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HIGH RESOLUTION DOA ESTIMATION IN FULLY COHERENT ENVIRONMENTS

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Abstract—A novel method for direction-of-arrival (DOA) estimation is proposed. This technique employs the excellent performance of Bartlett method in coherent environments as well as high resolution and low computational complexity of Beamspace MUSIC. Simulation results show that the use of Beamspace MUSIC with Bartlett yields significantly improved performance compared to the original MUSIC especially in highly correlated situations.

1. INTRODUCTION

Array signal processing has been a very active research area for several decades. One of its most important fields is Direction-Of-Arrival (DOA) estimation which is very applicable in wireless and mobile communication (e.g., smart antenna [1–5]).

In many applications, we need high resolution estimation of DOAs. For this purpose, many high resolution subspace-based algorithms such as MUSIC [6] and ESPRIT [7] have been proposed in the past. However, they have poor performance in fully or partially correlated situation which is very common in communication system (e.g., in multipath phenomenon [8]). Many methods have been suggested to improve the performance of algorithms in fully correlated situation. But they suffer from some difficulties such as needing extra elements [9] or limitation on the number of correlated sources [10].

In this paper, we propose an effective method for DOA estimation for coherent sources which is also computationally efficient. While the algorithm has no limitation on the number of coherent sources it does not need extra elements either. Hence it is expected that this algorithm performs better in multi-path situations when compared to MUSIC and ESPRIT. This method takes advantage of Bartlett and

Beamspace MUSIC methods and offers a low computational algorithm for locating the coherent sources.

In this paper at first Bartlett, MUSIC, and Beamspace MUSIC DOA estimation methods are briefly introduced, followed by our proposed algorithm. Finally simulation results are presented to compare this method with MUSIC algorithm.

2. BACKGROUND

2.1. Data Model

Narrowband processing is considered herein. Assume that there are D narrowband signals arriving to the array in directions $\theta \in \{\theta_1, \theta_2, \theta_3, \ldots, \theta_D\}$ and M sensors in the array have identical isotropic responses. In order to simplify the problem under consideration, a uniform linear array (ULA) is assumed with inter-element spacing of half wavelength of the narrowband signal frequency (There are many studies about DOA estimation with uniform circular arrays (UCA)[11–15]).

Data received at sensors can be described by a $M \times 1$ vector as:

$$\bar{x}(t) = A(\Theta)\bar{s}(t) + \bar{n}(t) \tag{1}$$

$$\bar{x}(t) = [x_1(t) \quad x_2(t) \quad \ldots \quad x_M(t)]^T \tag{2}$$

where $A(\Theta) = [\bar{a}(\theta_1) \quad \bar{a}(\theta_2) \quad \ldots \quad \bar{a}(\theta_D)]$ is the array manifold containing information on the steering vectors associated with the DOAs of the incident signals (Steering vector is a complex vector containing responses of all sensors of the antenna to a narrowband source of unit power). $\bar{s}(t)$ is the $D \times 1$ source signal vector and $\bar{n}(t)$ is the $M \times 1$ additive noise vector at sensors.

If we assume that noise at each sensor is uncorrelated and independent from both incident signals and the noise at other sensors, then the array covariance matrix can be written as:

$$\begin{aligned} R_{xx} &= E\left[x(t) \cdot x^H(t)\right] \\ &= A(\Theta)E\left[\bar{s}(t) \cdot \bar{s}^H(t)\right] A^H(\Theta) + E\left[\bar{n}(t) \cdot \bar{n}^H(t)\right] = ASA^H + \sigma^2 I \end{aligned} \tag{3}$$

where $S = E[\bar{s}(t) \cdot \bar{s}^H(t)]$ is the source covariance matrix and σ^2 is the noise power at array sensors.

From matrix theory, the array covariance matrix can be described by its eigenvalues and eigenvectors as:

$$R_{xx} = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H \tag{4}$$

where $E_s = [\bar{e}_1 \quad \bar{e}_2 \quad \dots \quad \bar{e}_D]$ and $E_n = [\bar{e}_{D+1} \quad \bar{e}_{D+2} \quad \dots \quad \bar{e}_M]$ are the so-called signal and noise subspaces respectively, and $\Lambda = \mathrm{diag}(\lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_M)$ is a diagonal matrix containing the eigenvalues. The D biggest eigenvalues form the signal eigenvalues and others form the noise eigenvalues.

