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THE SCATTERING FROM AN ELLIPTIC CYLINDER IRRADIATED BY AN ELECTROMAGNETIC WAVE WITH ARBITRARY DIRECTION AND POLARIZATION

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Abstract—The analytical expression of scattering field from a conductor elliptic cylinder is presented, as the electromagnetic wave propagating vertical to the axis of an elliptic cylinder with arbitrary incident angle and polarization. The obtained result is in agreement with that in the reference when we use this analytical expression to calculate the scattering field from a cylinder. Simulations show that the vertical size of the elliptic cylinder greatly affects the scattering field when we observe it in the direction perpendicular to the direction of the incident wave. The scattering field is strong as the polarization direction of incident wave parallel to the axis of the elliptic cylinder. The algorithm used in the article is valid to investigate the scattering characteristics of other elliptic cylinders. The obtained result offers a theoretical foundation for the practical applications such as electromagnetic remote sensing of target's size and shape.

1. INTRODUCTION

A large number of scatters in the fields ranging from industry production to medical imaging to halobios and radio wave propagation are all approximately considered as an elliptic cylinder. Thus many scholars in the world focus their energy on the investigation of the scattering characteristics of an elliptic cylinder based on Mathieu functions [1]. In 1881 Rayleigh investigated the scattering property of a uniform medium cylinder in a plane electromagnetic wave. In [3], the electromagnetic scattering of a non-uniform medium cylinder is well studied and a new algorithm for calculating scattering field is developed when the incident wave vertically irradiating it. Kong [4] has investigated the scattering field in detail of a cylinder as the direction of polarization parallel to the cylinder's axis. Furthermore the scattering property of the multilayer dielectric cylinders [5–7] is researched. The scattering peculiarity of cylinders in the Gaussian beam is presented in [10, 11]. BarabÀs [12] and Bussey [13] use the matrix iteration to study the scattering characteristic of multilayer concentric dielectric cylinder, but the cylinder's size and the number of layers are limited. Theoretically, cylinders considered as an electromagnetic scattering model have been investigated by many researchers [8, 9]. In practical applications, an elliptic cylinder being considered as an electromagnetic scattering model is better than that of the cylinder. The investigations of scattering field about this kind of model are seldomly found, which is a great inconvenience for many practical applications.

Based on the electromagnetic scales theory, the scattering field from an elliptic cylinder irradiated vertically to its axis by an electromagnetic wave with arbitrary direction and arbitrary polarization is presented in detail. The results obtained are in agreement with that in the references [4]. The method used not only can be used to investigate the subject of scattering of the multilayer dielectric elliptic cylinder, but also has the characteristics of simplicity and not being limited by the sizes of targets and the operating frequency.

2. THE SCATTERING CHARACTERISTIC OF AN ELLIPTIC CYLINDER IRRADIATED VERTICALLY BY A PLANE ELECTROMAGNETIC WAVE

There are respectively the elliptic cylinder's semi-axes As shown in Figures 1(a) and (b). Its symmetric axis is parallel to the z-axis of the coordinate system. The incident wave propagates to this elliptic cylinder in the direction of angle φ_0. The vector of wave number $\mathbf{k}$, the incident angle φ_0 and electric field $\mathbf{E}$ are respectively changed into $\mathbf{k}'$, φ_0' and $\mathbf{E}'$. Meanwhile the elliptic cylinder can be shaped into a cylinder after a series scales transformation. For the sake of simplicity, we take the conductor elliptic cylinder as an example, but the method is valid to research the scattering field from the other dielectric elliptic cylinders and multilayer medium elliptic cylinders.

2.1. The Scattering Characteristic of an Elliptic Cylinder as the Incident Wave Polarizing in the Direction of z-axis

Let the incident wave being the following expression in the Σ coordinate system

$$\mathbf{E}_i = \hat{u}_z E_0 \exp\left[-j\left(k_x x + k_y y\right)\right] \tag{1}$$

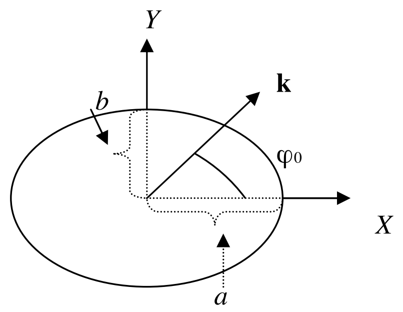

Figure 1. An elliptic cylinder.

