oscillator is its sensitivity to initial values of the oscillator parameters, which means the chaotic signals vary greatly with different parameters, this is useful in the multi-users condition such as car collision avoidance. To evaluate the ECCM capability of the Colpitts oscillator, we have calculated the cross-ambiguity function of the signal shown in Fig. 4 with the same circuit parameters but at a different time with a correlation interval $T = 10\,\mu\text{s}$ in Fig. 9(a), and that of two slight different parameters in Fig. 9(b). One circuit has the same parameters as listed in Fig. 1, the other changes value of R ($R = 27\,\Omega$). The cross-ambiguity function in Fig. 9(a) indicates the chaotic signal of Colpitts oscillator has good randomicity in time domain, thus signal samples at different time have low degree of correlation, the cross-ambiguity function in Fig. 9(b) is due to the sensitivity of chaotic signal of Colpitts oscillator to the circuit's parameters, that means slight change of the parameters will greatly change the character of the chaotic signal. Therefore, the chaotic signal of Colpitts oscillator has excellent capabilities in ECCM.

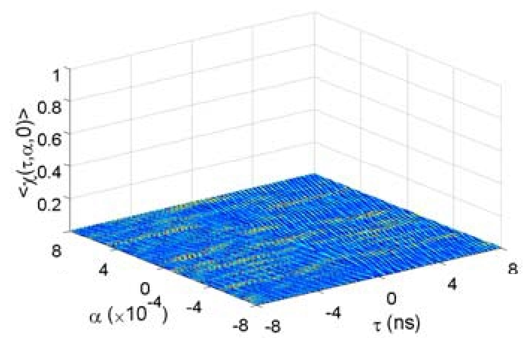
4. RESOLUTION CALCULATION FROM EXPERIMENTAL DATA

The above ambiguity function is calculated with simulation results, as it is difficult to achieve the time-domain wave form for such a wide-band signal, especially for its phase information. We will calculate the resolution of range with wave spectrum from experiment.

For n point targets, suppose $s(t)$ is the reference signal, and



(a)



(b)

Figure 9. Cross-ambiguity function of the Colpitts oscillator with 3.5 GHz fundamental frequency, correlation interval $T = 10\,\mu s$. (a) Cross-ambiguity function with different time. (b) Cross-ambiguity function with slight different parameters.

$s_r(t) = \sum_1^n \sigma_i s(t-\tau_i) = s(t) * \sum_1^n \sigma_i \delta(t-\tau_i) = s(t) * h(t)$ is the reflected signal from the targets, where σ_i is the reflection coefficient, $\tau_i = \frac{2R_i}{c}$ is the delay time between the reference and the i-th target received signals. σ_i can be regarded as one unit for simplicity. The target range resolution is obtained by correlating $s(t)$ with $s_r(t)$, i.e., $s_r(t) * s^*(-t) = s(t) * h(t) * s^*(-t)$, whose spectrum expression is $S(jw)H(jw)S^*(-jw) = |S(jw)|^2 H(jw)$, where $|S(jw)|^2$ is power spectrum obtained by experiment. The self-correlation for the case of two targets is calculated and the results are listed in Figs. 10 and 11. We see that for the chaotic signals with fundamental