2.2. Conventional Beamforming Method (Bartlett)

One of the earliest methods of spectral analysis of DOAs is the Bartlett method. The idea is to guide the antenna beam in one direction (by using weighting vector ($\bar{w}$) which acts like a spatial filter) and measure the output power. The directions which result in maximum power yield the DOA estimates. The average output power of N snapshots is given by:

$$P(\bar{w}) = \frac{1}{N}\sum_{t=1}^{N}|y(t)|^2 = \frac{1}{N}\sum_{t=1}^{N}\bar{w}^H\bar{x}(t)\bar{x}^H(t)\bar{w} = \bar{w}^H\hat{R}_{xx}\bar{w} \quad (5)$$

where $\hat{R}_{xx} = \frac{1}{N}\sum\limits_{t=1}^{N}\bar{x}(t)\bar{x}^H(t)$.

$\hat{R}_{xx}$ is the Maximum Likelihood estimation of $R_{xx} = E[x(t) \cdot x^H(t)]$.

Bartlett method maximizes the power of the beamforming output for a given input signal. In this method, weighting vector ($\bar{w}$) which maximizes the output power in direction θ is:

$$\bar{w}_B = \arg_{\bar{w}}\{\max\{P(\bar{w})\}\} \rightarrow \bar{w}_B = \bar{a}(\theta) \quad (6)$$

The above weighting vector can be interpreted as a spatial filter, which has been matched to the impinging signal. Intuitively, the array weighting equalizes the delays (and possibly attenuations) experienced by the signal on various sensors to maximally combine their respective contributions. By inserting the weighting vector Eq. (6) into Eq. (5), the classical "power spectrum" is obtained [16]:

$$P_B(\theta) = \bar{a}(\theta)^H\hat{R}_{xx}\bar{a}(\theta) \quad (7)$$

The limitation of this method is that the sources, whose electrical angles are closer than the beamwidth of the array, cannot be resolved by this method [17].

2.3. MUSIC Method

MUSIC is a popular high resolution eigenstructure method. The MUSIC algorithm is based on the assumption that the desired signal array response is orthogonal to the noise subspace (En) [17]:

$$E_n^H \cdot \bar{a}(\theta) = 0 \quad \theta \in \{\theta_1, \theta_2, \theta_3, \ldots, \theta_D\} \tag{8}$$

However, in correlated situation, the above equation is not valid anymore. Hence, MUSIC-based algorithms do not show good performance for correlated sources. In this method, the estimation of covariance matrix ($\hat{R}_{xx}$) is decomposed into the signal eigenvectors (Es) and noise eigenvectors (En). By using Eq. (8) the MUSIC power spectrum is defined as [17]:

$$P_{MU}(\theta) = \frac{1}{|\bar{a}(\theta)^H E_n E_n^H \bar{a}(\theta)|} \tag{9}$$

2.4. Beamspace MUSIC

As it was mentioned, MUSIC algorithm requires an eigenvalue decomposition, which is a computation that is difficult to implement in parallel and requires $O(M^3)$ multiplications for an $M \times M$ matrix, corresponding to M sensors. Therefore, increasing M leads to a cubic increase in computational complexity [18].

To achieve a significant reduction in computational time, researchers proposed a so-called beamspace approach, which first projects the original data into a subspace of lower dimension (i.e., the beamspace) and then processes the beamspace data by using well-known direction finding algorithms such as MUSIC [19]. Since the beamspace data are received from a pseudoarray of size B, the computational complexity is reduced to $O(B^3)$.

The procedure of beamspace DOA estimation algorithms can be divided into two stages, as shown in Fig. 1. In the beamforming process, the output data at the array sensors from sources in different directions pass through the multi-beamformers and the beam output data are obtained. Beams can be formed in different ways, such as using conventional beamforming techniques or adaptive beamforming methods. Then in the DOA estimation process, based on the beam outputs, source directions are determined by using high resolution algorithms.