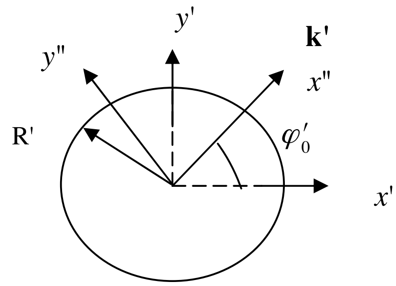

Figure 2. A cylinder.

For the sake of accurateness, we chose the scale factor in x-direction and the scale factor in y-direction, respectively

$$s_x = \frac{2a}{a+b}, \quad s_y = \frac{2b}{a+b} \tag{2}$$

We thus conclude that this elliptic cylinder [14, 15] is reconstructed as a cylinder of radius $R' = 0.5\,(a+b)$. The wave-vector is also changed as

$$k_x' = s_x k_x, \quad k_y' = s_y k_y \tag{3}$$

This new wave-vector has an angle φ_0' turning off x'-axis anticlockwise and the size of electric field is not changed. Aiming to use the available result of scattering field from a cylinder, we rotate the x'-axis an angle φ_0' around z'-axis, then we obtain another coordinate system, namely Σ'' coordinate system. It is easy to see that the relation of angles is given by

$$\varphi'' = \varphi' - \varphi_0' \tag{4}$$

The transformation of a vector between the system Σ' and Σ'' is

$$\mathbf{A}'' = \mathbf{T}\mathbf{A}' \tag{5}$$

where

$$\mathbf{A}'' = \begin{bmatrix} A_x'' \\ A_y'' \\ A_z'' \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} \cos\varphi_0' & \sin\varphi_0' & 0 \\ -\sin\varphi_0' & \cos\varphi_0' & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}' = \begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix}$$

Substituting (3) in (5) and using the results [15] of trigonometric function give

$$k_x'' = k/g_0, \quad k_y'' = 0 \tag{6}$$

where

$$g_0 = \left(\frac{\cos^2 \varphi_0}{s_x^2} + \frac{\sin^2 \varphi_0}{s_y^2} \right)^{\frac{1}{2}}$$

We can conclude from (6) that the incident wave propagates in the Σ'' system in the direction of x''-axis and polarizes in z''-axis. We assume the medium being isotropic one, thus the scattering field [16, 17] is obtained by

$$\begin{aligned} E''_{sz} &= E_0 \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)} \left(k\rho''/g_0 \right) e^{jn\varphi''} \\ a_n &= -\frac{J_n \left(kR'/g_0 \right)}{H_n^{(2)} \left(kR'/g_0 \right)} \end{aligned}$$

where $R'' = R'$ has been used. After utilizing the transformations of vector in the three coordinate systems Σ, Σ', Σ'' and the trigonometric function, we obtain the scattering field

$$E_{sz} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)} (k\rho) \left(\frac{\cos n\varphi}{s_x g_n} + j \frac{\sin n\varphi}{s_y g_n} \right) \left(\frac{\cos n\varphi_0}{s_x g_{n0}} - j \frac{\sin n\varphi_0}{s_y g_{n0}} \right) \quad (7)$$

where

$$g_n = \left(\frac{\cos^2 n\varphi}{s_x^2} + \frac{\sin^2 n\varphi}{s_y^2} \right)^{1/2}, \quad g_{n0} = \left(\frac{\cos^2 n\varphi_0}{s_x^2} + \frac{\sin^2 n\varphi_0}{s_y^2} \right)^{1/2}$$

Expression (7) is the analytical expression of scattering field from an elliptic cylinder in the original coordinate system. This expression implies that the scattering field is relative to the factors of observing point, the sizes of the elliptic cylinder and the incident wave's angle.