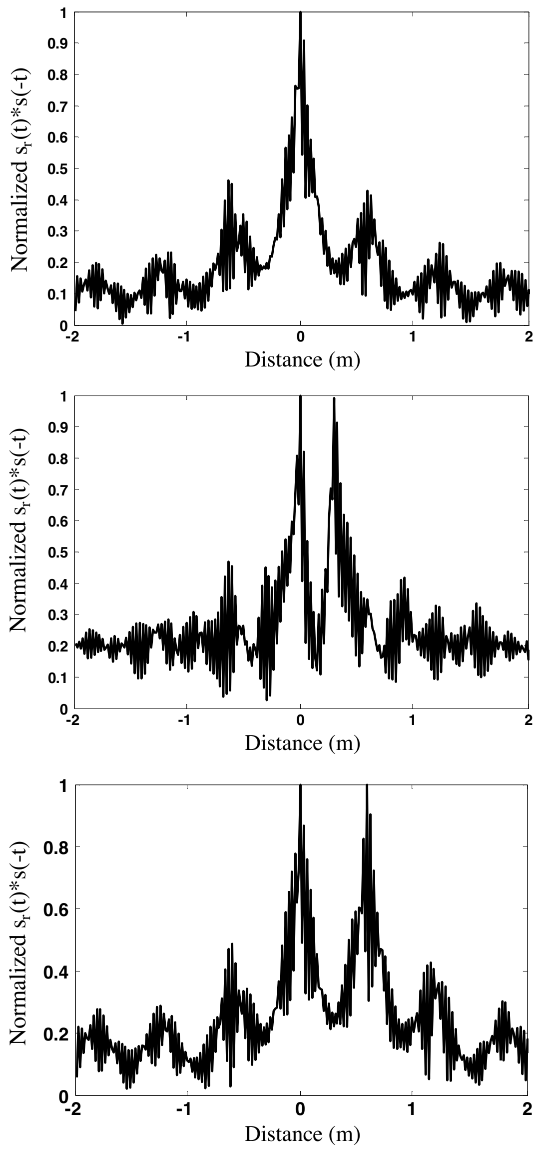


Figure 10. Calculated results of distance resolution of two objects using the experimental signal from the Colpitts oscillator with 1 GHz fundamental frequency. (a) Distance 0 m, (b) distance 0.3 m, (c) distance 0.6 m.

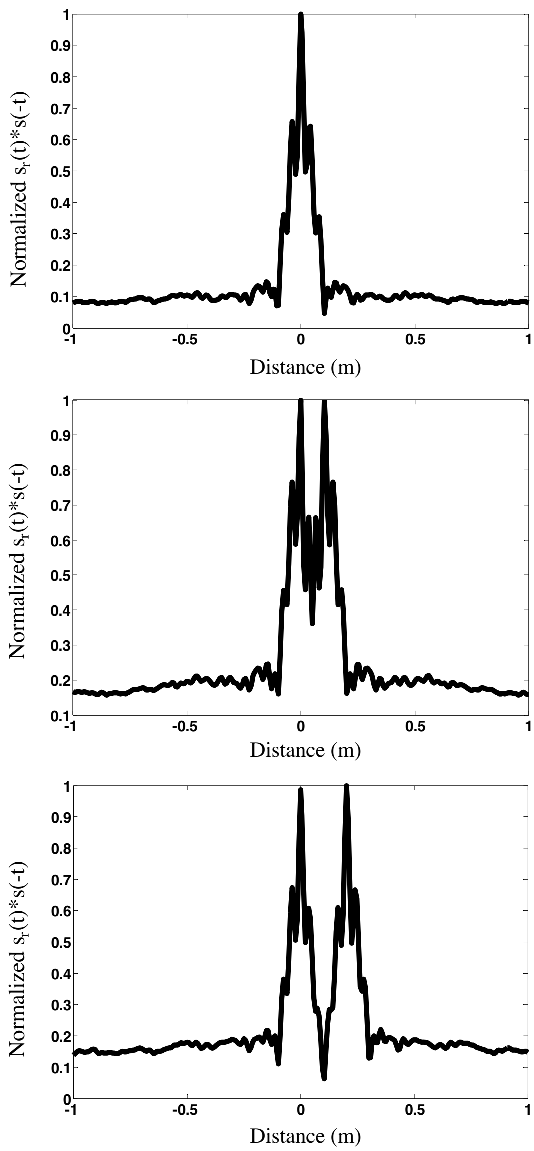


Figure 11. Calculated results of distance resolution of two objects using the experimental signal from the Colpitts oscillator with 4 GHz fundamental frequency. (a) Distance 0 m, (b) distance 0.1 m, (c) distance 0.2 m.

frequencies of 1 and 4 GHz, the resolution ability of range is 0.3 m and 0.1 m, respectively, which is a quite good result. However, such result only concerns the characteristics of the chaotic signal itself, without considering influences from other aspects, such as the noise inside the RADAR system, the interference during the RADAR signal propagating in free space, and etc.

5. CONCLUSION

In this paper, the ambiguity functions of microwave chaotic signal generated by a chaotic Colpitts oscillator have been studied. The time-domain, frequency domain chaotic signal and chaotic attractor of the Colpitts oscillator are presented for illustration. The chaotic signals with fundamental frequency of 1 GHz and 3.5 GHz have been studied and compared. The auto-ambiguity function of chaotic signals shows that the chaotic signal with low frequency possesses periodicity, too many side lobes makes the unambiguous range detection difficult, and the side lobe reduces when the fundamental frequency increases, so that the randomness characteristics of the microwave chaotic signal improves. It should be mentioned that the operating condition of the Colpitts oscillator with more abound dynamic characteristics can help improve the signal spectrum.

The cross-ambiguity functions of the direct radar system have also been investigated to evaluate the ECCM performance and the"multi-user" characteristic when the chaotic signal is used as anti-collision vehicle-borne radar. Rather excellent ECCM capability can be achieved with transmitting chaotic signals generated by circuits with same parameters but at different time or with slightly different circuit parameters.

Finally we present the self-correlation for the case of two targets with experimental spectral results, the resolution range is 0.3 m and 0.1 m for chaotic signals with fundamental frequencies of 1 GHz and 4 GHz, respectively.

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