The outputs of these B beamformers can be written as

$$\bar{y}(t) = W^H \cdot \bar{x}(t) \quad \bar{y}(t) = [y_1(t) \;\; y_2(t) \;\; \ldots \;\; y_B(t)]^T \tag{10}$$

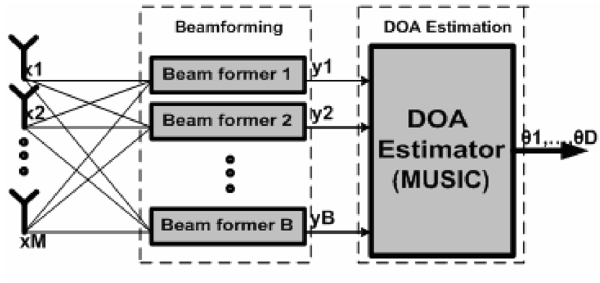

Figure 1. Beamspace MUSIC block diagram.

where W is a $M \times B$ beamforming matrix consisting of beamformer vector, one for each beam. The transform described in Eq. (10) converts the element domain outputs into the beam domain outputs.

Under the narrowband assumption and for the case of multiple incident sources, we have the beamformer outputs as follows:

$$\bar{y}(t) = W^H \cdot A(\Theta)\bar{s}(t) + \bar{n}_y(t) \quad \text{where} \quad \bar{n}_y(t) = W^H \cdot \bar{n}(t) \tag{11}$$

From Eq. (11), we have the steering vectors after beamforming defined as:

$$\bar{v}(\theta) = W^H \cdot \bar{a}(\theta) \tag{12}$$

This vector plays the same role in the beamspace processing as that of vector $\bar{a}(\theta)$ in the element-space processing.

Similar to element space domain, in the beam domain, the beam covariance matrix can be written as

$$R_{yy} = E\left[\bar{y}(t) \cdot \bar{y}^H(t)\right] = W^H R_{xx} W = W^H A S A^H W + \sigma^2 W^H W \tag{13}$$

R_{yy} also can be written in terms of eigenvalues and corresponding eigenvectors as:

$$R_{yy} = W^H A S A^H W + \sigma^2 I = E_{B-s}\Lambda_{B-s}E_{B-s}^H + E_{B-n}\Lambda_{B-n}E_{B-n}^H \tag{14}$$

where E_{B-s} and E_{B-n} represent the beam domain signal and noise eigenvectors, respectively and Λ_B is the beam domain eigenvalues matrix.

The MUSIC power spectrum in beamspace domain can be written as [20]:

$$P_{B-MU}(\theta) = \frac{1}{\left|\left(W^H \cdot \bar{a}(\theta)\right)^H \cdot E_{B-N} \cdot E_{B-N}^H \cdot \left(W^H \cdot \bar{a}(\theta)\right)\right|} \tag{15}$$

2.5. Comparison between MUSIC and Bartlett

As mentioned before, Bartlett cannot resolve sources closer than beamwidth of the array; however, MUSIC has the ability to resolve very close sources. Figs. 2 and 3 show the power spectrum of Bartlett and MUSIC for $\Delta\theta = 20°$ and $\Delta\theta = 10°$, where $\Delta\theta$ represents the angular difference between two incident sources. Fig. 3 shows that Bartlett cannot resolve two close sources while MUSIC can.

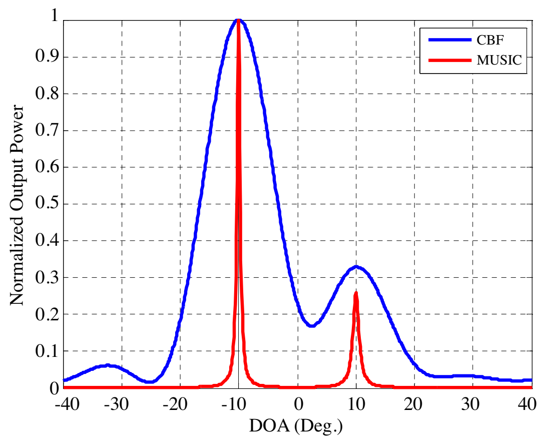

Figure 2. Normalized spectrum of the Bartlett, and the MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 0$, $\theta_1 = 10°$, $\theta_2 = -10°$.

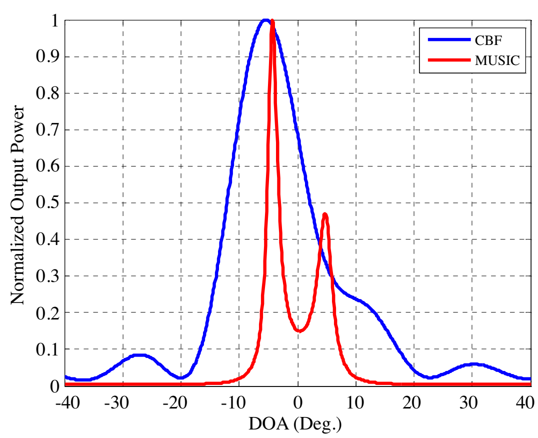

Figure 3. Normalized spectrum of the Bartlett, and the MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 0$, $\theta_1 = 5°$, $\theta_2 = -5°$.