2.2. The Scattering Characteristic of an Elliptic Cylinder as the Incident Wave Polarizing in the x-y Plane

Let the incident wave being the following expression in the Σ coordinate system

$$\mathbf{E}_i = \left(-\sin \varphi_0 \hat{u}_x + \cos \varphi_0 \hat{u}_y \right) E_0 \exp \left[-j \left(k_x x + k_y y \right) \right] \quad (8)$$

Utilizing the former results and putting (8) in (5), we conclude that the incident wave in the coordinate system Σ'' now propagates in the direction of x''-axis and polarizes in the direction of y''-axis. We assume the medium being isotropic one, thus the scattering field [16, 17] can be written as

$$E''_{s\varphi} = jE_0 \sum_{n=-\infty}^{\infty} j^{-n} b_n H_n^{(2)'} \left(k\rho''/g_0\right) e^{jn\varphi''}$$

$$b_n = -\frac{J'_n \left(kR'/g_0\right)}{H_n^{(2)'} \left(kR'/g_0\right)}$$

After utilizing the transformations of vector in the three coordinate systems Σ, Σ', Σ'' and the trigonometric function, and a length is an invariant in rotating system, we obtain the scattering field as

$$E''_{s\varphi} = jE_0 \sum_{n=-\infty}^{\infty} j^{-n} b_n H_n^{(2)'} (k\rho) \left(\frac{\cos n\varphi}{s_x g_n} + j\frac{\sin n\varphi}{s_y g_n}\right)\left(\frac{\cos n\varphi_0}{s_x g_{n0}} - j\frac{\sin n\varphi_0}{s_y g_{n0}}\right) \quad (9)$$

It is noticeable that the left of expression (9) is in Σ'' coordinate system and the right is all in the Σ coordinate system. Therefore, we must know the distributed characteristic of the scattering field in Σ coordinate system. For the sake of simplicity, we only present the algorithm. (a) We first obtain expression of the right angle components in system Σ'' by using the transformation of a vector between the spherical coordinate system and the right angle coordinate system. (b) Then scattering field in the Σ' system is derived by using expression (5). (c) Using the scales transformation of a vector in both systems Σ' and Σ to develop the right angle components of the field in Σ coordinate system. (d) Obtaining the expression of the spherical vector components in system Σ by using the transformation of a vector again between the spherical coordinate system and the right angle coordinate system. Now we can obtain the following expression

$$E_{s\varphi} = \frac{E''_{s\varphi}}{s_y g_1 g_0} \left[\frac{\sin\varphi \sin(\varphi - \varphi_0) + \cos^2\varphi \cos\varphi_0}{s_x^2} + \frac{\cos\varphi \sin\varphi \sin\varphi_0}{s_y^2}\right] \quad (10)$$

where

$$g_1 = \left(\frac{\cos^2\varphi}{s_x^2} + \frac{\sin^2\varphi}{s_y^2}\right)^{1/2}$$

Expression (10) demonstrates that the scattering field of an elliptic cylinder is a complicated function in both of the shape of the elliptic cylinder and the angle of the incident wave.

2.3. Discussions and Applications

We can use both expressions (7) and (10) to calculate the scattering field from cylinder. It is obvious that the scale factors $s_x = s_y = 1$ as $a = b$. In this condition, the elliptic cylinder is reconstructed into a cylinder with radius of $R = a$ and the symbols are changed at the same time as $g_n = g_0 = g_1 = g_{n0} = 1$. The results in (7) and (10) are simplified as

$$E_{sz} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)}(k\rho)\, e^{jn(\varphi-\varphi_0)} \tag{11}$$

$$E_{s\varphi} = jE_0 \cos\varphi_0 \sum_{n=-\infty}^{\infty} j^{-n} b_n H_n^{(2)\prime}(k\rho)\, e^{jn(\varphi-\varphi_0)} \tag{12}$$

The above expressions (11) and (12) are respectively the analytical expressions of scattering field from a cylinder in the electromagnetic wave of two kind of polarizations and propagating with the angle of φ_0 in space. When the angle $\varphi_0 = 0$, expressions (11) and (12) are entirely in agreement with that in the [16]. This agreement also indicates the rightness of the algorithm. The wave incidents in the direction that turns an angle of φ_0 off x-axis anticlockwise and polarized in the direction that turns an angle of θ_0 off the z-axis. The field components vertical to z axis and parallel to z-axis are respectively $E_0 \sin\theta_0$ and $E_0 \cos\theta_0$. Thus we can get the total scattering field as

$$\begin{aligned}\mathbf{E}_s = {} & \hat{z} E_0 \cos\theta_0 \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)}(k\rho) \left(\frac{\cos n\varphi}{s_x g_n} + j\frac{\sin n\varphi}{s_y g_n}\right) \\ & \left(\frac{\cos n\varphi_0}{s_x g_{n0}} - j\frac{\sin n\varphi_0}{s_y g_{n0}}\right) + \hat{\varphi}\frac{E''_{s\varphi}\sin\theta_0}{s_y g_1 g_0} \\ & \left[\frac{\sin\varphi\sin(\varphi-\varphi_0) + \cos^2\varphi\cos\varphi_0}{s_x^2} + \frac{\cos\varphi\sin\varphi\sin\varphi_0}{s_y^2}\right]\end{aligned} \tag{13}$$