Despite Bartlett low resolution, this algorithm is not sensitive to coherent sources in contrast to MUSIC high sensitivity. Figs. 4 and 5 show the power spectrum of Bartlett and MUSIC for two different correlation coefficients ($\rho = 0$, $\rho = 1$), and Fig. 6 compares standard deviation of Bartlett and MUSIC algorithm with respect to different correlation coefficients. The results show the robustness of Bartlett to coherent sources, while resolution of MUSIC decreases with the increase of correlation coefficient.

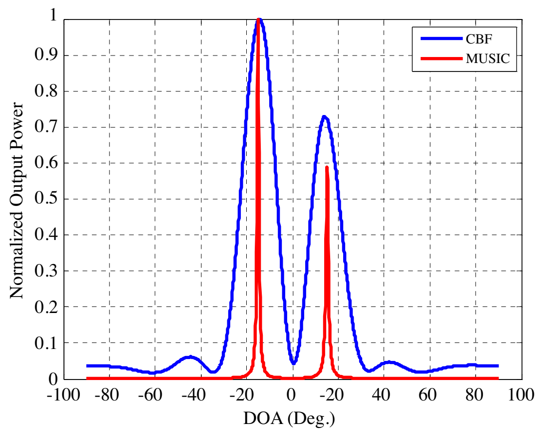

Figure 4. Normalized spectrum of the Bartlett, and the MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 0$, $\theta_1 = 15°$, $\theta_2 = -15°$.

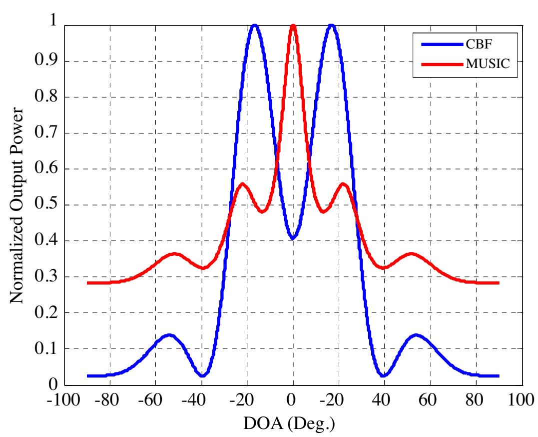

Figure 5. Normalized spectrum of the Bartlett, and the MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 1$, $\theta_1 = 15°$, $\theta_2 = -15°$.

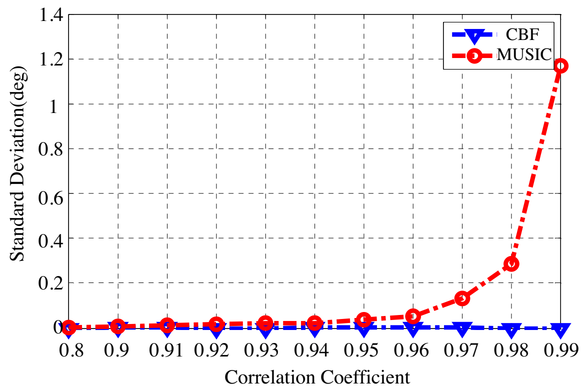

Figure 6. Standard deviation of Bartlett, and MUSIC versus correlation coefficient — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\theta_1 = 10°$, $\theta_2 = -10°$.

3. BARTLETT-MUSIC METHOD

In the previous section, we see that we can reduce the computational burden by working in beamspace domain instead of element space domain. If we know the approximate location of targets, we can create beams just near expected region and increase the resolution. The placements of beams are chosen in order to cover the region which the target is expected to be. The initial approximation of target location can be obtained by Bartlett algorithm. The advantages of using Bartlett are low computational complexity and robustness to coherent signals. Fig. 7 shows the block diagram of this algorithm. The diagram shows that Bartlett is used as initial DOA estimator for beamspace MUSIC method. Beamformer section forms a small number of beams in the adjacent region of the Bartlett spectrum peaks. Then the outputs of beamformer are applied to MUSIC algorithm and finally DOAs are determined with high resolution.

This algorithm reduces the computational complexity in two ways: 1) by reducing the dimensions of search from M to B by using beamspace method, 2) by searching only near the peaks of Bartlett spectrum, instead of sweeping the whole space.

In addition, by use of Bartlett algorithm as an initial target location estimator, we can resolve fully coherent sources which cannot be differentiated by other algorithms such as MUSIC and ESPRIT.

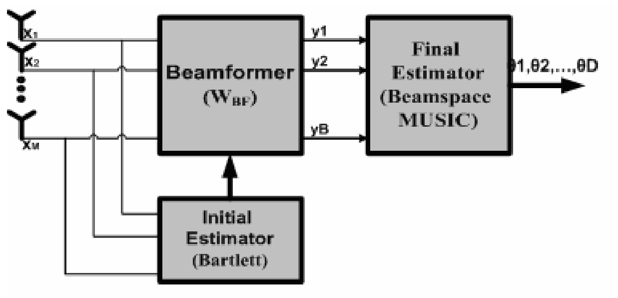

Figure 7. Bartlett-MUSIC block diagram.

4. SIMULATION RESULTS

In this section, results of our simulation are presented, which help the better understanding of the performance of proposed algorithm.

Simulations were run with MATLAB. The data received on the arrays are obtained by FEKO software. Eight monopole antennas ($M = 8$) with an interelement spacing of a half wavelength are placed in a ULA structure (Fig. 8). The method of open circuit mutual coupling cancellation is used to mitigate the effect of mutual coupling [21]. Two fully correlated sources with SNR $= 10\,\text{dB}$ are placed far from the array elements.

In the first case (Fig. 9 and Fig. 10) the two sources are located in $\theta_1 = +2^\circ$ and $\theta_2 = -2^\circ$. Neither Bartlett nor MUSIC can resolve these

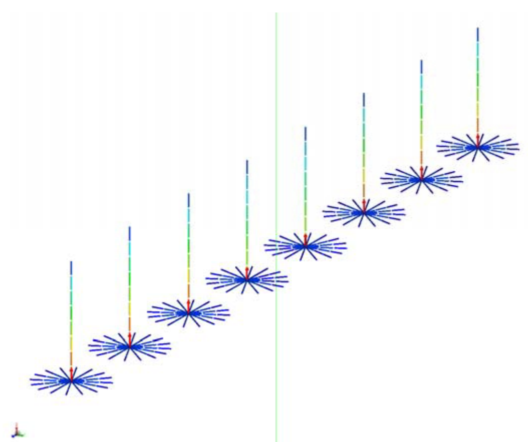

Figure 8. Eight monopole antenna in a ULA structure with an interelement spacing of a half wavelength (FEKO software).

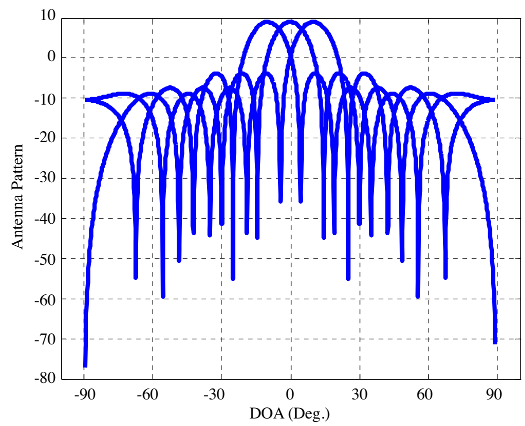

Figure 9. Antenna pattern of 3 beams in $\theta_1 = 5°$, $\theta_2 = -5°$, $\theta_3 = 0° - M = 8$.

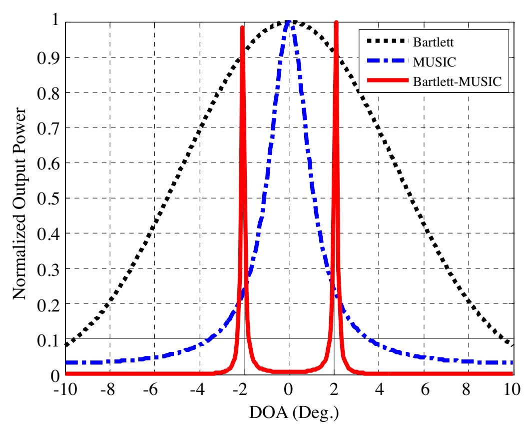

Figure 10. Normalized spectrum of the Bartlett, the MUSIC and the Bartlett-MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 1$, $\theta_1 = 2°$, $\theta_2 = -2°$.

sources (Fig. 10). But by generating of three beams in the vicinity of Bartlett peak (0°, ±5°) (Fig. 9) and applying MUSIC for the output of beams, we can estimate the location of sources (Fig. 10, Bartlett-MUSIC algorithm).