The differential scattering width is generally used to investigate the scattering subject of two dimensions and is defined as [16, 17]

$$K(\varphi) = \lim_{\rho\to\infty} \rho \left|\frac{E_s}{E_i}\right|^2 \tag{14}$$

Substituting expressions (7), (10), (13) in above gives the concrete differential scattering width. Followings are partial simulation.

The changes of scattering width are shown in Figure 3 to Figure 5 when the polarization direction is parallel to the z-axis and the operating frequency is 30 MHz. In Figure 3, the radius of the cylinder is $R = 1$ m and the incident angle is $\varphi_0 = \pi/4$. The solid line denotes the result using expression (7) and the plus line is the result of [16]. It is shown that the two results are in agreement. In Figure 4, the frequency is the same as that in Figure 3 and the incident angle is $\varphi_0 = \pi/6$. We can conclude that the scattering effect in the forward direction is the strongest and the sizes of the elliptic cylinder obviously influence the scattering in this forward region. In the backward region, this kind of influence is not very remarkable. We assume that the observing

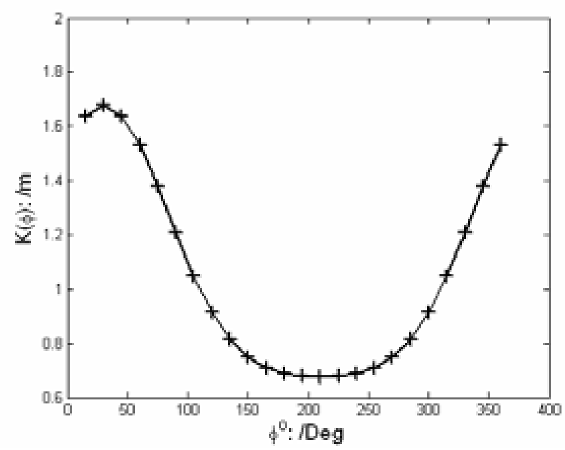

Figure 3. Scattering width of a cylinder.

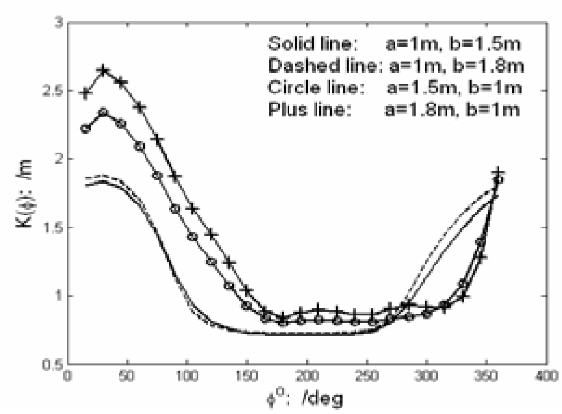

Figure 4. The influence induced by sizes on scattering width.

position is fixed, the change of the scattering width versus the incident angle is shown in Figure 5. This curve shows that the scattering width is bigger in the forward region and is influenced remarkably by the observing point. But this influence is not obvious in the backward area. It is demonstrated from Figure 6 and Figure 7 that the scattering width is the biggest when there is a difference of π between the incident angle and observing angle. Thus it is conjectured that for the scattering

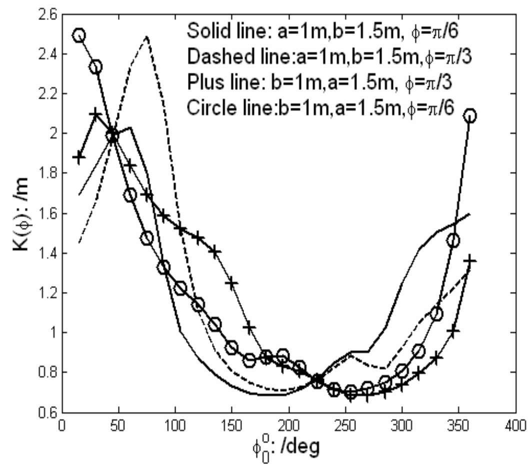

Figure 5. Scattering width versus angle φ_0.

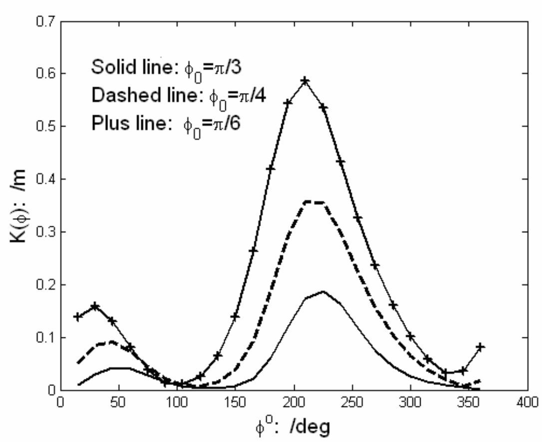

Figure 6. Scattering width versus observing angle.

of the elliptic cylinders there is an enhancement effect in backward direction. From Figure 8 to Figure 10, the incident angle is $\pi/4$, it can be seen that the two axes of a and b have about the same impact on $K(\varphi)$. The vertical size of b has more effect when measured in the direction perpendicular to the incident direction and the horizontal size of a has an obvious impact on the backward scattering. The changes of $K(\varphi)$ versus φ_0 and θ_0 are shown in Figure 11 and Figure 12. We conclude that the polarization of wave has a great influence on

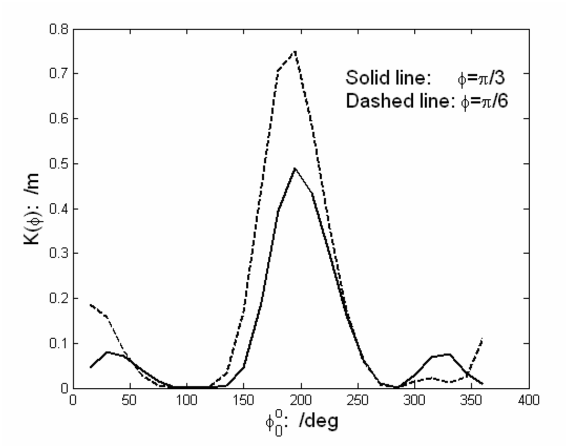

Figure 7. Scattering width versus angle φ_0.

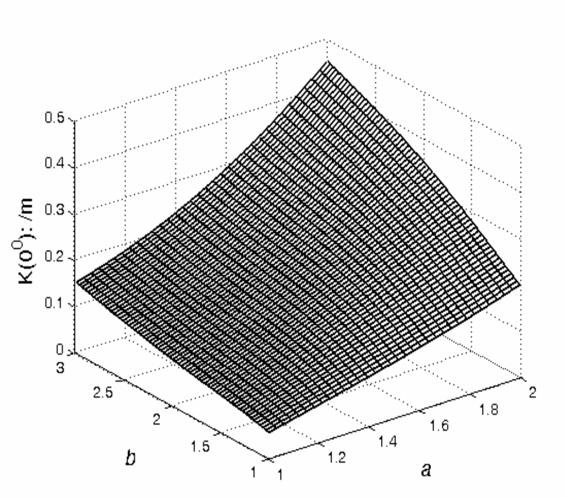

Figure 8. The influence induced by sizes on scattering width in forward direction.

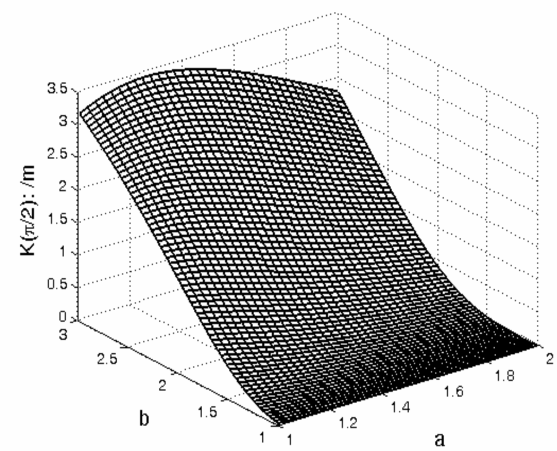

Figure 9. The influence induced by sizes on scattering width in vertical direction.