It should be noted that in this case we only need to search near Bartlett peak (−10° to +10°) and we do not have to sweep the entire space.

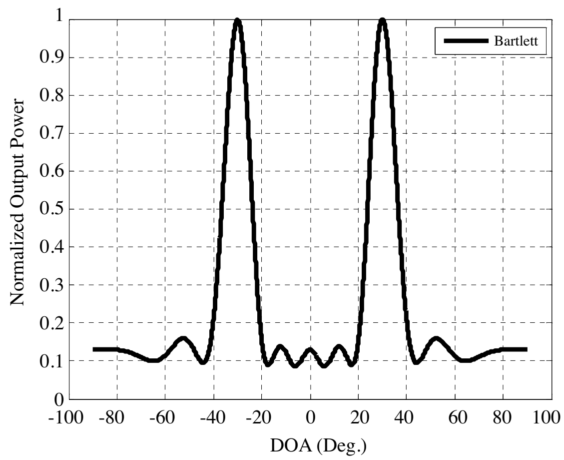

Figure 11. Normalized spectrum of the Bartlett — $M = 8$, $D = 2$, SNR $= 10\,\mathrm{dB}$, $\rho = 1$, $\theta_1 = 30°$, $\theta_2 = -30°$.

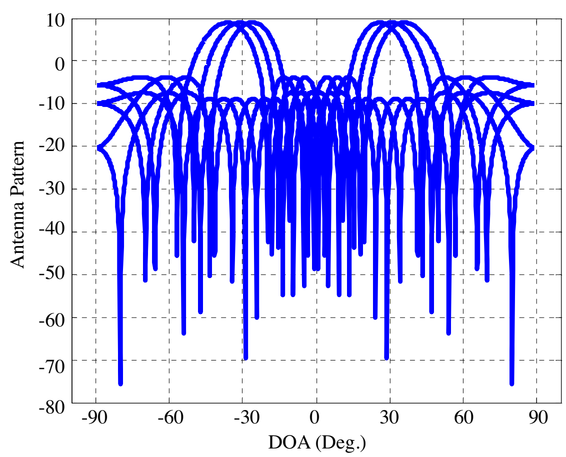

Figure 12. Antenna pattern of 2 set of 3 beams near $\theta_1 = 30°$, $\theta_2 = -30° - M = 8$.

In the second case (Figs. 11, 12, and 13), the two fully correlated sources are located in $\theta_1 = +30°$ and $\theta_2 = -30°$. Fig. 11 shows the initial estimation by Bartlett algorithm. By generating two set of beams near each peak (Fig. 12), DOA of each source can be obtained precisely (Fig. 13).

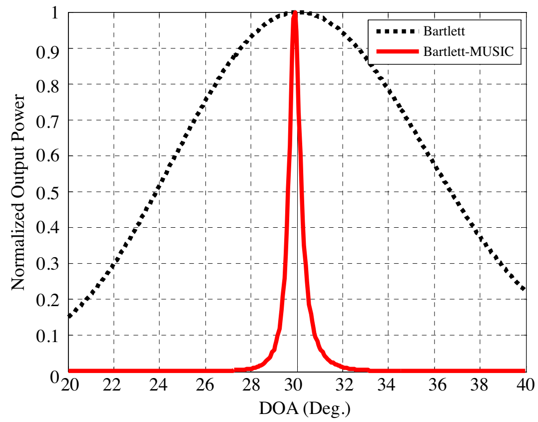

Figure 13. Normalized spectrum of the Bartlett and the Bartlett-MUSIC — $M = 8$, $D = 2$, SNR $= 10\,\text{dB}$, $\rho = 1$, $\theta_1 = 30°$, $\theta_2 = -30°$.

5. CONCLUSION

In this paper, we have presented a novel DOA estimator, which employs the benefits of Bartlett and MUSIC algorithms. Simulation results show that Bartlett-MUSIC algorithm is not affected by the correlations in the signals. Moreover, the algorithm has less computational complexity when it is compared to MUSIC algorithm.

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