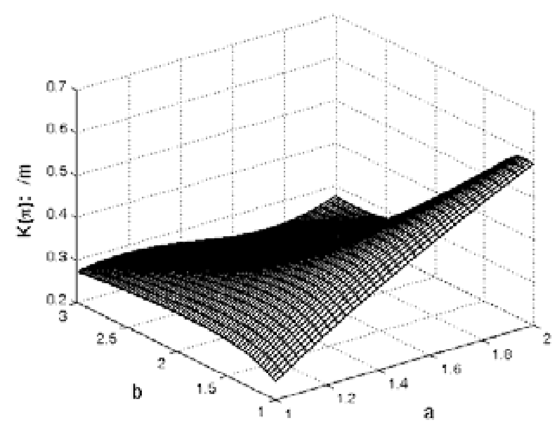

Figure 10. The influence induced by sizes on scattering width in backward direction.

$K(\varphi)$ and $K(\varphi)$ and will arrive its maximum as the polarization is horizontal. When the incident angle is of $\pi/2$, $K(\varphi)$ has arrived its biggest. Since the irradiated size is the biggest and the relative second radiating sources on the target are maximum.

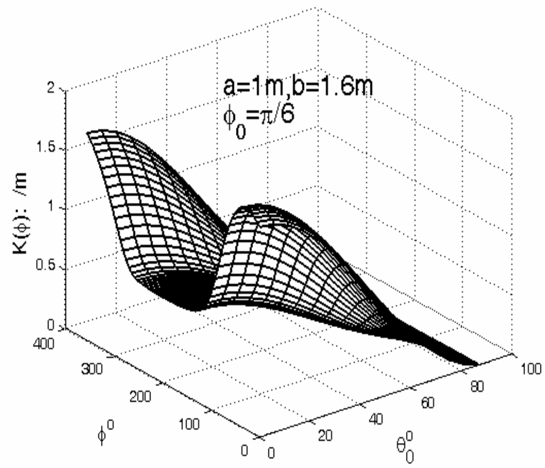

Figure 11. The influence induced by polarized angle on scattering width.

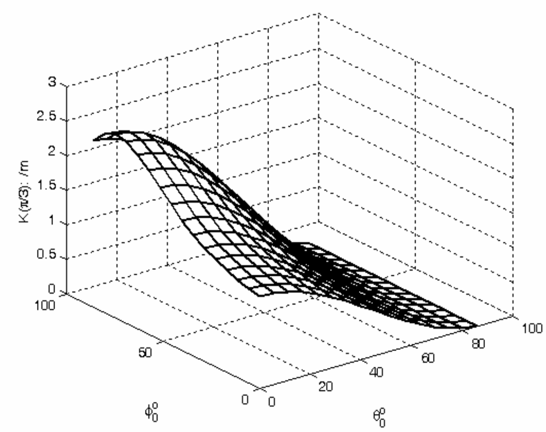

Figure 12. The influence induced by angles of φ_0^o and θ_0^o on scattering width.

3. CONCLUSION

Based on the scales transformation of electromagnetic theory and a conductor elliptic cylinder is taken as an example, the analytical expression of scattering field from an elliptic cylinder is presented when it lies in electromagnetic wave with the arbitrary polarization and random propagating direction. The obtained results are in agreement with that in the references when we use it to calculate the scattering field from cylinder. The scattering width of an elliptic cylinder is obtained and simulations of relation among $K(\varphi)$, a, b, φ_0, θ_0 and φ are presented. The used method can not only be used to research the scattering field from a conductor elliptic cylinder, but also be valid to investigate the scattering field from other elliptic cylinder such as medium elliptic cylinder, the coated elliptic cylinders etc. Since the analytical relation among the scattering field with the sizes of a and b, φ_0 etc. is obtained exactly, the obtained results can provide the theoretical foundation for the Radar target identification and the electromagnetic remote sensing of target's size and shape in space. How to use the obtained result to determine the sizes and the attitude of an elliptic target will be the target of our next work.

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